JPMC Quant Finance Mentorship Program 2023

General Instructions:

This document contains two case studies each with equal score (60 marks each). You are expected to attempt both case studies and submit the solution by the deadline **11:59 PM 29th January 2023**.

1. Each of these case studies introduce the basics of a problem and pose a few questions on them. A question might have multiple sub-parts, all of which need to be answered. The scores for the sub-parts have been indicated against the question. The solution format for each question has been specified alongside the question and the final submission format is described at the next page of this document.

2. You are free to use online resources to improve your understanding of the problem statement.

3. Hand-written answers are also accepted but need to be collated in a single pdf while submitting. Loose snapshots will not be accepted.

4. You are supposed to work individually on this case study. Collusion/collaboration of any kind will not be tolerated.

5. Only 1 submission per candidate is allowed.

Plagiarism of any kind will not be tolerated.

Final Submission Format:

Create a well formatted word/pdf document describing your solution to each question, clearly enumerated and in the same order as the questions. Include your personal details such as name, institute name, branch and year in the first page.

For each question, if the expected solution is a written answer/ plot / table or pseudo code, include it in line in this document. Name this document in the format

{FirstName}_{LastName}_{CollegeName(short)}_JPMQuantMentorshipCaseStudy

If the expected solution is a program, state the file name of the program.

Instructions for program submissions:

Any programming language you are comfortable with is permitted. You are permitted to use standard libraries in your chosen language.

a. The program submitted must compulsorily have a main function that must call all the test cases described in the question when run.

b. The program must be commented and intelligible.

c. The program's file name must be of the format:

{CANDIDATE_NAME}_{QUESTION_NO_SUB_PART}_MAIN.

d. If there are additional modules that are imported in the script with the main function, name those files as {CANDIDATE_NAME}_Module_{ SCRIPT_NAME}.

e. You may include a short description of your program in the solution document if you wish.

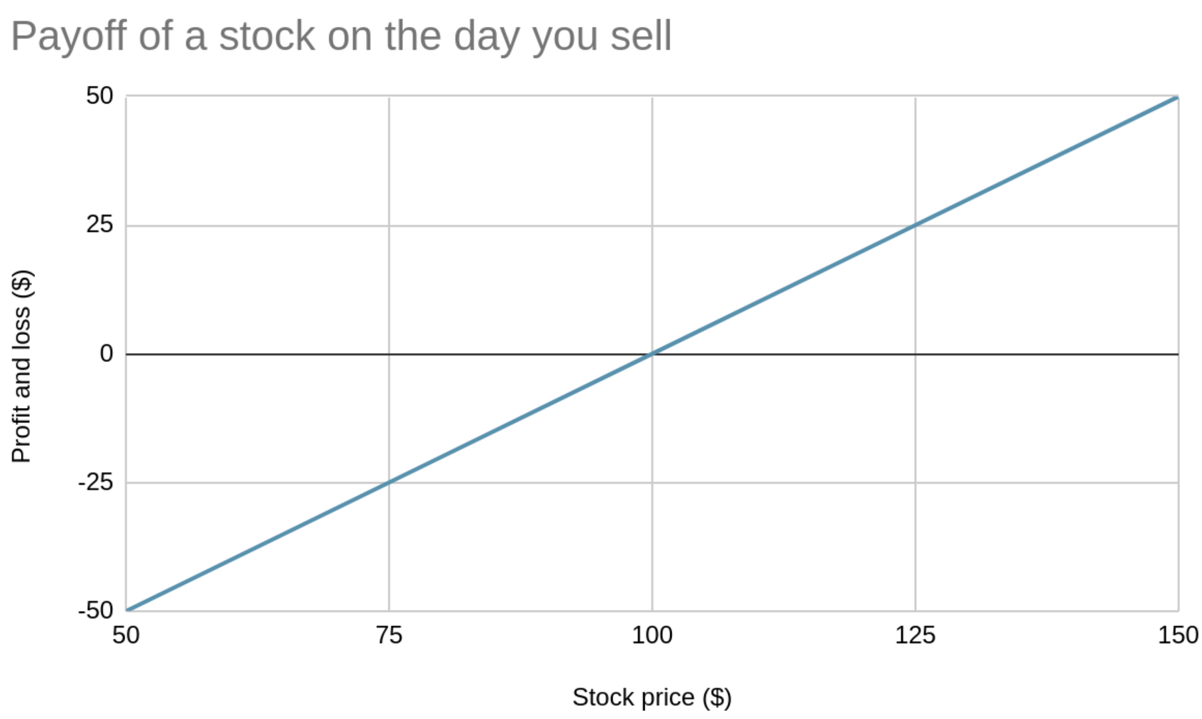
f. Compress the PDF and all code files in a zip and name it 'JPMC_Quant_Mentorship_CaseStudy_<*YourFirstName*>_<*YourLastName*>' and mail it to jpmqrmentorship.mumbai@jpmorgan.com with the subject '**JPMC_Quant_Mentorship_CaseStudy_Submission**_<*YourFirstName*>_<*YourLastName*>'

Derivative hedging

Theory:

Stocks:

A stock (or a share) is a small ownership of a company that can be freely traded (allows you to buy and sell from other participants like you). Like any other asset, the price of the stock can fluctuate based on the forces of supply and demand. Higher the demand, greater the price; and higher the supply, lower the price. If you buy a stock at, for example, $100 today, and after 3 months, its value rises to $120, you would have made a profit of $20. Likewise, if it went down to $80, you would have made a loss of $20.



Are stocks the only way you can participate in the market? Read further sections to find out!

