Problem Solving and Search

Chapter 3, Sections 1–4

Outline

♢ Problem-solving agents

♢ Problem types

♢ Problem formulation

♢ Example problems

♢ Basic search algorithms

Problem-solving agents

Restricted form of general agent:

```
function Simple-Problem-Solving-Agent( $p$) returns an action
    inputs: $p$, a percept
    static: $s$, an action sequence, initially empty
            $state$, some description of the current world state
            $g$, a goal, initially null
            $problem$, a problem formulation

    $state \leftarrow$ Update-State($state, p$)
    if $s$ is empty then
        $g \leftarrow$ Formulate-Goal($state$)
        $problem \leftarrow$ Formulate-Problem($state, g$)
        $s \leftarrow$ Search( $problem$)
    $action \leftarrow$ Recommendation($s, state$)
    $s \leftarrow$ Remainder($s, state$)
    return $action$
```

Note: this is *offline* problem solving.
Online problem solving involves acting without complete knowledge of the problem and solution.

Example: Romania

On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:

be in Bucharest

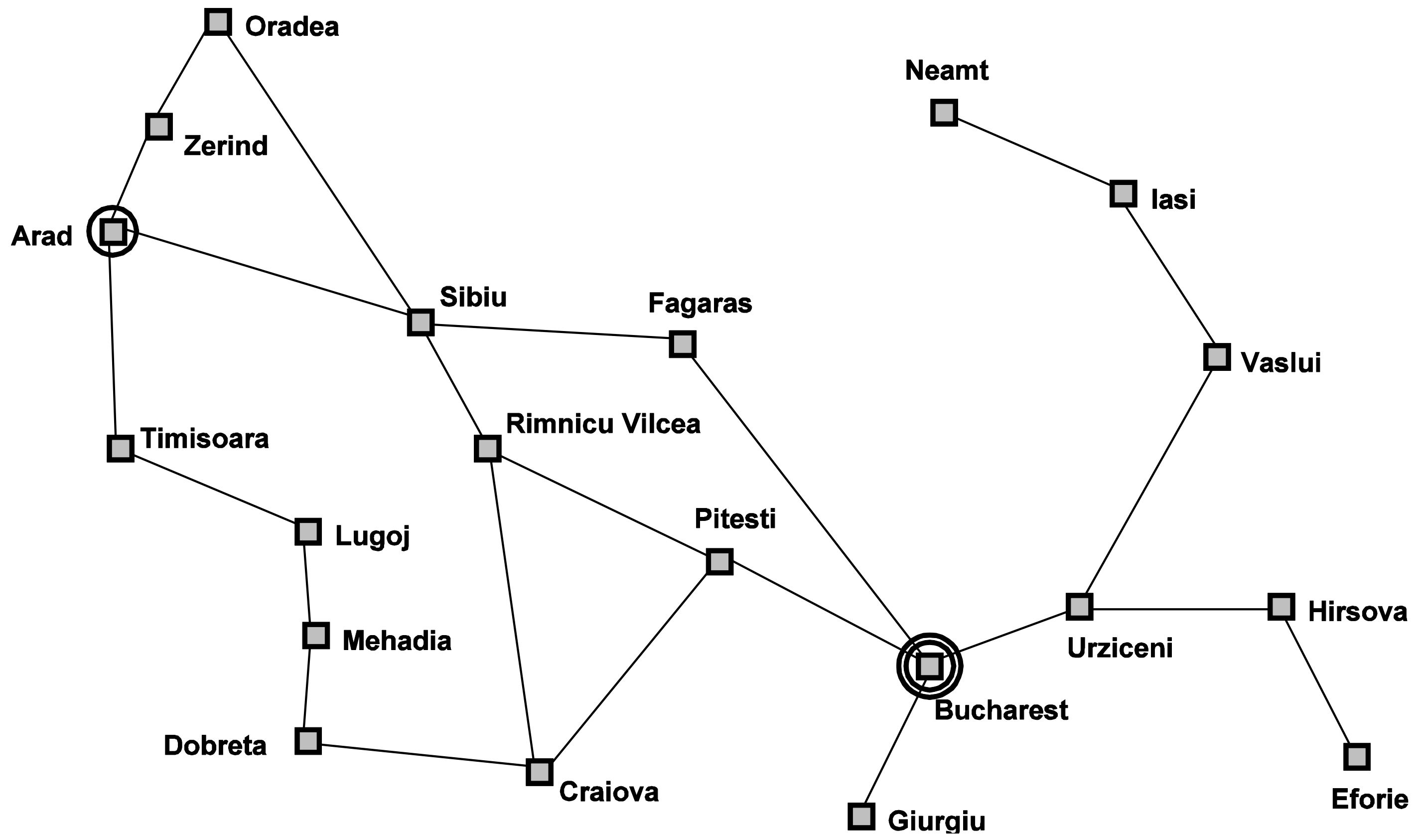
Formulate problem:

