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Abstract

The global rise in temperatures presents serious challenges to the durability and functionality of asphalt pavements, including roads and urban streets. Persistent high temperatures accelerate structural wear, shorten pavement lifespan, and exacerbate the Heat Island effect in cities, emphasizing the need for innovative cooling strategies. Existing solutions are designed mainly based on engineering experience, and the quantitative relationship between design objective and variable is not constructed during the design process. This study tackles these shortcomings by employing numerical methods, such as interpolation, to model and analyze the relationship between the design objective (lower pavement temperature) and critical design variables (thermal conductivity, heat capacity, and surface reflectivity). Using discrete data sets, generalized functional relationships were developed to systematically evaluate the influence of these variables on cooling efficiency. The mechanisms driving pavement cooling were explored through thermal effect analysis and thermal resistance calculations. Results demonstrate that the optimized cooling pavement exhibited enhanced cooling performance, particularly at night, by reducing net heat absorption, lowering heat transfer efficiency, and increasing internal thermal resistance [my results] through controlled variations in heat flux. Compared to conventional asphalt pavements, the optimized design achieved reductions in surface and interior temperatures of up to [insert results]. This work underscores the value of integrating numerical simulations with theoretical modeling to solve complex engineering challenges. By confirming the effectiveness of the proposed optimization approach, this study contributes to the development of more efficient, sustainable cooling pavement systems.

Key words: Asphalt pavement, Topology optimization, Cooling effect, Thermal effect analysis

Introduction

The rapid urbanization and global rise in temperatures have made asphalt pavements, a common choice for roads and streets, increasingly vulnerable to high thermal stress. Asphalt, while offering benefits such as a smooth, joint-free surface and efficient construction, suffers from high solar radiation absorption and poor heat dissipation. These characteristics lead to prolonged high temperatures in pavements during summer, causing accelerated structural degradation, reduced lifespan, and a significant contribution to the urban Heat Island effect (Shamsaei et al. [2022]). High pavement temperatures also elevate urban energy consumption and release harmful volatile organic compounds (VOCs), creating health and environmental risks. Thus, reducing the temperature of asphalt pavements has become a critical area of research. (Zhao et al. [2024]). Several approaches to cooling pavements have been proposed, broadly categorized

into reflective, evaporative, and thermal resistance pavements. Reflective pavements reduce temperature by enhancing solar reflectance, achieving reductions of up to 5°C-15°C (Kappou et al. [2022], Anak Guntor et al. [2014] and Wardeh et al. [2022]). However, their reliance on light-colored materials raises concerns about cost and durability, limiting their practical applications. Evaporative pavements utilize water retention to cool surfaces, but this method is constrained by the requirement for significant water retention capacity and its potential impact on local humidity (Mullaney and Lucke [2014]). Thermal resistance pavements improve cooling by incorporating materials with low thermal conductivity. For instance, studies have shown that replacing traditional fillers with materials like glass microspheres can reduce thermal conductivity by over 20% (Zhao et al. [2024]). Despite these advancements, thermal resistance methods face challenges such as insufficient cooling performance and difficulty in balancing internal and surface temperature reductions.

Most existing designs rely heavily on experimental trial-and-error approaches and engineering experience, which lack a systematic framework for optimizing the relationship between design objectives (e.g., lower pavement temperature) and variables (e.g., thermal conductivity). This absence of a quantitative connection often leads to suboptimal designs and inefficient material usage. Furthermore, current methods fail to establish a precise range of thermal conductivity values that simultaneously achieve optimal surface and internal temperature reductions (Wardeh et al. [2022]). To address these limitations, the present work introduces a data-driven approach using numerical methods, particularly interpolation techniques, to model the relationship between design objectives and critical variables. This methodology transforms the cooling pavement design problem from an empirical challenge into a mathematical optimization problem. By applying interpolation methods such as Lagrange and Newton's Divided Difference to discrete data sets, generalized functions will be derived to explore the effects of parameters like thermal conductivity, heat capacity, and surface reflectivity on pavement cooling performance.

