

# Planning IV

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#### 50.021 Artificial Intelligence

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### Faster Heuristics: Overall Idea

Concise representation using facts (F) and actions (A)

$$^{\circ} F_0 = x_1, x_2$$

$$\circ$$
 A<sub>0</sub> = O<sub>1</sub>, O<sub>2</sub>

$$\circ$$
  $F_1 = X_1, X_2, X_3, X_4, X_5$ 

$$\circ$$
 A<sub>1</sub> = O<sub>3</sub>, O<sub>4</sub>

$$\circ$$
  $F_2 = x_1, x_2, x_3, x_4, x_5, x_6, x_7$ 

$$^{\circ}$$
 A<sub>2</sub> = o<sub>5</sub>

$$F_3 = X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$$

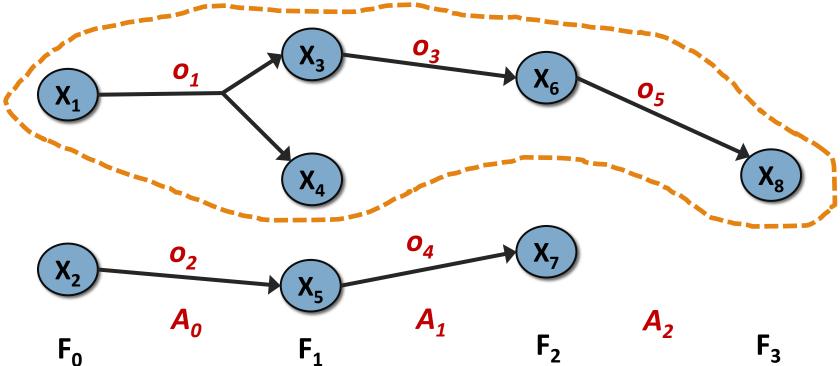
- Why do we care about the level/index of a fact?
  - It tells us the <u>number of actions (i.e., cost) required</u> to achieve that fact
  - This is useful as we examine the h<sub>max</sub>, h<sub>add</sub> heuristics



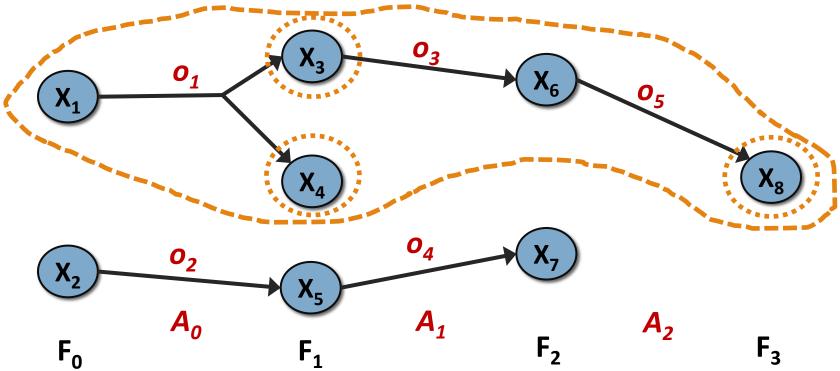
- h<sub>max</sub> is the cost of the single most costly goal fact (out of all goal facts)
  - i.e., the max number of actions needed for achieving one of the goal facts.