states: various cities
operators: drive between cities

Find solution:

sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



Single-state problem formulation

A *problem* is defined by four items:

<u>*initial state*</u> e.g., "at Arad"

<u>*actions*</u> (or *successor function* $S(x)$)
e.g., Arad $\rightarrow$ Zerind   Arad $\rightarrow$ Sibiu   etc.

<u>*goal test*</u>, can be
explicit, e.g., $x =$ "at Bucharest"
implicit, e.g., $Checkmate$ in chess

<u>*path cost*</u> (additive)
e.g., sum of distances, number of actions executed, etc.

A *solution* is a sequence of actions
leading from the initial state to a goal state

Note: we sometimes refer to actions as "operators"

Selecting a state space

Real world is absurdly complex
 $\Rightarrow$ state space must be *abstracted* for problem solving

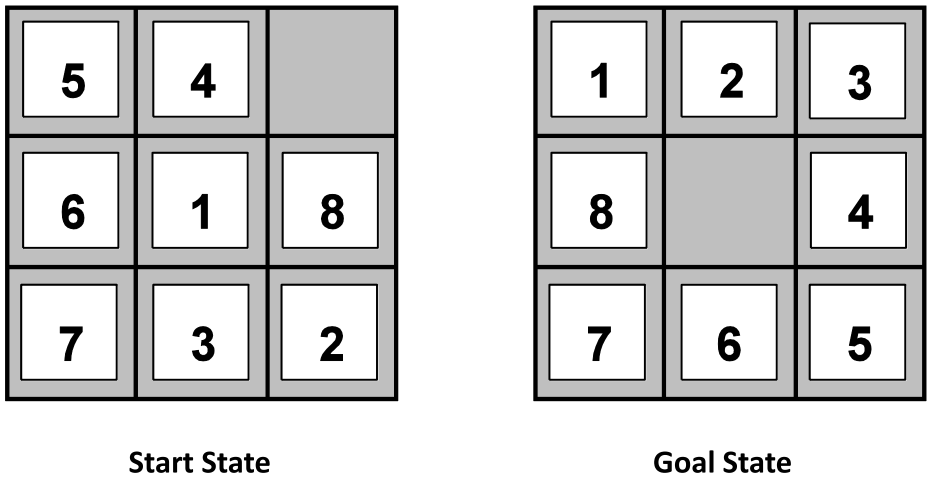
(Abstract) state $=$ set of real states

(Abstract) action $=$ complex combination of real actions
 e.g., "Arad $\rightarrow$ Zerind" represents a complex set
  of possible routes, detours, rest stops, etc.
For guaranteed realizability, <u>any</u> real state "in Arad"
 must get to *some* real state "in Zerind"

(Abstract) solution $=$
 set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