Derivatives:

In the previous example, you have already purchased the stock, and plan to book your profit or loss after 3 months. Obviously, you have purchased it in the hope that the price rises so that you can sell at a higher price than what you bought for, but nobody can guarantee on what happens 3-months from now, right?!

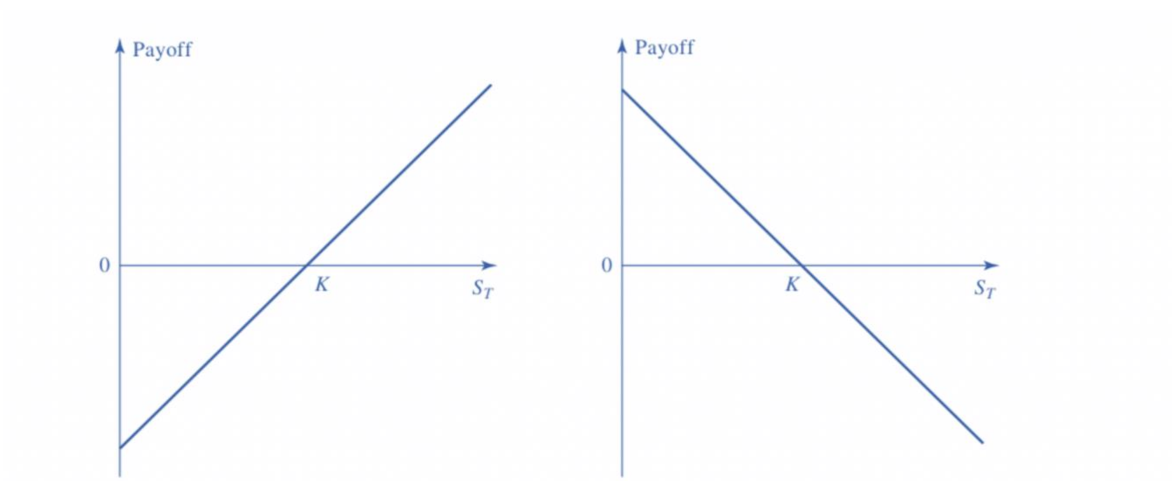
Theoretically speaking, the stock price can go to 0 at the worst case and you will lose all your money. Now what if there was some way which you could use to floor your losses? This is where derivatives come in. '*Options, Futures and Other Derivatives',* an extremely popular book on derivatives by John C. Hull, defines a derivative to be a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. Very often the variables underlying derivatives are the prices of traded assets. A stock option, for example, is a derivative whose value is dependent on the price of a stock. However, derivatives can be dependent on almost any variable, from the price of hogs to the amount of snow falling at a certain ski resort! Let's look at some famous derivatives -

Futures:

A future contract is an agreement between two parties to buy or sell an asset at a certain time in the future at a certain price. Once you enter a future contract, you are obligated to buy (i.e. take a long position) or sell (i.e. take a short position) at a pre-specified price (called futures price) on a certain date in the future (3 months in our example), irrespective of the prevailing market price (also called the spot price) on the expiry date.

Let's say you entered a long future contract to sell the stock at $100, 3 months from now even if the spot price is greater than $100. If the stock price ends up >= $100 or <$100, you still buy the contract at $100 and realize a profit of the difference in the price.

In general, the payoff from a long position in a future contract on one unit of a stock is S_T – K where K is the delivery price and S is the spot price of the stick at maturity of the contract (left). Similarly, the payoff from a short position in a future contract on one unit of stock is K – S_T (right). Both these pay offs can be positive or negative as can also be seen in the diagrams below:



Call Option:

Going back to our earlier example, your goal is to place a bet that the price of the stock goes up after 3 months, and at the same time, you want to floor your losses. A call option is a contract that gives you the right, but not obligation, to buy the stock at a pre-specified price (called strike price), on a certain date in the future (3 months, in our example), irrespective of the prevailing market price on the expiry date. You can *buy* a call option by paying a small premium to the seller of the option.

Imagine you bought a call option after paying a small premium. As the holder of the call option, you now have the right to buy the stock at $100, 3 months from now, even if the market price is greater than $100. If the stock price ends up <= $100, you can let your option expire without any action, and the maximum amount you can lose is the initial premium that you paid to buy the option (so the loss is floored)!

In the above example, you bought a call option in the hope that the prices of the underlying stock will go up. What if you wanted to place a bet on the stock price going down? In that case, you can *sell* a call option! You will get paid a small premium by the buyer of the call option, and if your estimate is correct and the stock price does go down, the buyer won't exercise the option and you will get to keep the initial premium paid to you!