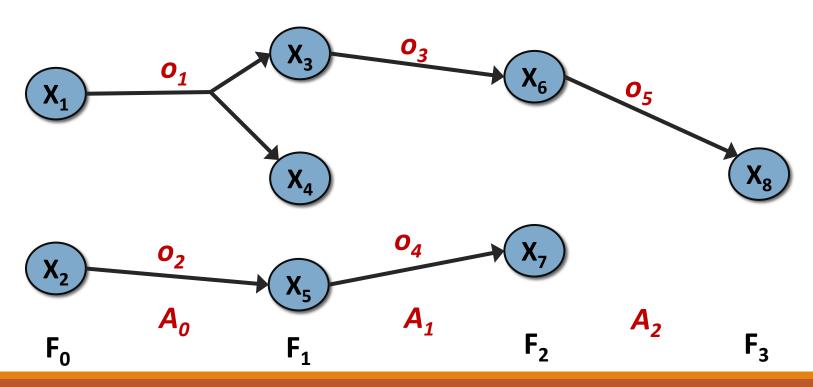
- h<sub>max</sub> is the cost of the single most costly goal fact (out of all goal facts)
  - Given goals  $x_3$ ,  $x_4$ ,  $x_8$ ,  $h_{max} = 3$



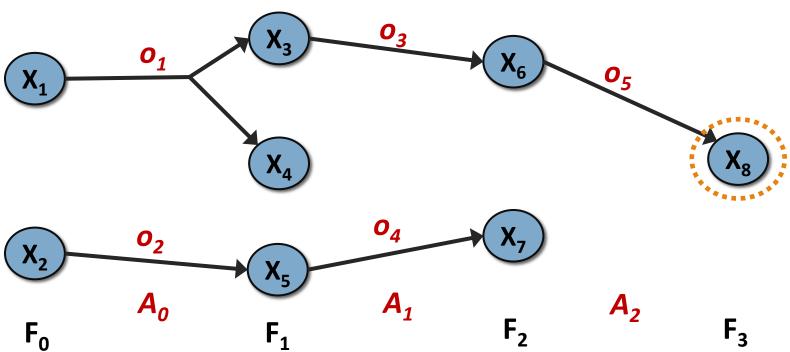
- An optimistic heuristic
  - Implicit assumption is that an action can set multiple facts to true



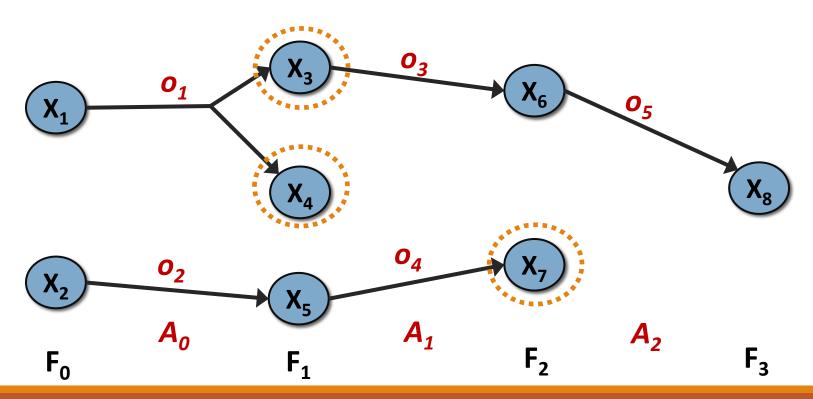
o Is h<sub>max</sub> admissible?



- o Is h<sub>max</sub> admissible? Yes
  - Given goals  $x_8$ ,  $h_{max} = h^*$

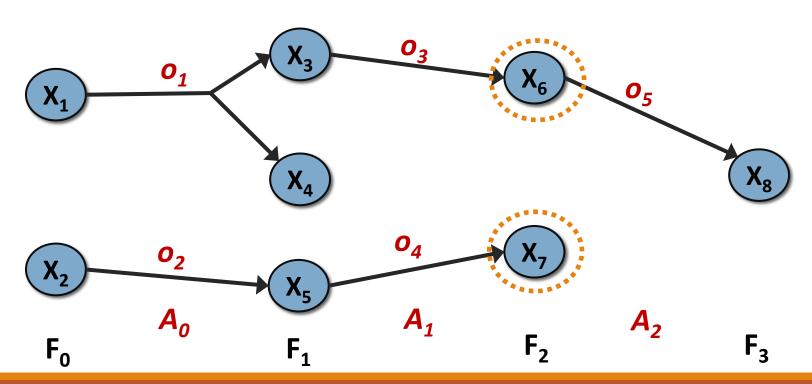


- o Is h<sub>max</sub> admissible? Yes
  - Given goals  $x_3$ ,  $x_4$ ,  $x_7$ ,  $h_{max} < h^*$