The advantages of the proposed method include its ability to reduce reliance on experimental iterations, provide precise functional relationships for key variables, and guide the design process more efficiently. However, the limitations of numerical methods, such as sensitivity to data accuracy, will also be addressed in this study. This research fills a critical gap by integrating theoretical modeling and numerical simulation to achieve a scientifically robust framework for pavement cooling design. The following sections will discuss the methodologies employed, including the interpolation techniques and thermal effect analysis. The results will provide insights into the cooling mechanisms and propose an optimized pavement design. Finally, the study will conclude with recommendations for future research and practical implementation strategies (Zhao et al. [2024]).

Heat Transfer Model

Cooling asphalt pavements primarily hinges on optimizing the structural design, as altering external environmental factors is a much more challenging task. To achieve this, the focus is on modifying the thermal properties of the pavement's structural layers. The design approach divides the pavement into three distinct sub-domains: the upper layer, the middle layer, and the lower layer, each with its unique role in managing thermal behavior and enhancing cooling performance.

For the asphalt mixture design, stones with a maximum nominal particle size of 13.2 mm (AC-13) were selected. AH-70 asphalt served as the binding material. Basalt was used as the coarse aggregate, categorized into two size ranges: 9.5–13.2 mm and 2.36–9.5 mm. Limestone, with a particle size of 0–2.36 mm, was utilized as the fine aggregate. Following the Marshall mix design method, the optimal asphalt-to-aggregate ratio was determined to be 6.0% (Zhao et al. [2024]).

In this model, solar radiation stands as the primary heat source, captured mathematically in Eq. (1). To fully describe the thermal dynamics, we incorporate the governing equations for heat conduction, convection, and effective radiation on the road surface, represented by Eqs. (2)–(4) (Qin and Hiller [2014]). These equations collectively provide a comprehensive framework for analyzing the heat transfer mechanisms in asphalt pavements.

1. Solar radiation:

$$G_{\text{amb},i} = F_{\text{amb},i} E_{\text{amb}} e_b(T_{\text{amb}}) FE P_i(T_e)$$
 (1)

where, $G_{\text{amb},i}$ is the solar radiation intensity, W/m², $F_{\text{amb},i}$ is the environmental angle coefficient, e_b is the blackbody emissive power, W/m², $F_{\text{EP},i}$ is the partial radiated power, and T_e is the environment temperature, K.

2. Heat conduction:

$$\rho c_v T \frac{\partial T}{\partial t} + \rho c_p \mathbf{u} \cdot \nabla T + \nabla \cdot \dot{q} = Q + Q_{\text{ted}}$$

$$\dot{q} = -k \nabla T$$
(2)

where, ρc_v is the volumetric heat capacity, c_p is the heat capacity of pavement, J/(kgK), T is the temperature of pavement, K, Q is the heat source, W/m^3 , k is the thermal conductivity of pavement, W/(mK), and \dot{q} is the conductive heat flux, W/m^2 .

3. Heat convection:

$$-n \cdot \dot{q} = q_0$$

$$q_c = h(T_e - T)$$
(3)

where, q is the convective heat flux, W/m2; h is the convective heat transfer coefficient, W/(m2K), which is calculated by h=5.6+4v, v is wind speed, m/s; Te is the environment temperature, K; T is the temperature of asphalt pavement, K.

4. Effective radiation of road surface:

$$k\frac{\partial T}{\partial y} = \sigma \epsilon (T^4 - T_e^4) \tag{4}$$

where, σ is 5.67 × 10⁸ W/(m² · K⁴); ϵ is the surface emissivity; T_e is the environment temperature, K; T is the surface temperature of asphalt pavement, K.

The topology optimization model was developed as an extension of the heat transfer framework, with carefully defined objective functions and design variables guiding the process. The entire workflow, illustrated in Fig. 1, begins by building on the heat transfer model, setting clear objectives, and identifying critical variables. Through iterative computations, the optimization results were systematically refined, showcasing the power of this structured approach.

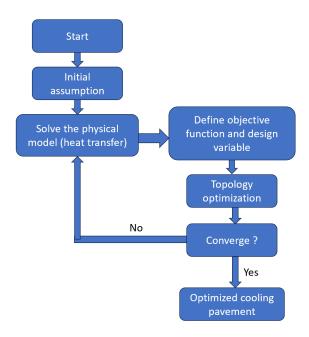


Fig. 1: Flow chart of topology optimization.