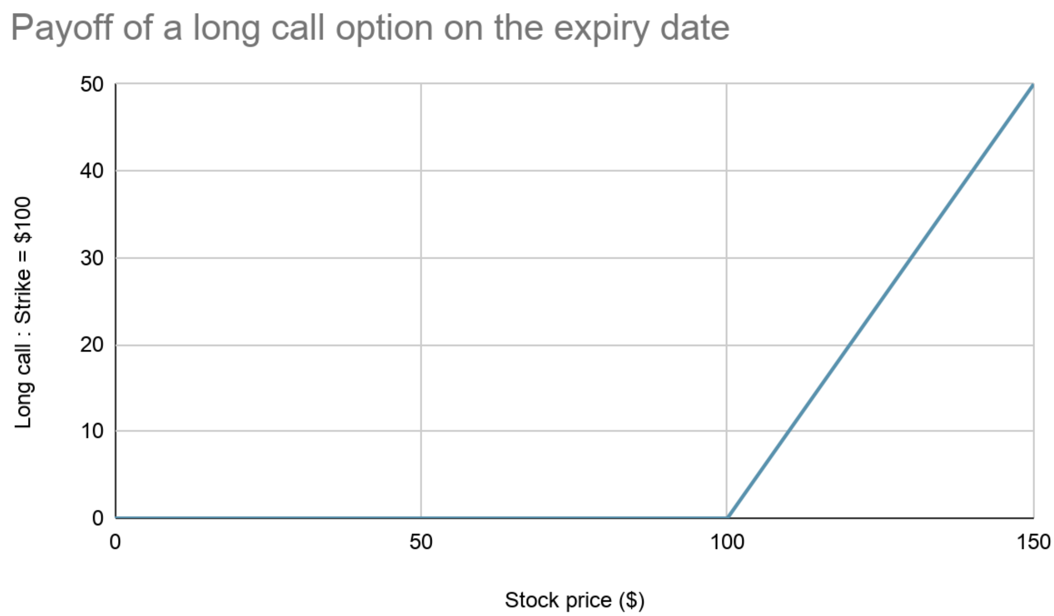
One fascinating point to note here is that you don't actually need to have any stock with you while selling a call option. When you sell a call option, your only obligation is to give the buyer the underlying stock at expiry at the predetermined strike price, that too only if the buyer chooses to exercise the call option. And so you have the liberty to not have the underlying stock with you until the expiry date arrives!

Since you don't need to have the stock with you when you are buying or selling the option, your initial investment is very minimal unlike the initial case where you were buying the stock at the current market price today and hoping that it would increase in price 3 months later.

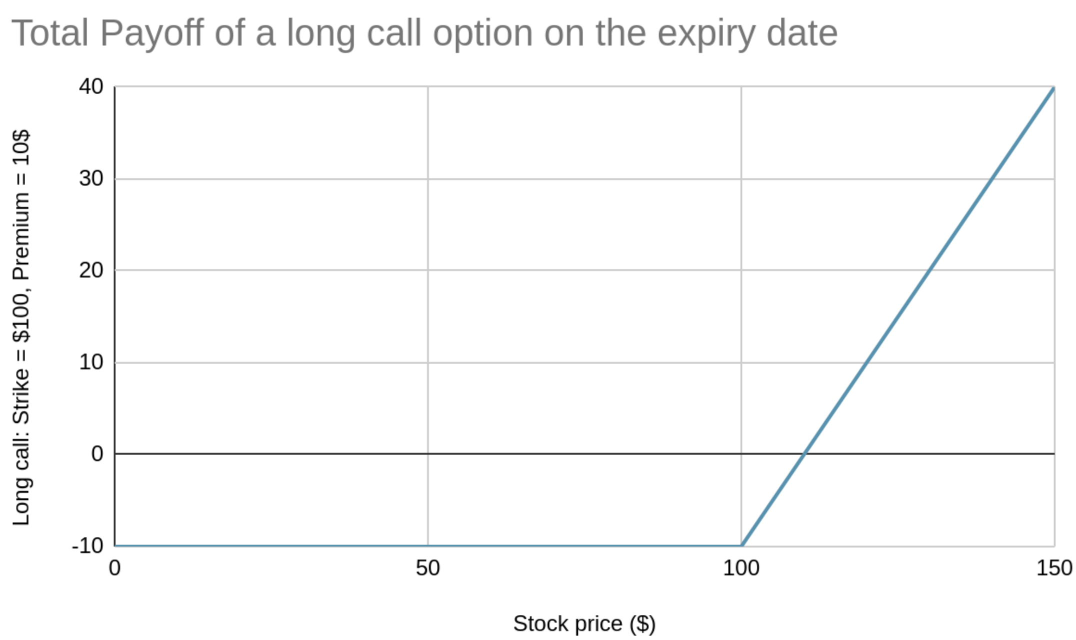
Note: 'Buying' of an asset is termed as a 'long position' and 'Selling' of an asset is termed as a 'short position'.

See the below diagrams to see how the payoff looks like on the date of expiry for the call option:

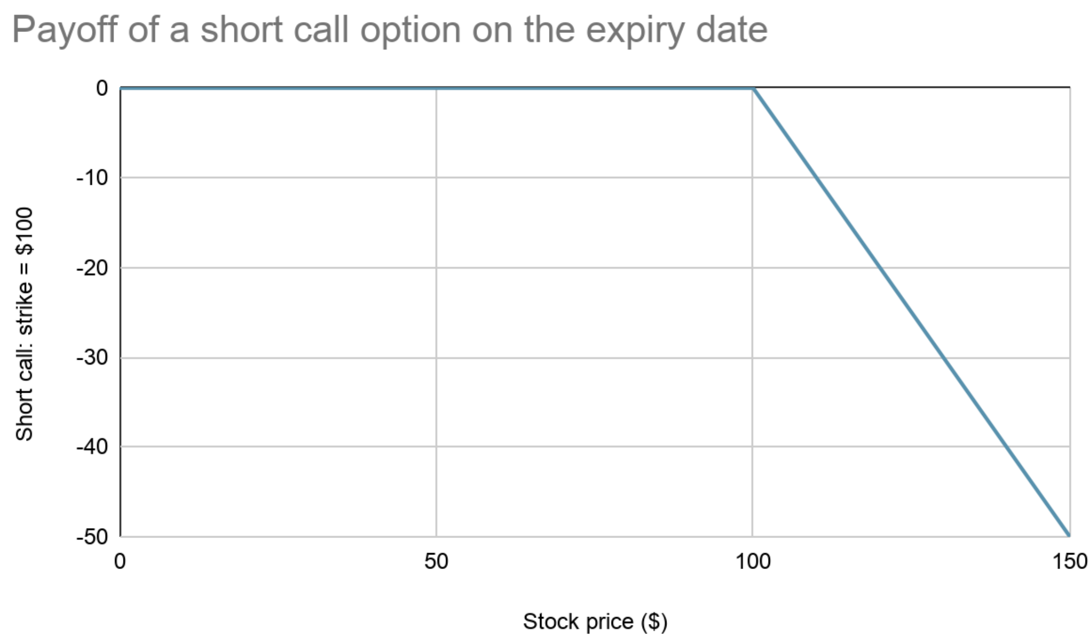
1. A long call with Strike = \$100 has the payoff as C = max (stock price on expiry date - strike, 0).



