Informed Search Algorithms

Chapter 3, Sections 5–6

Outline

◇ Best-first search

◇ A^* search

◇ Heuristics

◇ Hill-climbing

Review: General search

function GENERAL-SEARCH(*problem*, QUEUING-FN) **returns** a solution, or failure

    nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[*problem*]))
    loop do
        if *nodes* is empty **then return** failure
        node ← REMOVE-FRONT(*nodes*)
        if GOAL-TEST[*problem*] applied to STATE(*node*) succeeds **then return** *node*
        nodes ← QUEUING-FN(*nodes*, EXPAND(*node*, OPERATORS[*problem*]))
    end

A strategy is defined by picking the *order of node expansion*

Best-first search

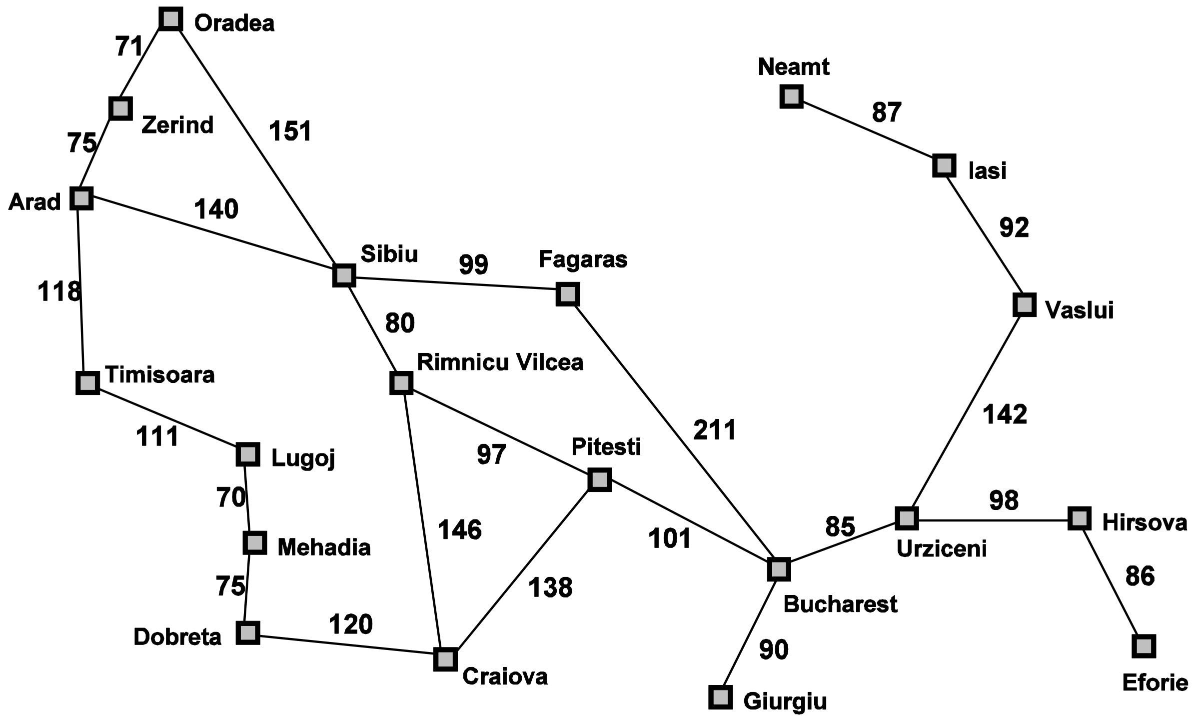
Idea: use an *evaluation function* for each node
– estimate of "desirability"

$\Rightarrow$ Expand most desirable unexpanded node

Implementation:
$\textsc{QueueingFn}$ = insert successors in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

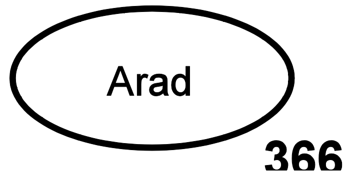
Greedy search

Evaluation function $h(n)$ (heuristic)
$= $ estimate of cost from n to $goal$

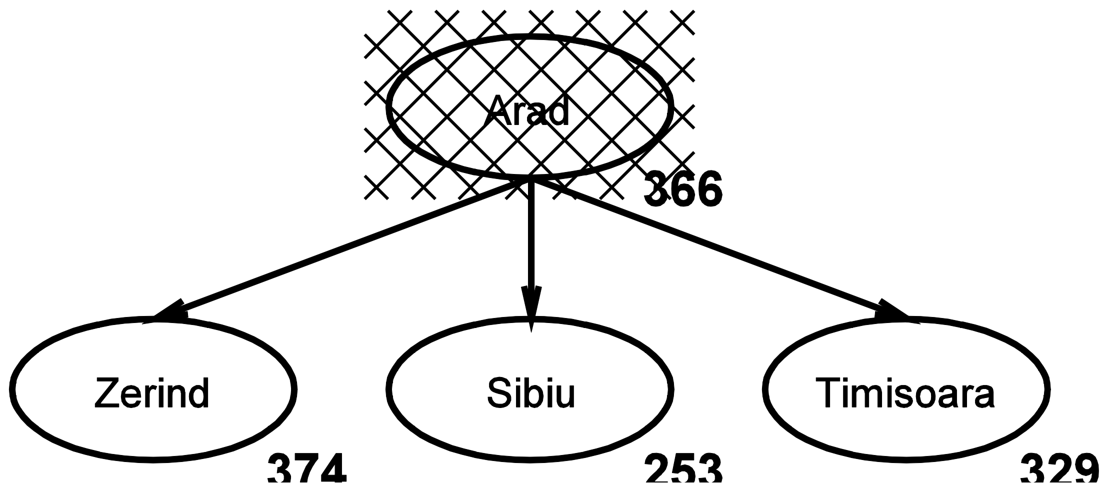
E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

