Date : / /

7b) 3D point : $X = (x, y, z, 1)^T$

let two unknown cameras be : $P = R_{3\times4}$, $P' = R_{3\times4}$

let image points be :

$$x = (u, v, 1)^T \sim PX, \quad x' = (u', v', 1)^T \sim P'X$$

The epipolar constraint for the corresponding pair (x, x'):

$$x'^T F x = 0$$

$$\tilde{x} = Hx = (\tilde{u}, \tilde{v}, 1)^T, \quad \tilde{x}' = H'x' = (\tilde{u}', \tilde{v}', 1)$$

$\tilde{v} = \tilde{v}'$ (sam row)

$$d = \tilde{u} - \tilde{u}'$$

Now, we know focal length : 'f'

Base line : 'B'

we also know that $d = \frac{fB}{z} \Rightarrow z = \frac{fB}{d}$

If uncalibrated,

$$z = \frac{\alpha}{d}, \quad \alpha = fB \text{ (unknown Constant)}$$

$$\hat{z} = \frac{1}{d} \Rightarrow \boxed{z = \alpha \hat{z}}$$

$P_0 = [I \mid 0]$, $P_0' = [I \mid b]$ $\Rightarrow$ first camera in rectified pair

where,

$$b = (B, 0, 0)^T.$$

$$\hat{x} = \frac{\tilde{u}}{d}, \quad \hat{y} = \frac{\bar{v}}{d}, \quad \hat{z} = \frac{1}{d} \quad (d = \tilde{u} - \tilde{u}') \quad \left[\text{from pinhole model}\right]$$

$$\hat{p} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \frac{1}{d} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ 1 \end{pmatrix}$$

True distance between A & B is L_{AB}^{true}

$$d_A = \tilde{u}_A - \tilde{u}'_A, \quad d_B = \tilde{u}_B - \tilde{u}_B'$$

$$\hat{p}_A = \frac{1}{d_A} \begin{pmatrix} \tilde{u}_A \\ \tilde{v}_A \\ 1 \end{pmatrix}, \quad \hat{p}_B = \frac{1}{d_B} \begin{pmatrix} \tilde{u}_B \\ \tilde{v}_B \\ 1 \end{pmatrix}$$

$$\hat{L}_{AB} = \left| \hat{p}_A - \hat{p}_B \right|$$

Scale factor 's' $= \dfrac{L_{AB}^{true}}{\hat{L}_{AB}}$

& Once we compute $\hat{p}_i$, $\hat{p}_j$, we can get the projective distance $\Rightarrow \hat{L}_{ij} = |\hat{p}_i - \hat{p}_j|$

$$L_{ij} = s\,\hat{L}_{ij} = \frac{L_{AB}^{true}}{\hat{L}_{AB}}\, \hat{L}_{ij} \Rightarrow L_{ij} = s\, \| \hat{p}_i - \hat{p}_j \|$$