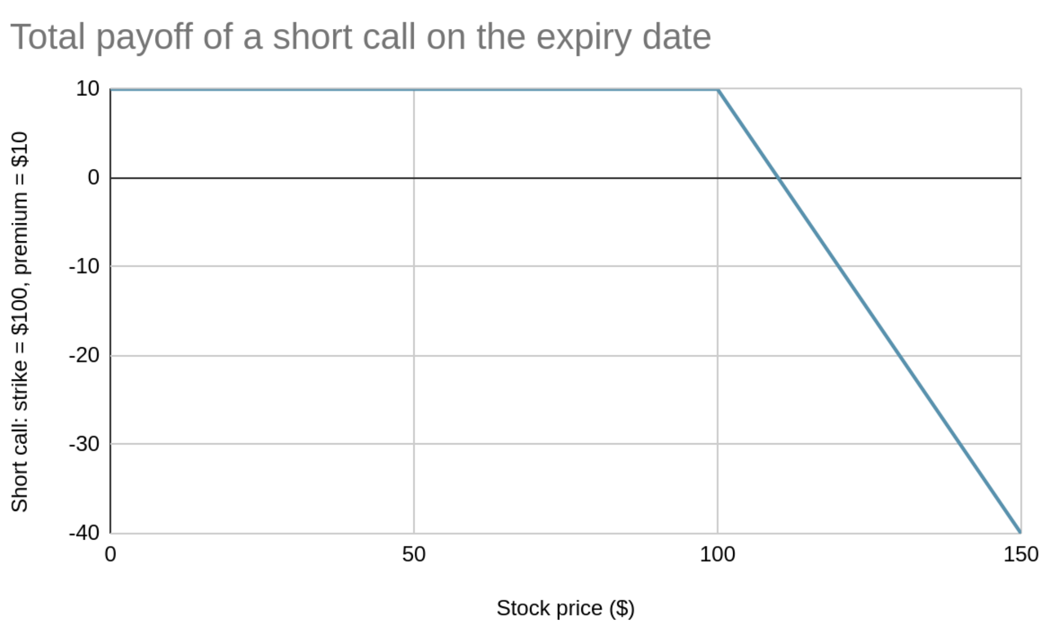
2. As you paid an initial premium (assume \$10) to buy the call option, the above payoff graph actually gets shifted down by the premium amount.



3. As you guessed, short call (i.e., selling a call option) payoff with Strike = $100 has the payoff of long call reflected on the x-axis.



4. As this time, you received the initial premium (assume $10) from the buyer of the option, the payoff graph will get shifted up by the premium amount.



Put option:

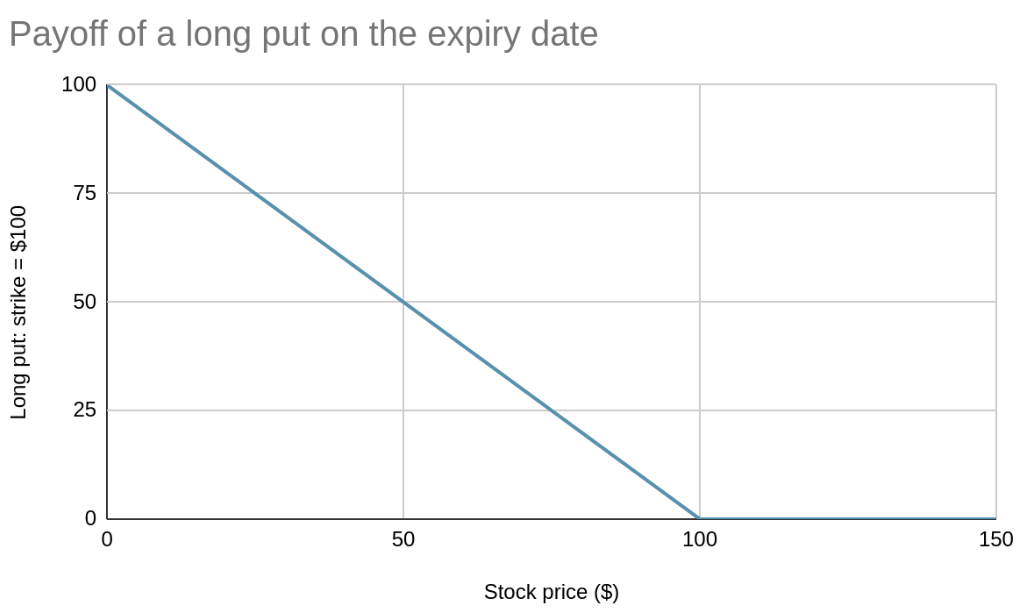
Just as the call option gave the buyer of the option the right to buy the stock at a predetermined price (strike) on the expiry date, a put option will give the buyer of the option the right to sell the stock at a predetermined price (strike) on the expiry date, irrespective of the prevailing market price on the expiry date. You can buy a put option by paying a small premium to the seller of the option.