Greedy search expands the node that $appears$ to be closest to goal

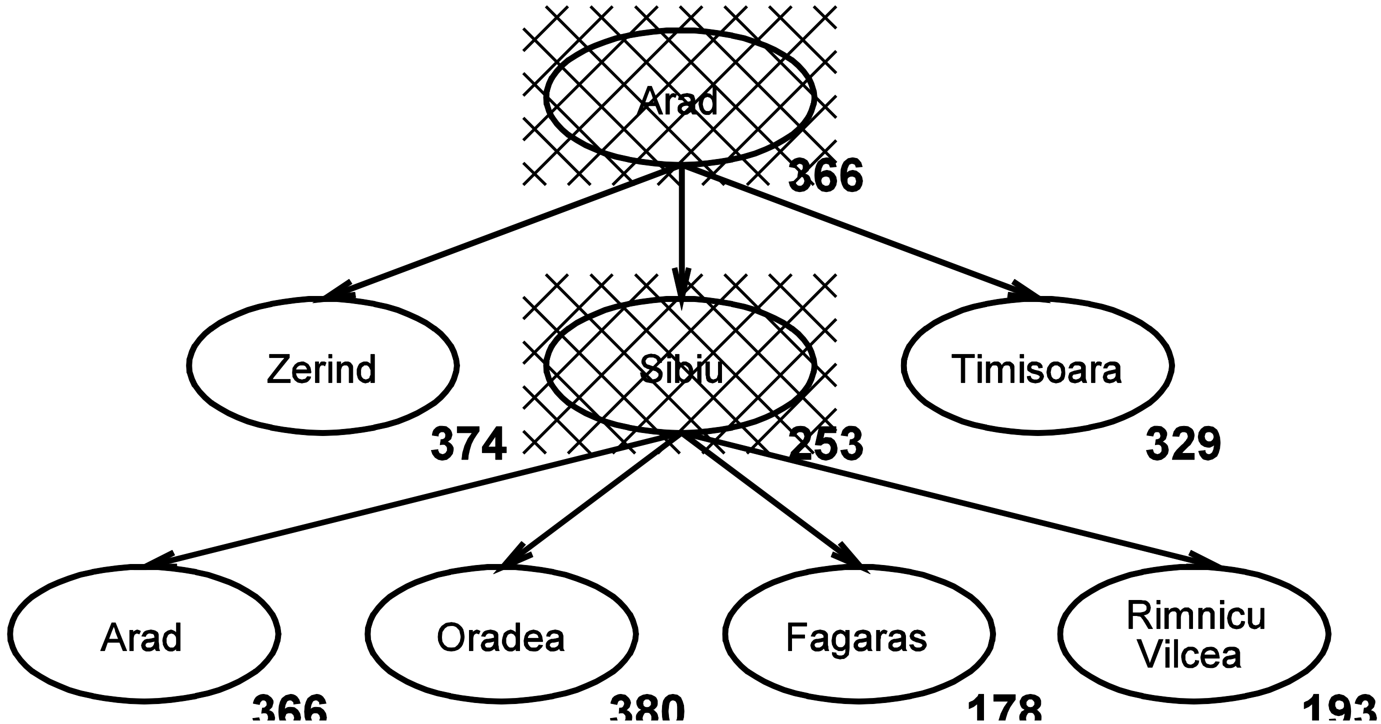
Greedy search example



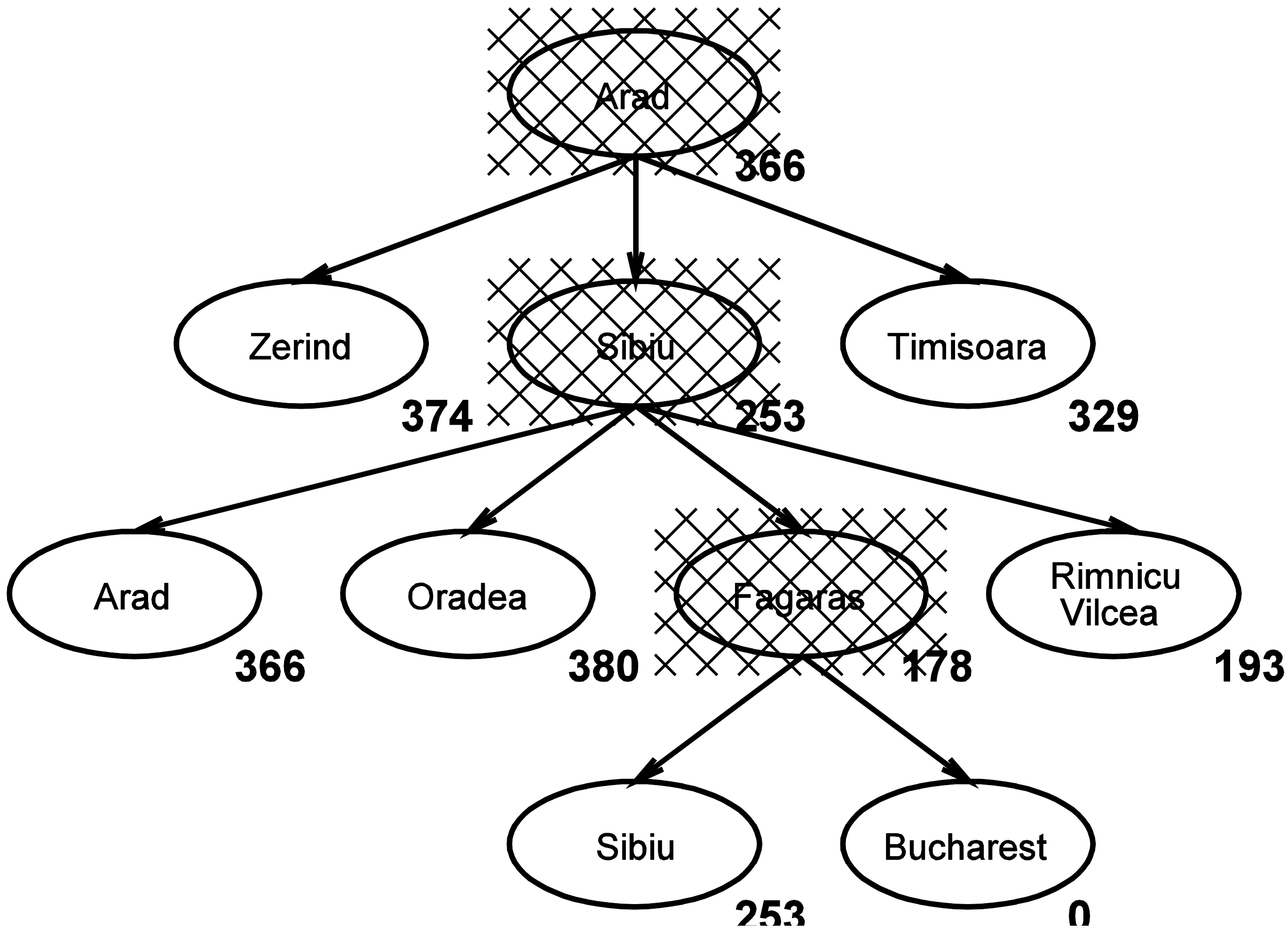
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Time??

Space??

Optimal??

Properties of greedy search

Complete?? No – can get stuck in loops, e.g., Iasi to Fagaras

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

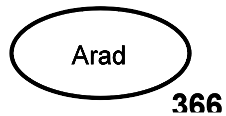
$g(n)$ = cost so far to reach n (path cost)
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an $admissible$ heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the $true$ cost from n.

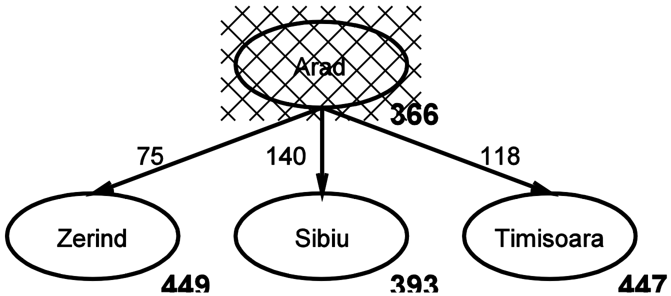
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