Since the simulation represents a three-dimensional unsteady process influenced by multiple factors and conditions, certain discrepancies between the numerical model and real-world scenarios are inevitable. For instance, temperature variations can alter some parameters over time. To address these complexities, specific assumptions were established for the simulation model:

- Each structural layer is treated as uniform, continuous, and isotropic.
- The analysis focuses solely on the longitudinal distribution of the pavement's temperature field, neglecting any transverse variations.
- The thermophysical parameters defined in the model are considered constant, unaffected by temperature changes.

These assumptions provide a simplified yet effective framework to streamline the simulation process and focus on critical aspects of the heat transfer dynamics.

Problem Formulation

Designing optimized cooling pavements is a multifaceted challenge that requires careful consideration of various variables influencing their performance. These variables, ranging from thermal conductivity and heat capacity to surface reflectivity play crucial roles in determining the efficiency of cooling solutions. The relationship between these variables and the overarching objective function (minimizing pavement temperatures) or improving material durability, is often intricate and demands a structured approach.

Instead of relying solely on discrete experimental data points, we can enhance the accuracy of our analysis by interpolating the dataset. This allows us to generate continuous curves, which provide the objective function for any value of the design variable, rather than limiting us to specific data intervals. By doing so, we

can gain a more refined and comprehensive understanding of how the design variables influence the pavement's performance.

The influence of thermal parameters on temperature

Research has identified two primary approaches to lowering asphalt pavement temperatures: enhancing surface reflectivity and modifying the pavement's thermal conductivity (Qin et al. [2015] and Zhao et al. [2024]). To pinpoint the key factors influencing pavement temperature and establish design variables for the topology optimization process, the average temperature of the upper layer was chosen as the evaluation metric.

Two critical parameters were analyzed to assess temperature variations in the upper layer of asphalt pavement. Using the orthogonal analysis method, the impact of these parameters on pavement cooling was systematically evaluated. Each parameter was tested across five levels, as detailed in Table-1(Zhao et al. [2024]), to determine their influence and optimize the design process.

Table 1. Levels of all factors

Level	Thermal conductivity	Road surface reflectivity
	$(\lambda) [W/(m \cdot K)]$	$(1-\alpha)$
1	4.2	0.18
2	3.6	0.23
3	3.0	0.28
4	2.4	0.33
5	1.8	0.38

A heat transfer simulation was conducted for 25 groups in the orthogonal experiment, allowing for the analysis of how each parameter impacts the pavement temperature using Range method analysis. The extreme difference (RD) was calculated to quantify the influence of each parameter on temperature. A higher RD indicates a greater impact, suggesting that the corresponding parameter is more significant. In this method, [T ij] represents the sum of temperatures at the same level for each column, where i refers to the level and j denotes the column. The heat transfer model was then applied to calculate the temperature variations under various conditions. The calculation method is shown in Eq. (5).

$$RD_j = T_{ij\max} - T_{ij\min} \tag{5}$$

where, T_{ij} is the sum of the temperatures at the same level for each column, i is the level, and j is the column. The heat transfer model was used to calculate the temperature variation under different conditions.

Numerical Methods

In this study, we employ standard interpolation techniques, namely Newton's Divided Difference formula and Lagrange interpolation, to generate polynomials. Using design variables as x and the corresponding objective function values as y, we create a dataset consisting of 5 data points. By applying these methods, we derive a 4th-degree polynomial. The resulting polynomial's coefficients are obtained as a linear array, which will be used for further analysis and optimization in the model.