Imagine you bought a put option after paying a small premium. As the holder of the put option, you now have the right to sell the stock at $100, 3 months from now, even if the market price is < $100. If the stock price ends up >= $100, you can let your option expire without any action, and the maximum amount you can lose is the initial premium that you paid to buy the option.

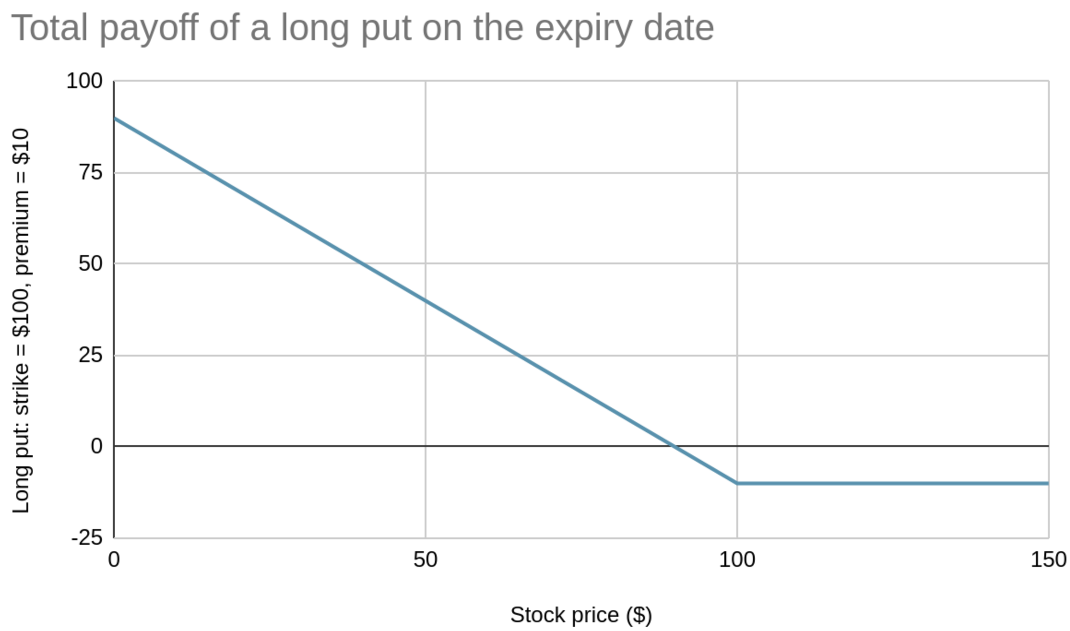
Q. What is the difference between buying a put option and selling a call option??
A.
(a) Different names, duh.
(b) Buying a put option will need you to pay a premium, while selling a call option will earn you a premium.
(c) A call option gets exercised when the stock price at expiry is greater than the strike price. As a seller of a call option, the most you can possibly get from the deal is the initial premium you were initially paid. A put option gets exercised when the stock price at expiry is smaller than the strike price. As a buyer of a put option, your upside gets maxed out at [strike – premium] (if the stock price goes down to $0 at expiry, you can buy the stock for free from the market on the expiration date and your put option will give you the ability to sell the stock at the pre-agreed strike price to the seller of that put option).

See the below diagrams to see how the payoff looks like on the date of expiry for the put option:

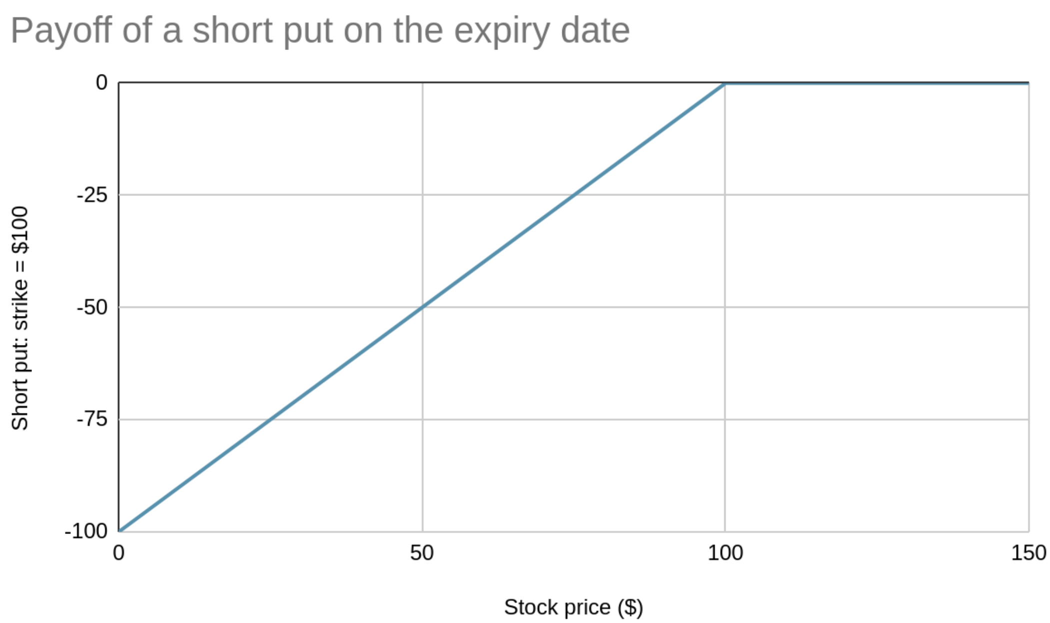
1. Buying a put option with Strike = $100 has the payoff as P = max (strike - stock price on the expiry date, 0).