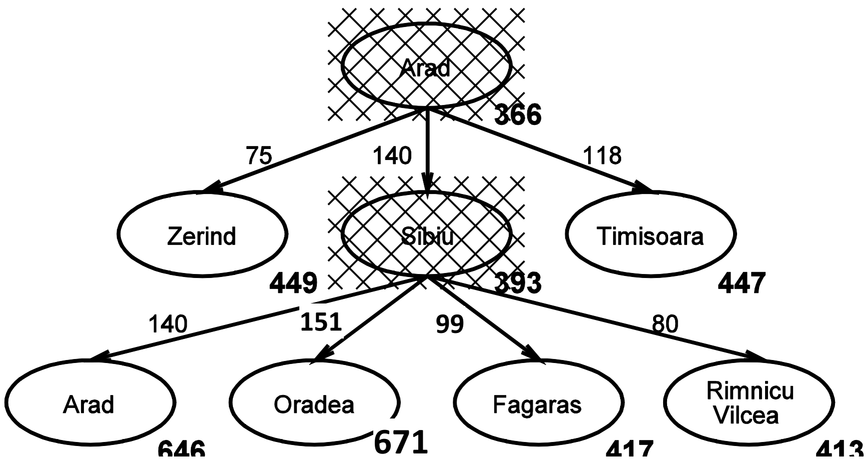
A* search example



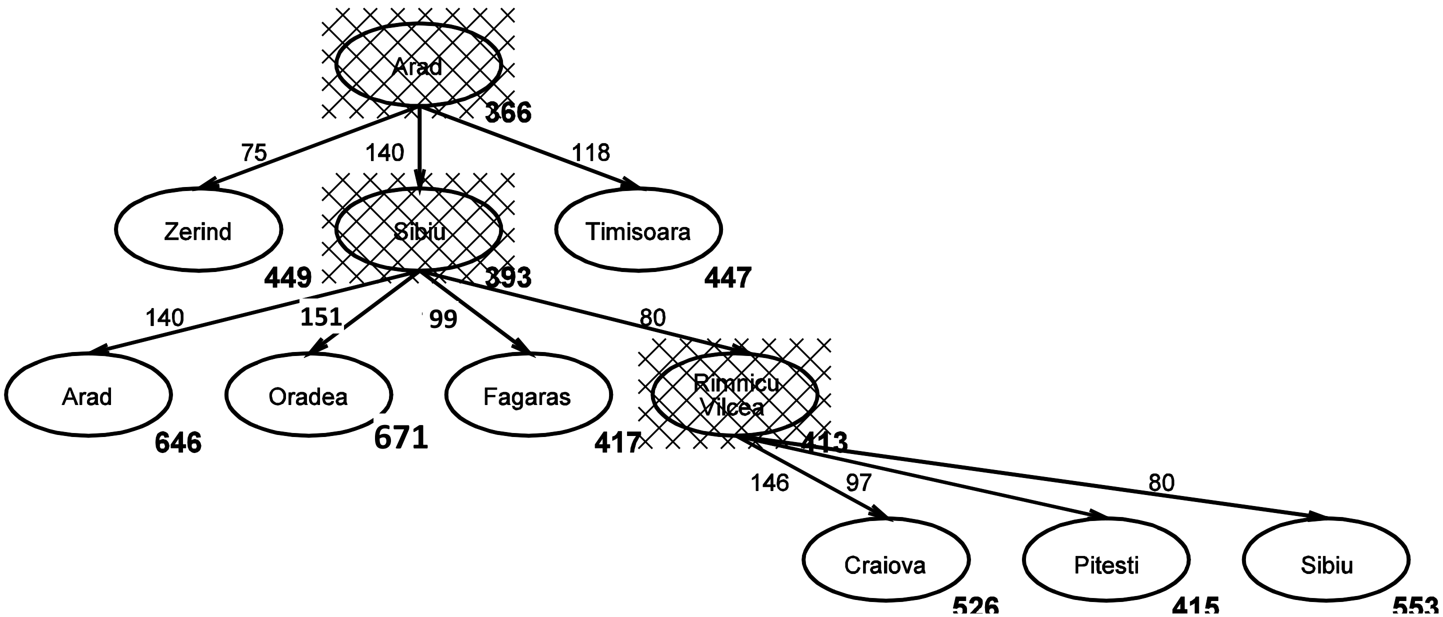
A* search example



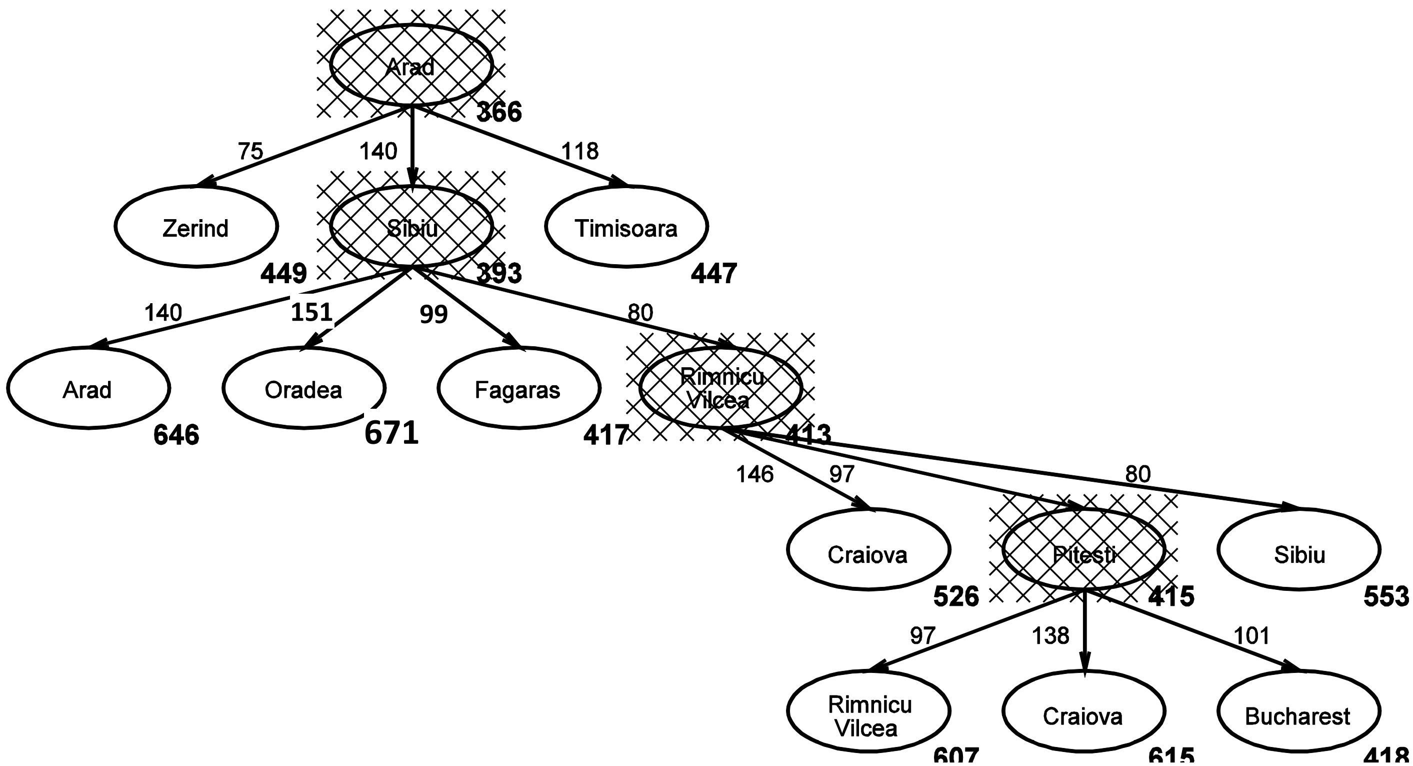
A* search example