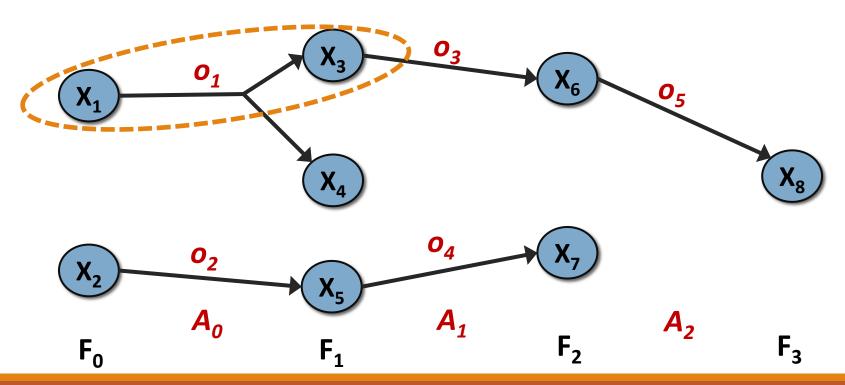


- However, h<sub>max</sub> could under-estimate the true cost (by a lot)
  - What if the goals were x<sub>6</sub>, x<sub>7</sub>?

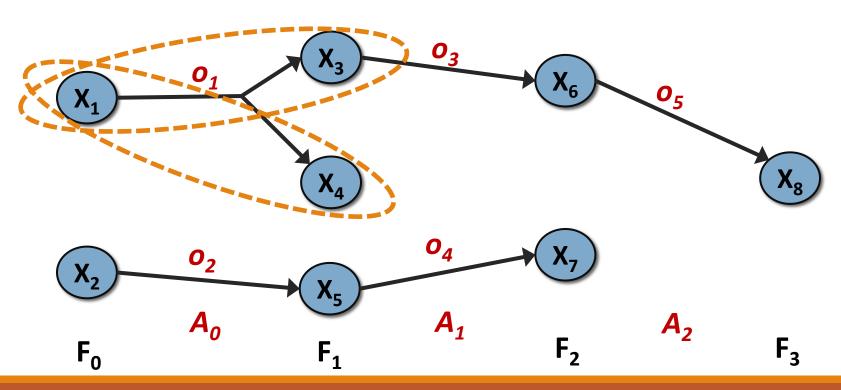


- h<sub>add</sub> is the summed cost of all goal facts
  - Adds up the number of actions needed to set a fact to true

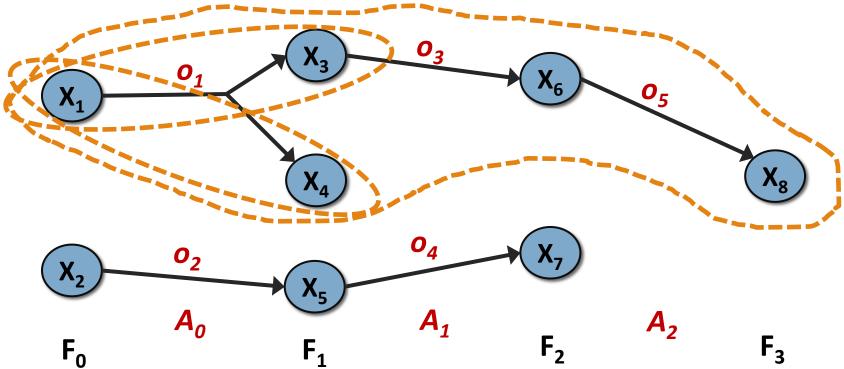
- h<sub>add</sub> is the summed cost of all goal facts
  - Given goals  $x_3$ ,  $x_4$ ,  $x_8$ ,  $h_{add} = 1$