For Newton's Divided difference formula,

$$P(x) = b_0 + b_1(x - x_0) + \dots + b_n(x - x_0)(x - x_1), \dots, (x - x_{n-1}).$$
(6)

Where coefficient b can be calculated as,

$$b_n = f[x_n, x_{n-1}, \dots, x_1, x_0]$$

And Finite Divided Difference for general nth order,

$$f[x_n,x_{n-1},x_1,x_0] = \frac{f[x_n,x_{n-1},\ldots,x_1] - f[x_{n-1},\ldots,x_1,x_0]}{x_n - x_0}$$

Also, The Lagrange interpolation polynomial is given by:

$$P(x) = \sum_{i=0}^{n} y_i \ell_i(x) \tag{7}$$

where the Lagrange basis polynomial $\ell_i(x)$ is defined as:

$$\ell_i(x) = \prod_{\substack{0 \le j \le n \\ i \ne i}} \frac{x - x_j}{x_i - x_j}$$

The data-set for our analysis is as follows in Table -2 and Table -3 (Zhao et al. [2024]):

Table 2. Thermal conductivity v/s Summation of Temperature.

Thermal conductivity	Summation of temperature
$(\lambda) [W/(m \cdot K)]$	(°C)
1.8	202.938
2.4	200.08
3.0	261.12
3.6	260.89
4.2	259.51

Table 3. Road Surface Reflectivity v/s Summation of Temperature.

Road surface reflectivity	Summation of temperature
$(1-\alpha)$	(°C)
0.18	238.775
0.23	233.39
0.28	235.5
0.33	237.16
0.38	239.72

Results

The obtained results display the cumulative temperature variations under different parameters, highlighting how each factor influences the pavement temperature. From the derived polynomials, it becomes evident that the two parameters—thermal conductivity and road surface reflectivity—affect the temperature in distinct ways.

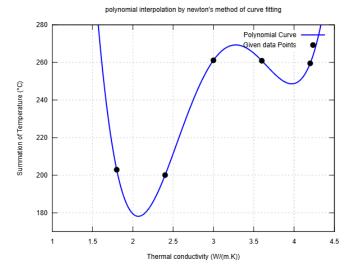
First, increasing thermal conductivity does not support the cooling of the upper layer. In fact, when the thermal conductivity (λ) was doubled, the pavement temperature rose significantly. On the other hand, the effect of road surface reflectivity on the temperature was comparatively less pronounced. For instance,

when the reflectivity was 0.38, the upper layer temperature was recorded at 239.72°C, which was just a 2.71% increase from the value observed at a reflectivity of 0.23.

Additionally, the Range Difference (RD_j) was computed. The analysis revealed that among the primary factors affecting the upper layer temperature, thermal conductivity and road surface reflectivity, the former has way more impact on the objective function. Since the average upper layer temperature was selected as the objective for topology optimization, thermal conductivity was ultimately chosen as the design variable for further investigation.

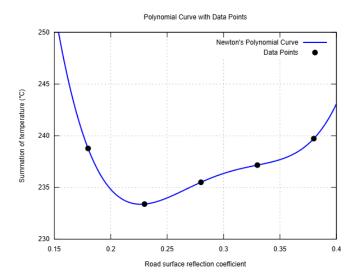
Upon analyzing the polynomial relationship between thermal conductivity and temperature, we observe a significant change in temperature, which is reflected in the high Range Difference (RD_j) value. This large variation in temperature underscores the strong impact that thermal conductivity has on the pavement's thermal behavior. The high RD_j value indicates that thermal conductivity is a crucial parameter in controlling the temperature of the upper layer, and its variation plays a dominant role in influencing the cooling efficiency of the asphalt pavement. This reinforces the selection of thermal conductivity as a key design variable for the topology optimization process.

$$f(x) = 680.06 - 6289.42x + 32597.40x^2 - 73799.33x^3 + 61966.67x^4$$



When we dive into the polynomial relationship between surface reflectivity and temperature, it's clear that the temperature variation is relatively minimal, which is reflected in the low RDj value. This tells us that changes in surface reflectivity have a smaller influence on pavement temperature compared to other factors, such as thermal conductivity. In other words, while reflectivity does play a role in cooling, its impact is more subtle, making it a secondary consideration in our optimization process.

$$f(x) = 4625.90 - 6412.23x + 3336.12x^2 - 739.94x^3 + 59.57x^4$$



Using the polynomial obtained from the interpolation process, we can now generate new original datasets by simply feeding different values of the design variables (e.g., thermal conductivity) into the equation. This allows us to predict the corresponding temperatures for various design scenarios.