A* search example



A* search example



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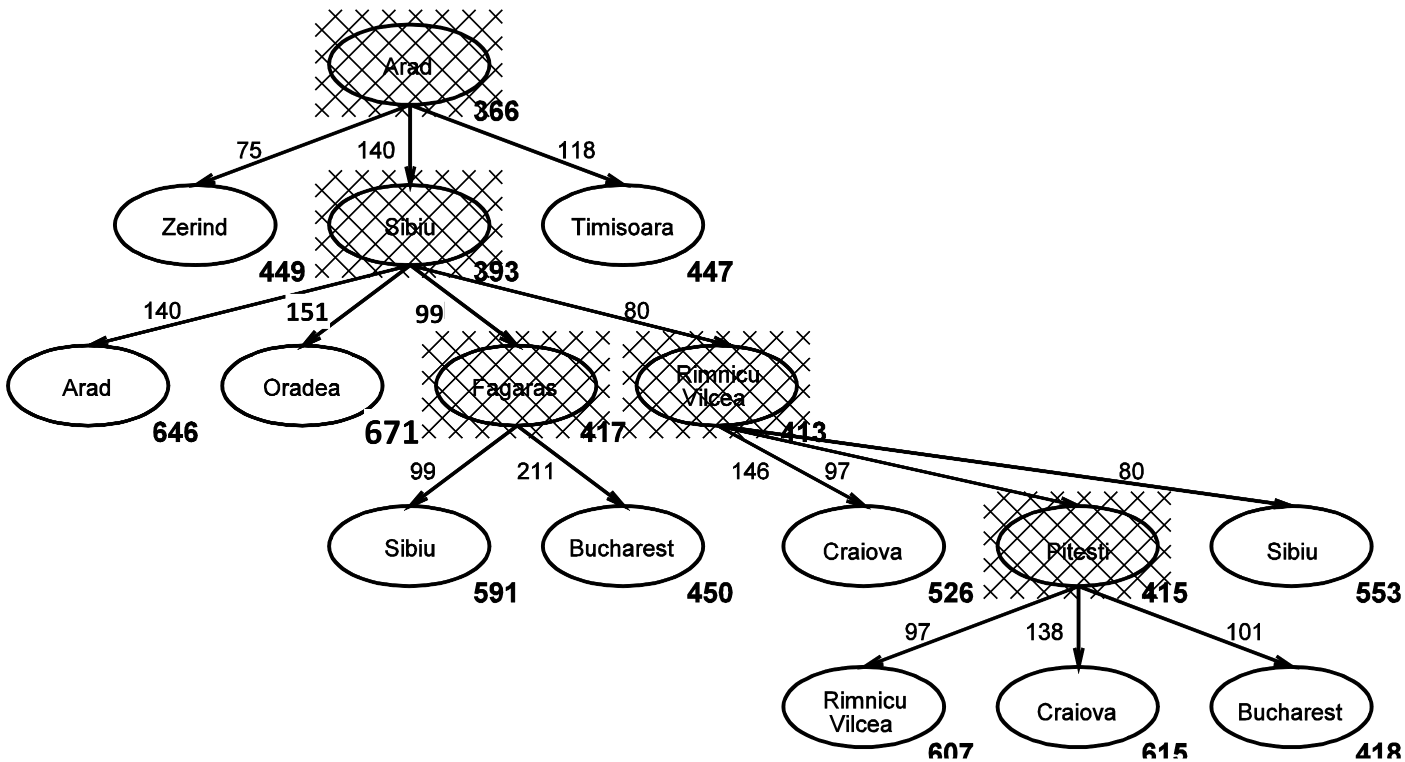
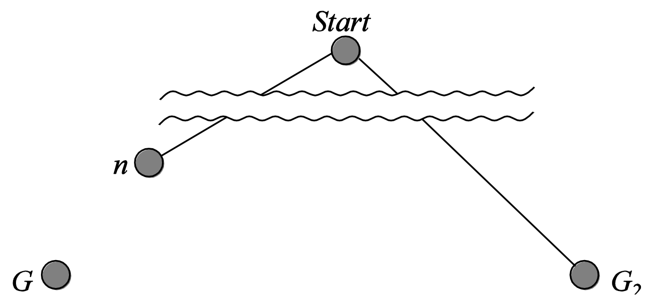


Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G.



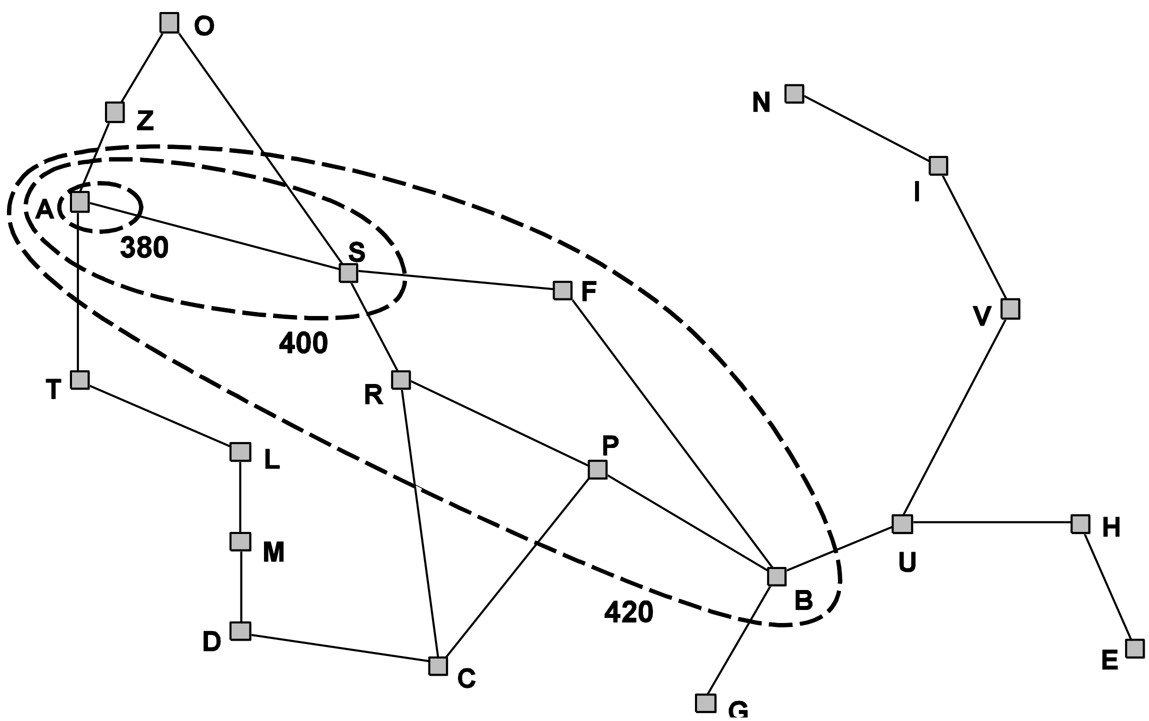
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

The heuristic can control A*'s behaviour

♢ If h(n) is very high relative to g(n), then only h(n) plays a role, and A* turns into ...

♢ If h(n) is 0, then only g(n) plays a role, and A* turns into ...

♢ If h(n) is always lower than (or equal to) the cost of moving from n to the goal, then A* ...

♢ If h(n) is sometimes greater than the cost of moving from n to the goal, then A* ...

♢ If h(n) is exactly equal to the cost of moving from n to the goal, then A* ...

The heuristic can control A*'s behaviour

♢ If h(n) is very high relative to g(n), then only h(n) plays a role, and A* turns into Greedy Best-First-Search.

♢ If h(n) is 0, then only g(n) plays a role, and A* turns into Uniform Cost Search, which finds the optimal solution.

♢ If h(n) is always lower than (or equal to) the cost of moving from n to the goal, then A* is guaranteed to find a shortest path. The lower h(n) is, the more node A* expands, making it slower.

♢ If h(n) is sometimes greater than the cost of moving from n to the goal, then A* is not guaranteed to find a shortest path, but it can run faster.

♢ If h(n) is exactly equal to the cost of moving from n to the goal, then A* will only follow the best path and never expand anything else, making it very fast.

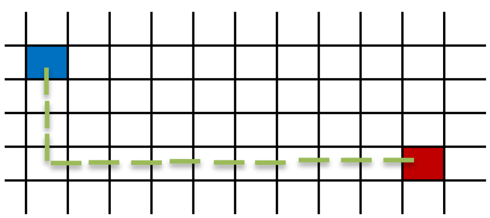
Examples of well-known heuristic functions

♢ Manhattan distance (L_1): On a square grid that allows 4 directions of movement.