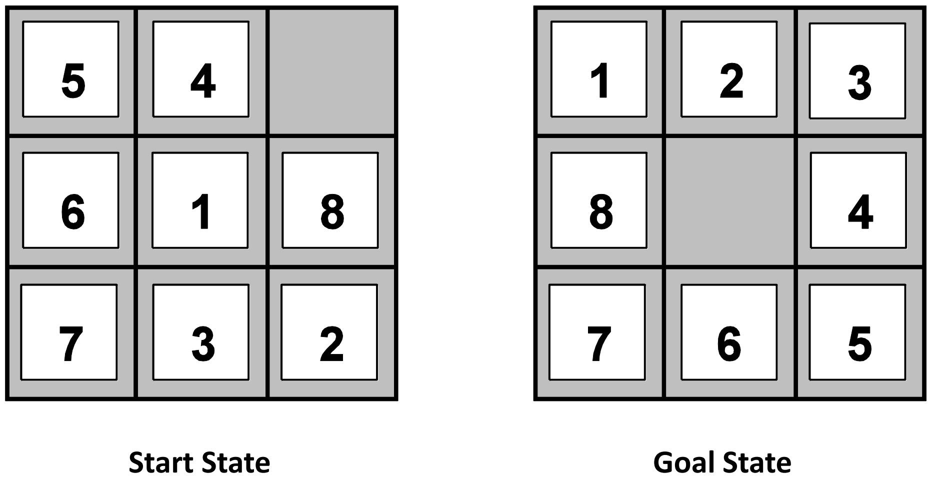
Example: The 8-puzzle



states??
actions??
goal test??
path cost??

[Note: optimal solution of n-Puzzle family is NP-hard]

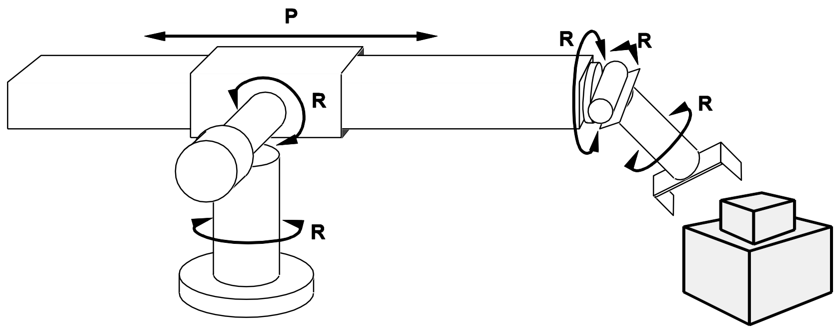
Example: The 8-puzzle



states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
goal test??: $=$ goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: robotic assembly



states??: real-valued coordinates of
robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly *with no robot included!*

path cost??: time to execute

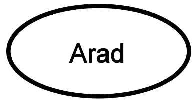
Search algorithms

Basic idea:

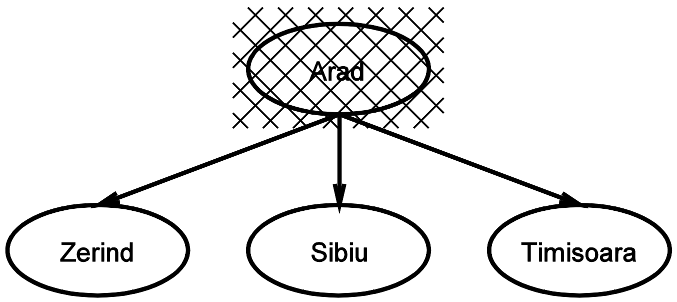
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. *expanding* states)

```
function GENERAL-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

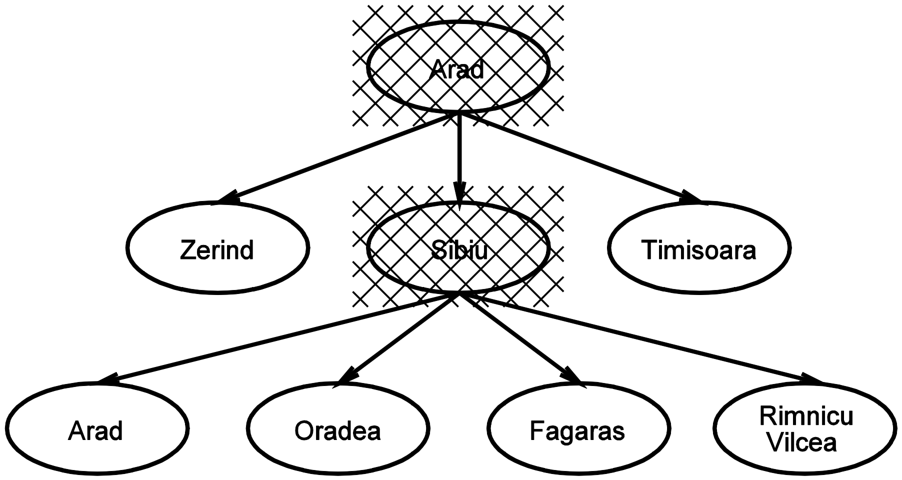
General search example



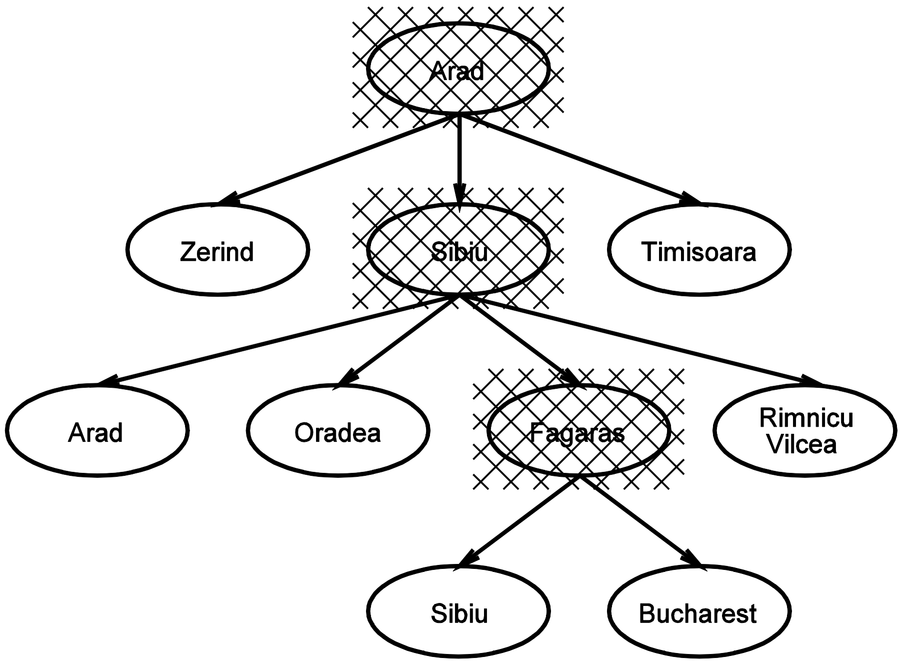
General search example



General search example



General search example

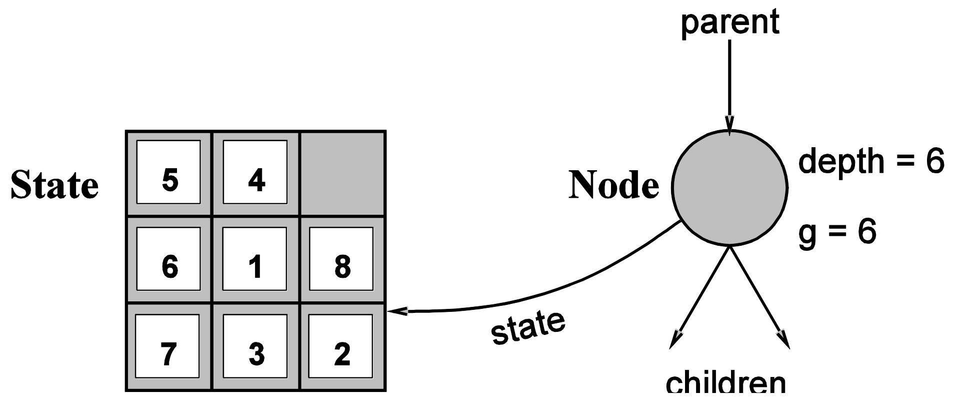


Implementation of search algorithms