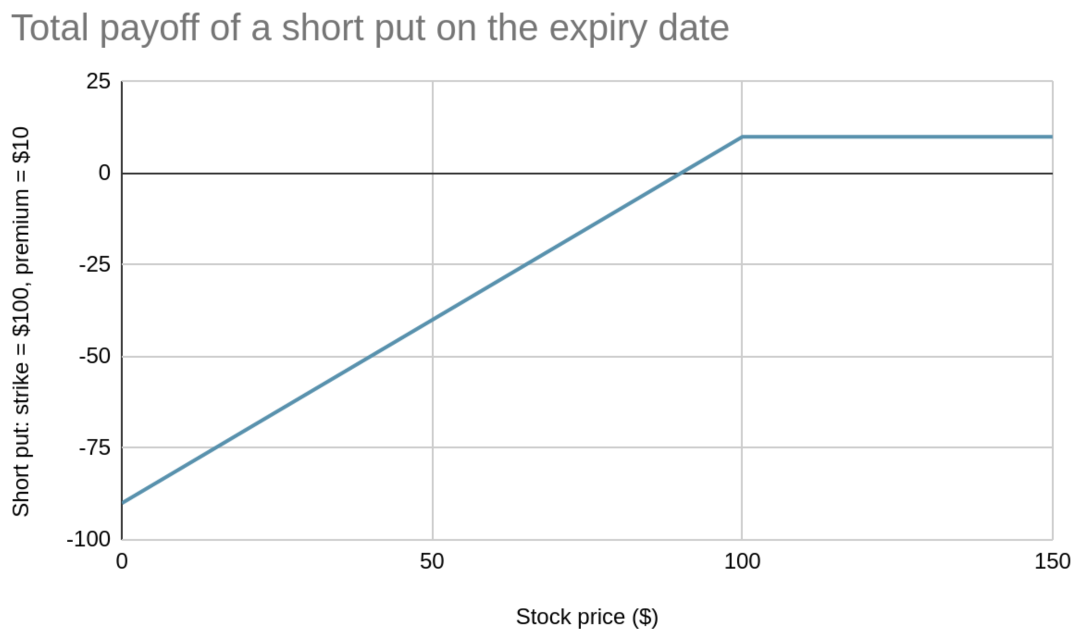
2. As you paid an initial premium (assume $10) to buy the put option, the payoff graph gets shifted down by the premium amount.



3. Selling a put payoff with Strike = $100 has the payoff of buying a put reflected on the x-axis.



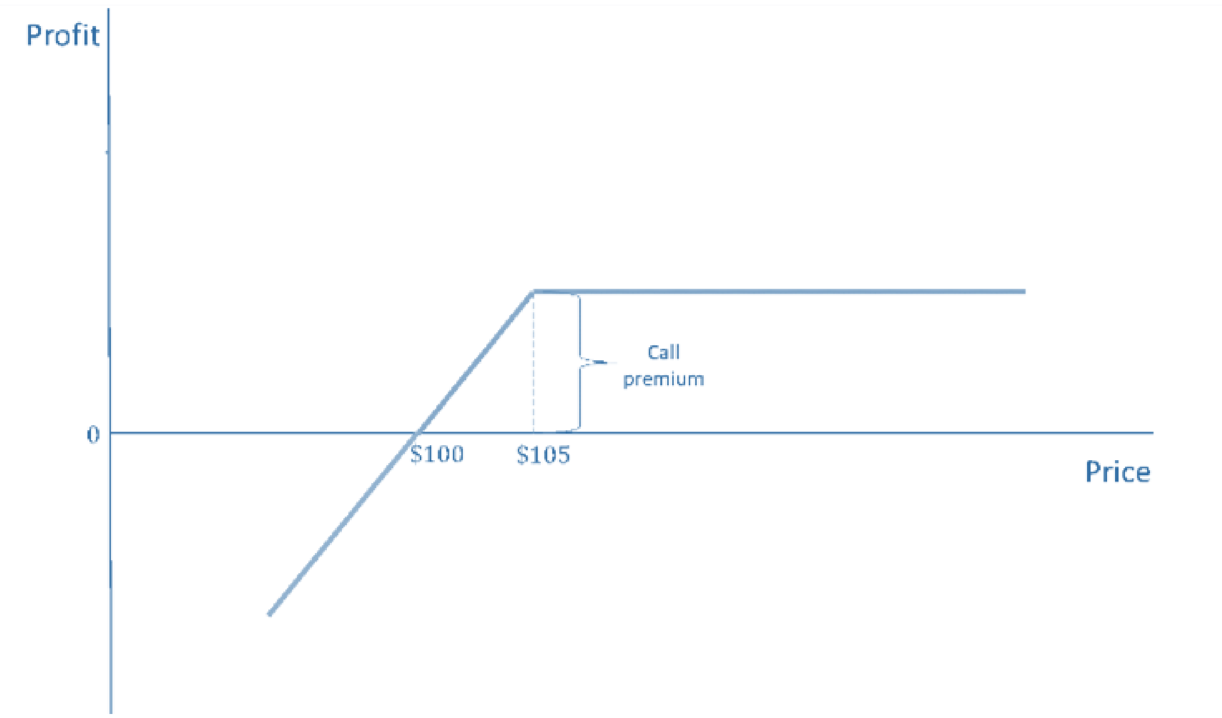
4. As you received the initial premium (assume $10) from the buyer of the put option, the payoff graph gets shifted up by the premium amount.