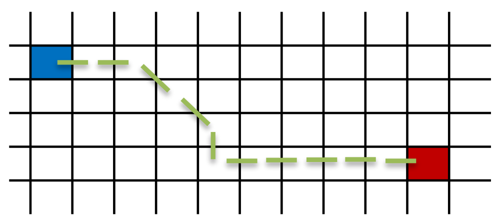
♢ Diagonal distance (L_∞): On a square grid that allows 8 directions of movement.

♢ Euclidean distance (L_2): On a square grid that allows any direction of movement.

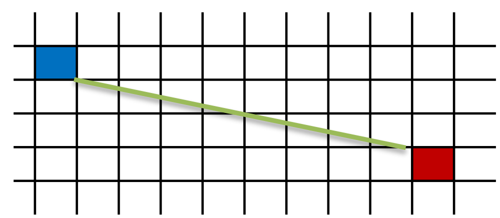
Manhattan Distance



Diagonal Distance



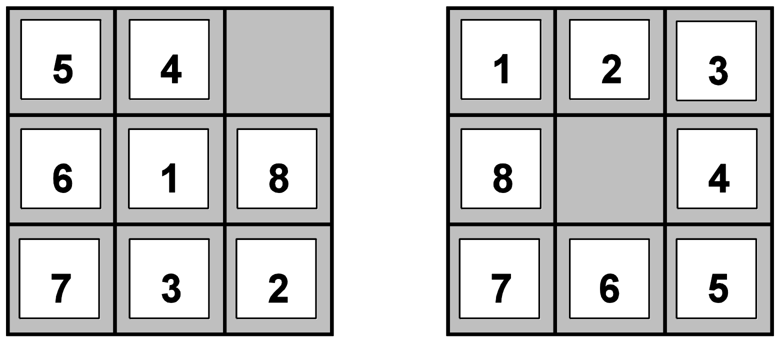
Euclidean Distance



Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$h_1(S) =$??
$h_2(S) =$??

Admissible heuristics

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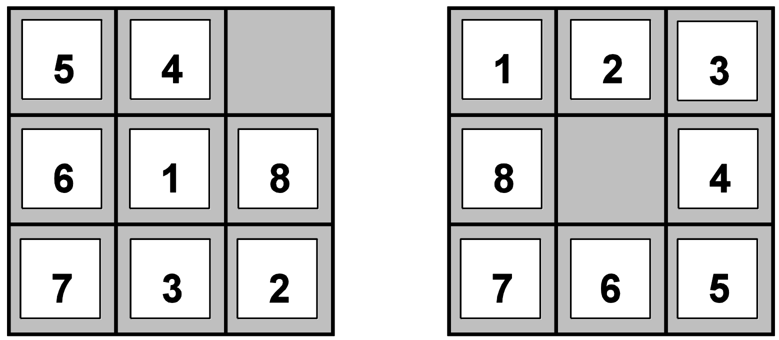


Start State Goal State

$h_1(S) =$?? 7
$h_2(S) =$?? 2+3+3+2+4+2+0+2 = 18

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes
$d = 24$ IDS = too many nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution

Iterative improvement algorithms

In many optimization problems, $path$ is irrelevant;
the goal state itself is the solution

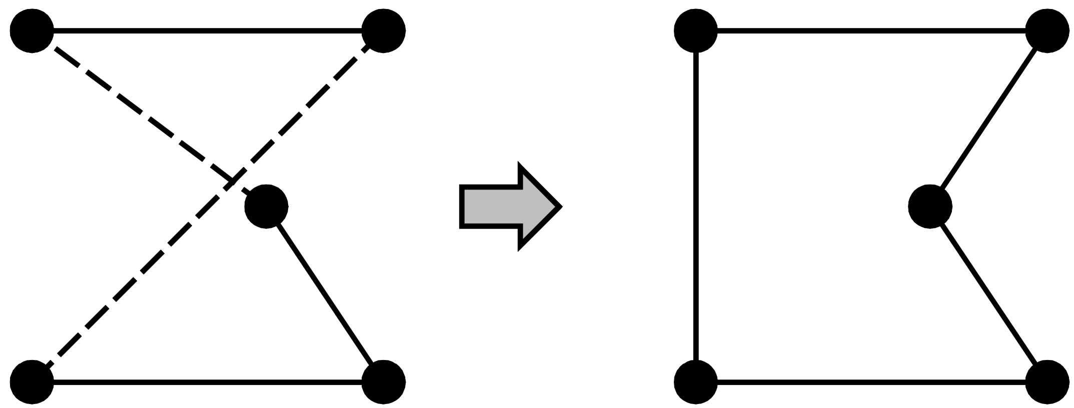
Then state space = set of "complete" configurations;
find $optimal$ configuration, e.g., Travelling Salesperson Problem
or, find configuration satisfying constraints, e.g., n-queens

In such cases, can use $iterative\ improvement$ algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