```
function GENERAL-SEARCH( problem, QUEUING-FN) returns a solution, or failure
    nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
    loop do
        if nodes is empty then return failure
        node ← REMOVE-FRONT(nodes)
        if GOAL-TEST[problem] applied to STATE(node) succeeds then return node
        nodes ← QUEUING-FN(nodes, EXPAND(node, OPERATORS[problem]))
    end
```

Implementation contd: states vs. nodes

A *state* is a (representation of) a physical configuration
A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$
States do not have parents, children, depth, or path cost!



The EXPAND function creates new nodes, filling in various fields and using OPERATORS (or ACTIONS) of problem to create the corresponding states.

Search strategies

A strategy is defined by picking the *order of node expansion*

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

Uninformed search strategies

Uninformed strategies use only the information available
in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

Depth-limited search

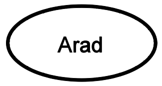
Iterative deepening search

Breadth-first search

Expand shallowest unexpanded node

Implementation:

QueueingFn = put successors at end of queue

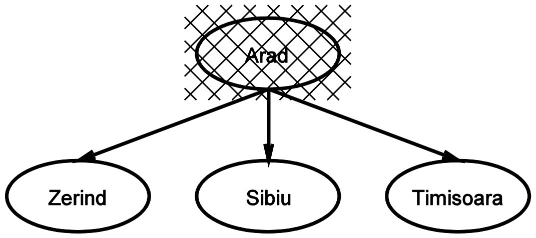


Breadth-first search

Expand shallowest unexpanded node

Implementation:

QueueingFn = put successors at end of queue



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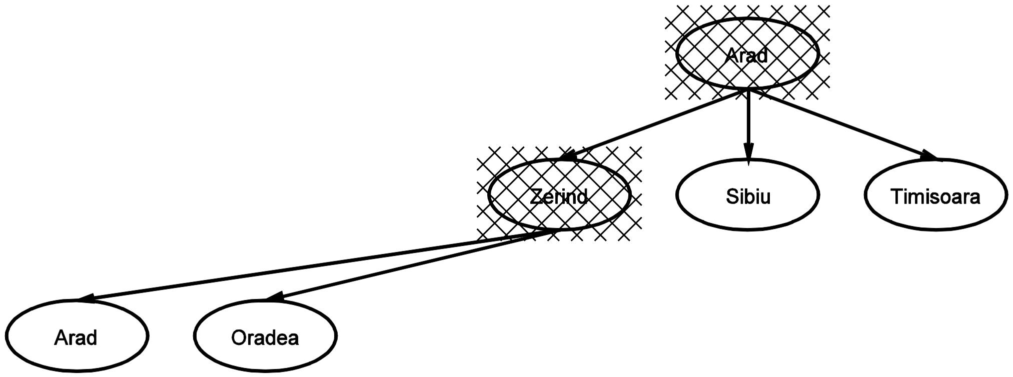

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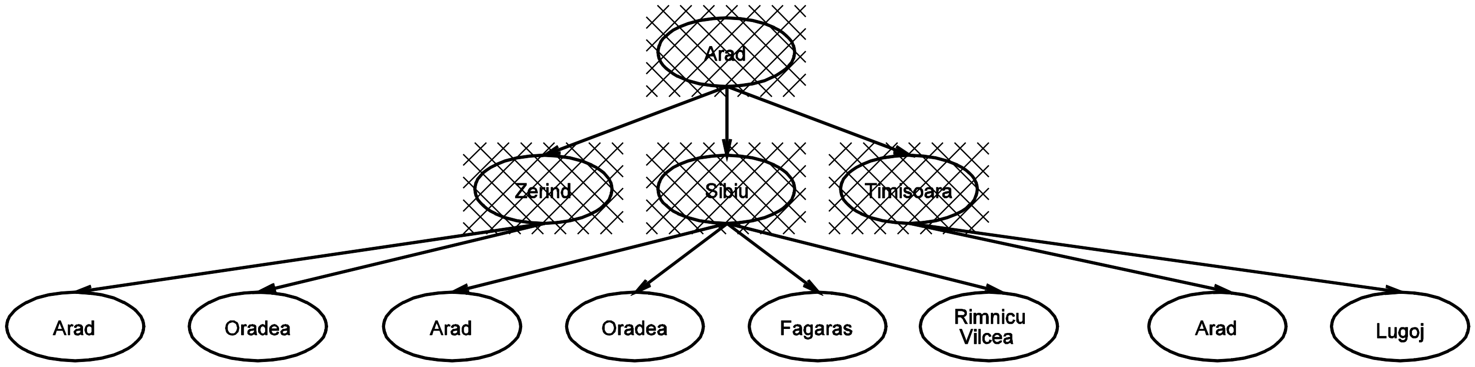


Properties of breadth-first search

Complete??

Time??

Space??

Optimal??

Properties of breadth-first search

Complete?? Yes (if b is finite)

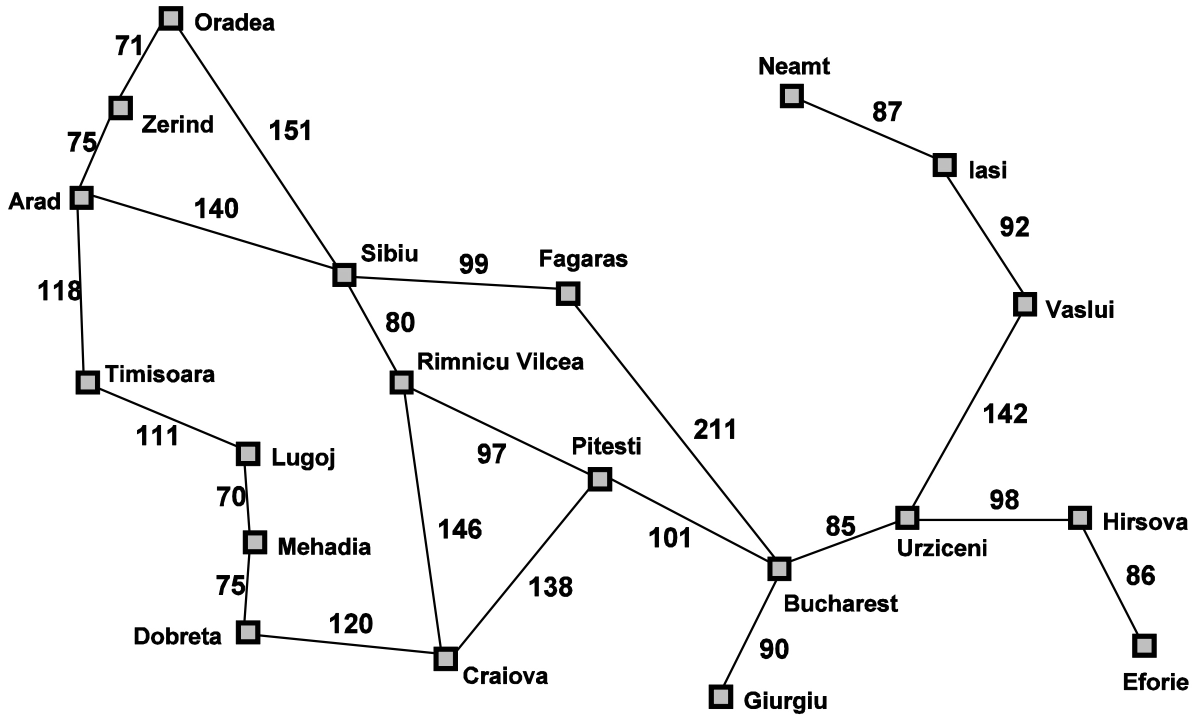
Time?? $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d

Space?? $O(b^d)$ (keeps every node in memory)

Optimal?? Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at 1MB/sec
so 24hrs = 86GB.

Romania with step costs in km



Straight-line distance
to Bucharest

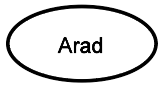
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Uniform-cost search

Expand least-cost unexpanded node

Implementation:

QueueingFn = insert in order of increasing path cost



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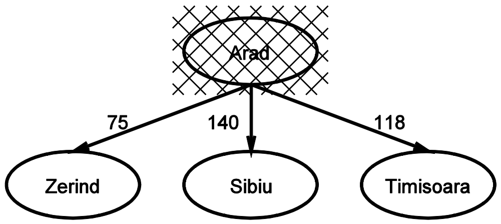

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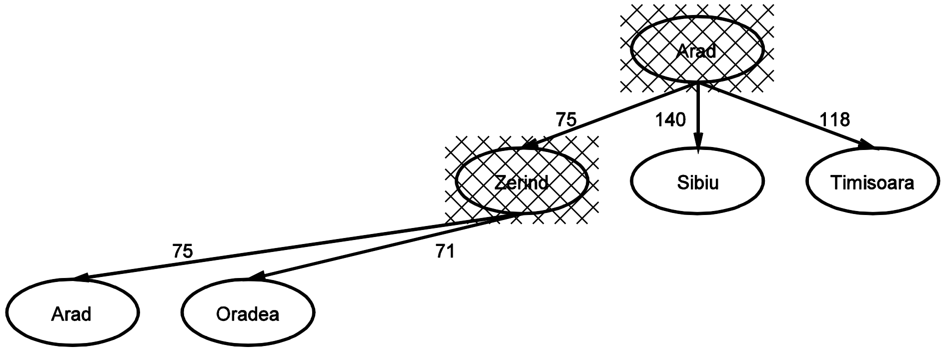