To take this a step further, we can differentiate the polynomial with respect to thermal conductivity (λ) to identify the minima in the Conductivity vs. Temperature relationship. By finding the derivative of the polynomial, we can determine the value of thermal conductivity that results in the lowest temperature for the pavement, which is essential for optimizing the cooling efficiency. The point where the derivative equals zero will give us the optimal thermal conductivity value, thus providing a critical parameter for the design of the most effective cooling pavement.

$$f'(x) = -6289.42 + 65194.80x - 221398.00x^2 + 247866.68x^3$$

The derivative function has a critical point at x=0.227 (approximately), giving 233.37 the minimum summation temperature.

Path forward

Our findings can be extended to different types of asphalt mixtures by adapting the same data-driven optimization approach to account for variations in material properties, such as aggregate type, binder composition, and mix proportions. Different asphalt mixtures may exhibit distinct thermal characteristics, which would influence their temperature response under solar radiation and environmental conditions. For example, mixtures with higher mineral content or modified binders may have different thermal conductivities, heat capacities, and reflectivity, which could impact their cooling performance. By applying our numerical model to the specific thermal properties of these mixtures, we can identify the optimal combination of parameters for each type of asphalt.

The solution code developed in this study is versatile and can be directly applied to datasets of other asphalt mixtures, allowing for tailored optimization of cooling properties. Whether designing pavements for border roads, highways, bridges, or roads in snowy regions, the model can accommodate diverse cooling requirements. Depending on the specific needs—whether we aim for rapid cooling or prefer a pavement that retains heat—the code offers the flexibility to optimize asphalt mixtures accordingly, ensuring that road performance aligns with environmental conditions and functional goals.

Conclusion

To conclude with, this study has highlighted the critical role that thermal conductivity plays in influencing the temperature of asphalt pavements. Through our numerical analysis and interpolation techniques, we have demonstrated that the temperature variations are significantly affected by thermal conductivity, with a clear and substantial relationship between the two. The results show that increasing thermal conductivity leads to a rise in pavement temperature, emphasizing its importance as a key design variable for cooling pavements.

Conversely, while road surface reflectivity contributes to temperature reduction, its effect is less pronounced compared to thermal conductivity. This suggests that, for optimal pavement cooling, modifying the thermal properties of the material—particularly thermal conductivity—should be prioritized over reflectivity adjustments.

The findings of this study provide valuable insights into the design and optimization of cooling pavements, offering a data-driven approach to understanding the impact of various factors on pavement temperature. By using numerical methods to generate precise relationships between design variables and performance, this work lays the foundation for more efficient, scientifically-backed pavement cooling solutions. Future research can build upon these results by further refining the optimization process and exploring additional factors that could enhance pavement performance under varying environmental conditions.

Self Assessment

The topic chosen for this term paper provided a valuable learning experience, allowing us to progress beyond simply reproducing existing results. Initially, we reached Level 1 by developing functional interpolation codes that helped generalize the temperature vs. conductivity curve. However, we aimed to push the boundaries and, through careful effort and extensive research, were able to achieve Level 2 by introducing original results and extending our understanding into new applications.

By identifying a new point on the temperature vs. conductivity curve, we explored how different conductivity values can optimize pavement design for various road types. This approach can help us to develop tailored asphalt mixtures that address specific needs such as faster cooling for urban roads or maintaining lower temperatures on highways. We also considered the implications of these findings for alternative materials, such as nanoparticles or phase change materials (PCMs), which can improve thermal performance and reduce long-term maintenance costs.

Our work also delved into practical applications for extreme climates, where the modified conductivity values could lead to better pavement designs for snowy or desert regions, ensuring roads either retain or dissipate heat as needed. Furthermore, the relationship between conductivity and energy usage pointed

to opportunities for energy-efficient road designs, contributing to sustainability efforts and urban planning strategies.

In conclusion, the integration of new points on the curve has not only broadened our understanding but also contributed to the development of innovative ideas for pavement cooling technologies, including self-cooling pavements and thermoelectric materials which may potentially empower the Transport Sector. These findings represent a significant step towards "newer engineering," showcasing the ability to apply theory to solve real-world challenges.

Ideating these original ideas/results felt satisfying enough for us to believe our work can be considered as reaching Level 2.

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