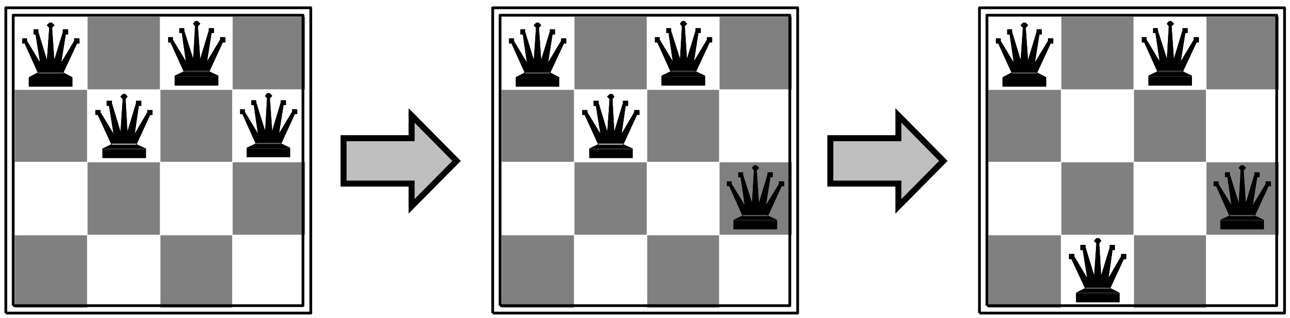
Find the shortest tour that visits each city exactly once



Relaxed problem: let path be *any* structure that connects all cities
$\Longrightarrow$ use minimum spanning tree as heuristic for the TSP

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



Hill-climbing (or gradient ascent/descent)

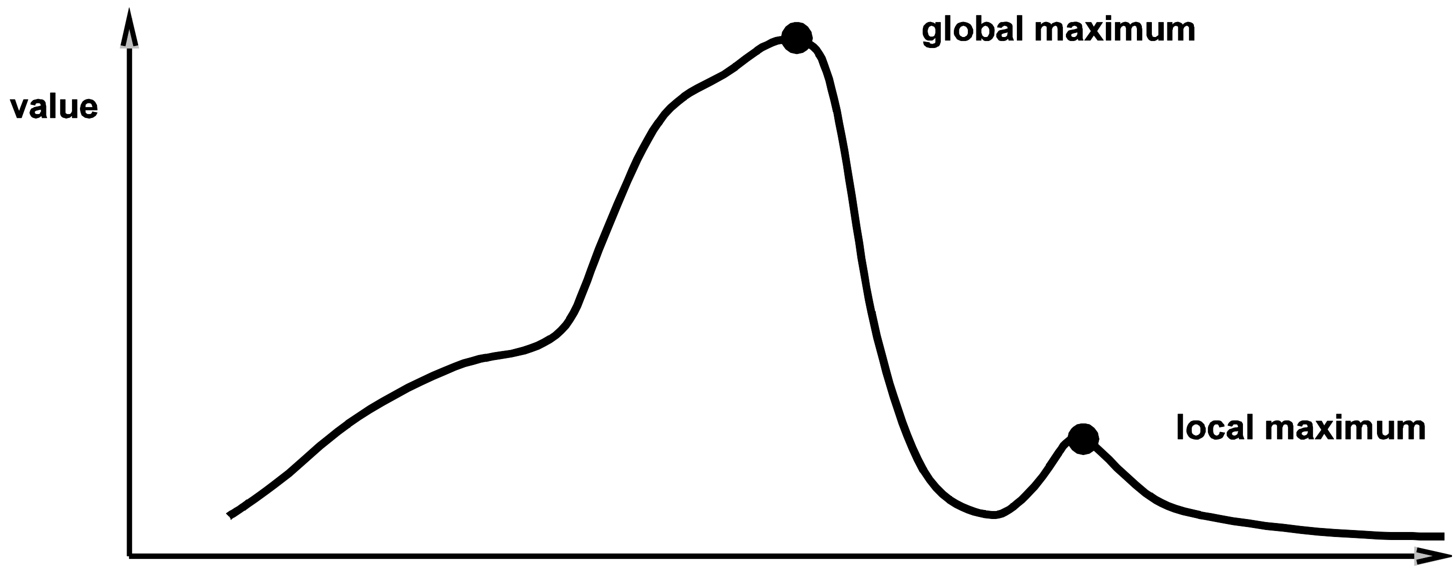
"Like climbing Everest in thick fog with amnesia"

function HILL-CLIMBING(*problem*) **returns** a solution state
  inputs: *problem*, a problem
  local variables: *current*, a node
    next, a node

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  loop do
    next ← a highest-valued successor of *current*
    if VALUE[next] < VALUE[current] **then return** *current*
    current ← *next*
  end

Hill-climbing contd.

Problem: depending on initial state, can get stuck on local maxima



Summary

Heurstics help reduce search cost,
however, finding an optimal solution is still difficult.

Greedy best-first search is not optimal, but can be efficient.

A* search is complete and optimal, but is prohibitive in memory.

Hill-climbing methods operate on complete-state formulations,
require less memory, but are not optimal.

Examples of skills expected:

◇ Demonstrate operation of search algorithms

◇ Discuss and evaluate the properties of search algorithms

◇ Derive and compare heuristics for a problem