Now let us consider a more realistic scenario where a trader holds more than one instrument in his portfolio. The trader holds a combination of different instruments to hedge against the market risk for different scenarios. Consider the below given combination to understand how the payoffs of the following combinations look like and then we will try constructing the payoffs of few combinations on our own.

Covered call is a strategy that involves the investor selling call options will hold an equivalent amount of the underlying security. The investor basically holding the long position in an asset sells call options on same asset to generate income. This strategy is often employed by those who wish to hold the underlying asset for long time but do not expect major increment in price in near future and is ideal for people not expecting major fluctuations in prices of the underlying asset.
Let us understand how we design the pay off of the covered call. Let us assume you have purchased 100 shares of XYZ with $100 per share with the hope that the share will go up to $105 in next 6 months with no significant volatility expected. To balance of the profits, you parallelly sell 1 call option with the strike price of $105 expiring in next 6 months. Assume that the premium on this call is $3 per share in contract. Your pay off in the future will depend on the price of stock in next 6 months.
a) Stock remains at $100 per share: In this case, the call option will not be exercised as the strike price exceeds market price. As the price remains same, we don't get any return from stock but earn the premium per share on the call.
b) Stock price increases to $110 per share: If the stock price increases to $110 per share after 6 months, the call will be exercised and you will receive $105 per share and $3 per share from premium.

c) Stock price decreases to $90 per share: This case is similar to the scenario 1. In this case we lose $10 per share but this loss will be reduced by the premium of $3 per share that we receive from the call option.
Thus, the payoff the strategy would look like below:



Problem Statements:

Total Marks: 60

There are two parts in this question with 3 questions each. The weightage for each question has been mentioned individually, along with the format of the solution that is expected for each question. A common instruction for all questions is to **show your work**; your journey is almost as important as your destination!

JPMC Quant Finance Mentorship Program 2023

Part A:

1. A farmer expects to sell 500,000 bushels of wheat at the end of a year. One wheat futures contract on the CME Group is for the delivery of 5,000 bushels of wheat. How should the farmer use future contracts for hedging?

(a) How many contracts of long or short hedge would he need? **(1 marks)**
(b) How will futures help him hedge the rise and fall of prices of wheat? **(1 marks)**

2. In April 2021, a company enters a series of futures contract to sell 100,000 barrels of oil to sell in June 2022 as follows:
April 2021 – Short 100 October 2021 futures contract
September 2021 – Close out the October 2021 futures contract, and short 100 March 2022 futures contract
February 2022 – Close out the March 2022 futures contract, and short 100 July 2022 futures contract
July 2022 – Close out the July 2022 futures contract

Given the futures and spot prices in the table below per barrel of oil, what is the final realized payoff per barrel of oil? (5 marks)

Date	April 2021	September 2021	February 2022	July 2022
Spot price	120.50			117.10
October 2021 futures price	119.80	119.20		
March 2022 futures price		118.80	118.00	
July 2022 futures price			117.60	

3. Trader X uses a short futures contract to sell an asset for $1,000 in one year. Trader Y buys a put option to sell an asset for $1,000 in one year at a premium of $100. How are the two strategies different? Plot a graph to show how the pay off varies with the price of the asset over the year and mark the following clearly:

i. Strike of the option
ii. Premium of the option
iii. Futures price

Also, mark the region in the graph when the futures contract outperforms the put option and the region when the option outperforms the futures contract clearly showing the points of intersection.
(8 marks)

We don't expect you to code each plot but how you came to the plot is most important so please make sure that you submit the idea behind the curve that you make. Solutions are expected in any format(pdf, images, handwritten, code files, etc)

JPMC Quant Finance Mentorship Program 2023

Part B:

The aim of this case study is to give you a glimpse into how the payoffs are calculated and used. Don't fret, let's start now to deep dive into the world of trading. Your task is to design the payoff the below three combinations. We don't expect you to code each plot but how you came to the plot is most important so please make sure that you submit the idea behind the curve that you make. Solutions are expected in any format (pdf, images, handwritten, code files, etc)

a) A trader holding a position in both Call and Put options which have the same expiry date, same strike price and same underlying asset and pays premium for both the instruments. **(15 marks)**

b) A trader trades by going long on ITM Call option with lower strike, long on a higher strike OTM Call option and simultaneously going short on two ATM call option. Assume that all the options have same expiration date, and distance between each strike price of constituent leg is same. **(15 marks)**

c) A trader trades by buying 1 ITM Put and selling 1 OTM put and the only difference between them is of strike price, with expiration date, underlying asset being the same. **(15 marks)**

Mathematics

PROBLEM 1

An ant is standing at the origin (0,0). It starts moving in the x-y plane such that at each step, it moves upwards (in +y direction) or eastwards (in +x direction) with equal probabilities. The length of the first step is 1 unit and the length of each subsequent step is half of its previous step.
Thus, after taking the first step, it can be at (1,0) or (0,1) with equal probabilities. The length of the second step would be 0.5 and so on.

Let (x_i, y_i) be the position of ant after i steps have been taken. Thus, $(x_0, y_0) = (0,0)$.
Also, $(x, y) = \lim_{i\to\infty}(x_i, y_i)$.