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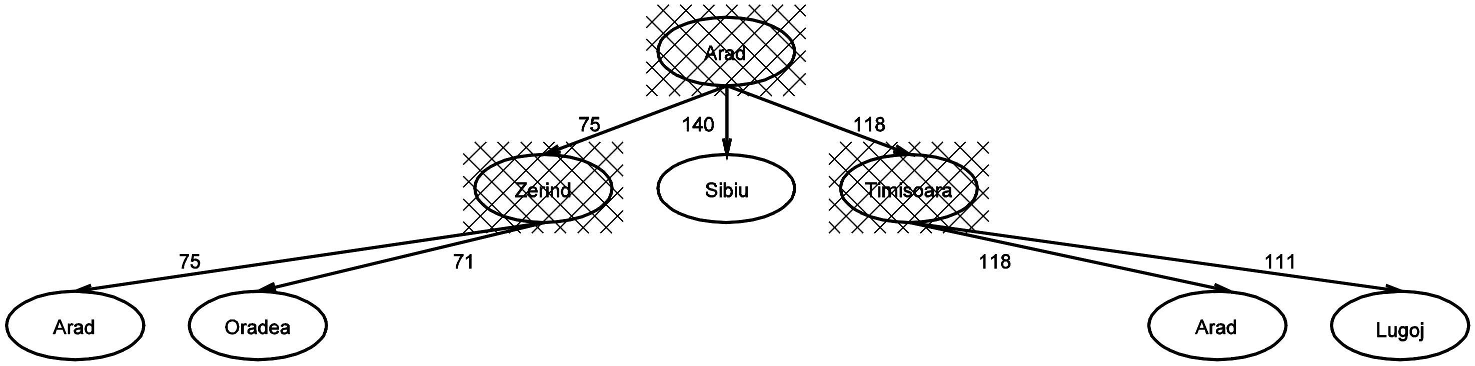


Properties of uniform-cost search

Complete?? Yes, if step cost $\geq \epsilon$

Time?? # of nodes with $g \leq$ cost of optimal solution

Space?? # of nodes with $g \leq$ cost of optimal solution

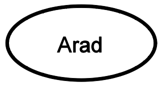
Optimal?? Yes

Depth-first search

Expand deepest unexpanded node

Implementation:

QueueingFn = insert successors at front of queue

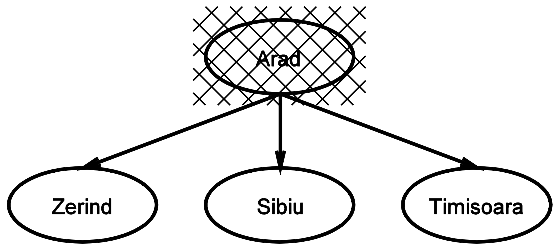


Depth-first search

Expand deepest unexpanded node

Implementation:

QueueingFn = insert successors at front of queue

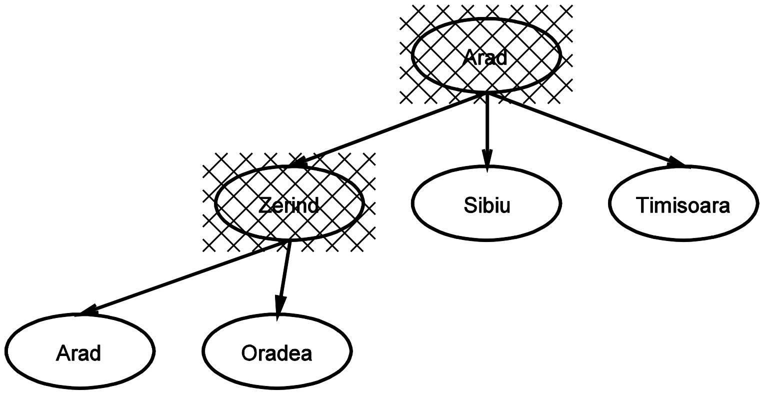


Depth-first search

Expand deepest unexpanded node

<u>Implementation</u>:

QueueingFn = insert successors at front of queue



Depth-first search

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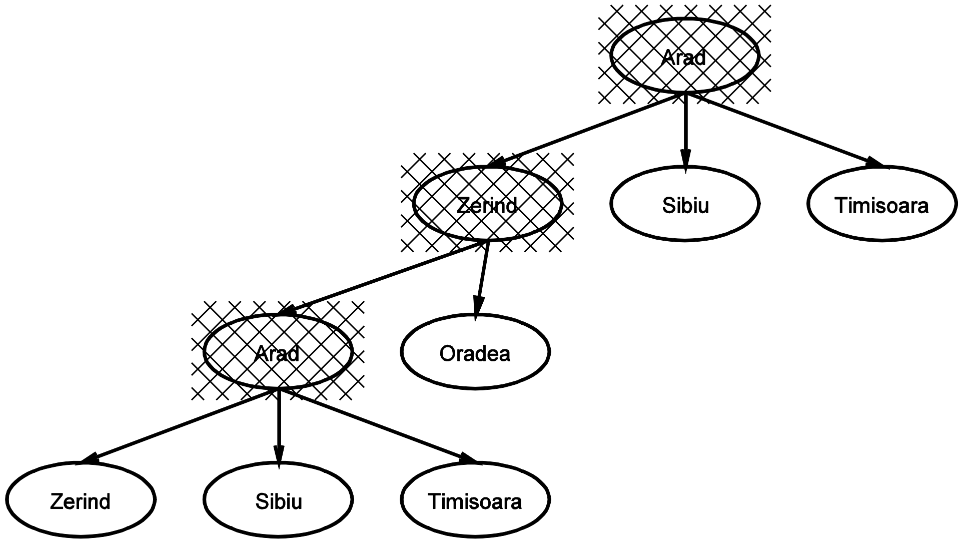

I.e., depth-first search can perform infinite cyclic excursions
Need a finite, non-cyclic search space (or repeated-state checking)

Properties of depth-first search

Complete??

Time??

Space??

Optimal??

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces

Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first

Space?? $O(bm)$, i.e., linear space!

Optimal?? No

Depth-limited search

= depth-first search with depth limit l

Implementation:

Nodes at depth l have no successors

Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution sequence
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

Properties of iterative deepening search

Complete?? Yes

Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$

Space?? $O(bd)$

Optimal?? Yes, if step cost $= 1$
Can be modified to explore uniform-cost tree

Bidirectional Search

Search simultaneously forwards from the start point, and backwards from the goal, and stop when the two searches meet in the middle.

Problems: generate predecessors; many goal states; efficient check for node already visited by other half of the search; and, what kind of search.

Properties of Bidirectional Search

Complete?? Yes

Time?? $O(b^{\frac{d}{2}})$

Space?? $O(b^{\frac{d}{2}})$

Optimal?? Yes (if done with correct strategy - e.g. breadth first).

Summary

Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

Variety of uninformed search strategies

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Examples of skills expected:

◇ Formulate single-state search problem

◇ Apply a search strategy to solve problem

◇ Analyse complexity of a search strategy