1.1 Find the quadrant on the x-y Cartesian plane in which (x,y) lies ? **(1 mark)**

1.2 The Manhattan distance(d_m) between points A(x_1, y_1) and B(x_2, y_2) is defined as $|y_1 - y_2| + |x_1 - x_2|$ where $|x|$ denotes the absolute value of x.
Find $E(d_m)$ where d_m denotes the Manhattan distance between (x, y) and $(0,0)$ and $E(d_m)$ denotes the expected value (mean) of d_m. **(3 marks)**

1.3 The Euclidean distance(d_e) between points A(x_1, y_1) and B(x_2, y_2) is defined as
$\sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$.
Find $E(d_e^2)$ where d_e denotes the Euclidean distance between (x, y) and $(0,0)$ and $E(d_e^2)$ denotes the expected value (mean) of d_e^2. **(4 marks)**

1.4 Find the equation of the locus representing all the possible values of (x,y). **(2 marks)**

Instructions for Problem 1:
1.1 The answer should be one of the four quadrants in the x-y plane.
1.2 The answer should be an integer. Marks will only be provided for an analytical solution.
1.3 The answer should be expressed in the form of p/q where p and q are whole numbers with no common factors. Marks will only be provided for an analytical solution.
1.4 Marks will only be provided for an analytical solution.

PROBLEM 2

As an aspirant for Quant Mentorship program, you are playing a game. You are provided with a 8 card deck which includes 4 identical red cards and 4 identical black cards.

2.1 What are the total number of possible configurations for this deck? **(2 marks)**

All these possible configurations are equally likely to show up when you play the game. But, in order to make this a bit easier for you (or may be not), the entire deck (order of cards of respective colors) is revealed to you.
After you have seen the entire deck, you start drawing cards one by one. Every time you draw a red card, you get one dollar and every-time you draw a black card, you lose one dollar. Once you have drawn any card, that particular card is not returned to the deck. You are allowed to stop any time you want.
As any rational person, you want to know your expected pay-off before you agree to play this game. Being someone who is adept with programming, you simply decide to generate all the possible configurations and calculate your maximum possible pay-off (if you play optimally) for each configuration.

2.2 Write a function in the programming language of your choice which generates all the possible configurations for this 8 card deck and calculate the maximum possible pay-off for each configuration. Using this information, calculate your expected pay-off for this game. **(13 marks)**

In the last sub-question, you simply used your knowledge of programming to calculate your expected pay-off for this game. But, life is not so easy always. What about the situation if you are provided a standard 52 card deck with 26 identical red cards and 26 identical black cards. It should be fairly evident that generating all possible configurations won't be an efficient solution here.

The subsequent parts of this problem would help you to calculate your expected pay-off if you play this game with a standard 52 card deck. For all the following sub-question, assume that the game is being with a standard 52 card deck.

2.3 If you are playing this game with a standard 52 card deck, what are the possible values of the pay-off which you can get? **(2 marks)**

2.4 Let n_0 denote the number of deck configurations for a standard 52 card deck such that your pay-off would be less than or equal to 0, prove that: $n_0 = \binom{2n}{n}$ - $\binom{2n}{n+1}$ where $n = 26$.
$\binom{2n}{n}$ denotes number of ways of selecting n objects from 2n objects where order of selection is irrelevant. **(6 marks)**
Hint: Something called the 'Reflection Principle' might come to your rescue.

2.5 In the last sub-question, we proved an expression for n_0. Using similar ideas ('Reflection Principle' would be helpful again), prove that: $n_k = \binom{2n}{n}$ - $\binom{2n}{n+k+1}$ where $n = 26$ and $0 \leq k \leq n-1$
n_k denotes the number of deck configurations for a standard 52 card deck such that your pay-off would be less than or equal to k. **(7 marks)**
$\binom{2n}{n}$ denotes number of ways of selecting n objects from 2n objects where order of selection is irrelevant.

2.6 Using the results proved before, find the cumulative distribution function for the possible pay-offs. **(5 marks)**

2.7 In order to calculate the expected pay-off of the game, numerical values of probability density function for possible pay-offs need to be calculated. Here's a possible approach which might help you to calculate these values without calculating those large factorials.
Let f_k denote the probability that the maximum possible pay-off is greater than or equal to k. Clearly $f_0 = 1$. Find a simplified expression for $\frac{f_k}{f_{k-1}}$ where $1 \leq k \leq 26$. (**(4 marks)**)

2.8 Using the expression derived in 2.7, find the numerical values for P(payoff = k) for $1 \leq k \leq 26$ where P(X=k) denotes probability of random variable X being k. **(6 marks)**

2.9 What is the most likely (one with the highest probability) payoff for the standard 52 card deck? **(2 marks)**

2.10 Using the values obtained in 2.8, find the expected pay-off if the game is played using a standard 52 card deck. **(3 marks)**

Instructions for Problem 2:

2.1 The answer should be an integer. Marks will only be provided for an analytical solution.

2.2 Please provide a code that runs without any bugs. Marks would be allocated based on the overall code correctness and readability. Pseudo code will be graded partially.

2.3 The answer should be a set.

2.4 All the intermediate steps should be thoroughly explained in the proof.

2.5 All the intermediate steps should be thoroughly explained in the proof.

2.6 There is no need to find numerical values for the cumulative distribution function. A correct expression in terms of factorials would work.

2.7 The final simplified expression should only be a function of n and k where n=26.

2.8 In order to find the numerical values, it is advisable to use programming. All the relevant steps and codes (if any) should be clearly mentioned.

2.9 The answer should be an integer.

2.10 In order to find the numerical values, it is advisable to use programming. All the relevant steps and codes (if any) should be clearly mentioned. The final answer should be correct till 2 decimal places.