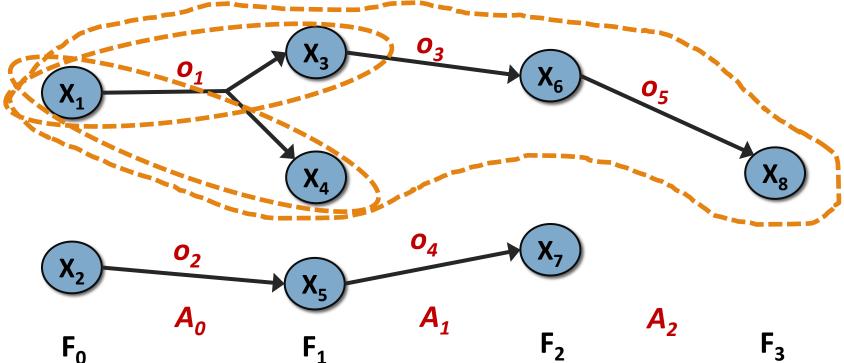
- h<sub>add</sub> is the summed cost of all goal facts
  - Given goals  $x_3$ ,  $x_4$ ,  $x_8$ ,  $h_{add} = 1 + 1$



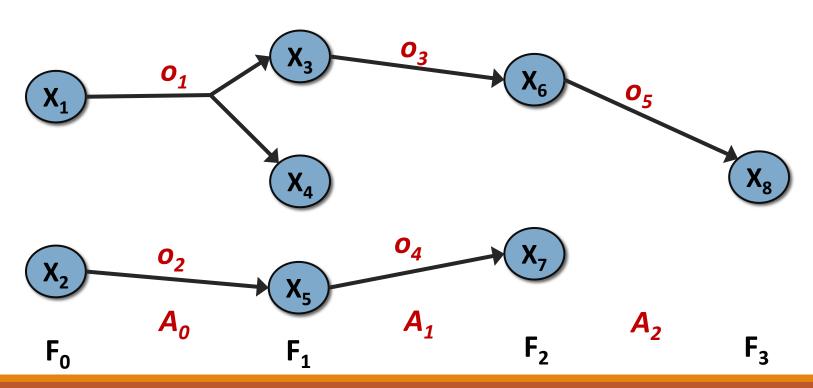
- h<sub>add</sub> is the summed cost of all goal facts
  - Given goals  $x_3$ ,  $x_4$ ,  $x_8$ ,  $h_{add} = 1 + 1 + 3 = 5$



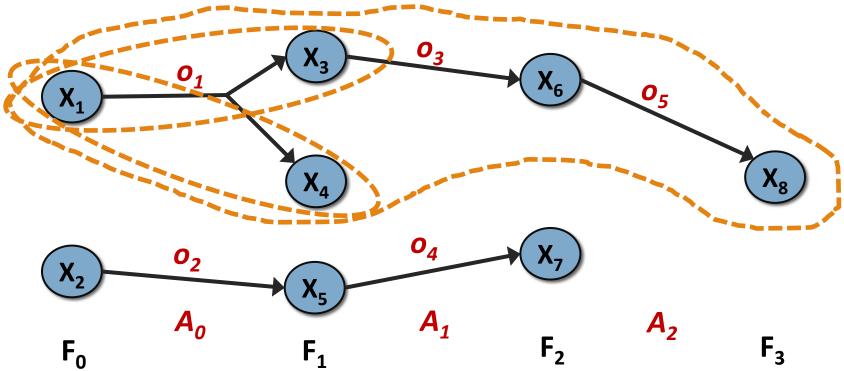
- A pessimistic heuristic
  - Implicit assumption is that an action can only set one fact to true, and vice versa



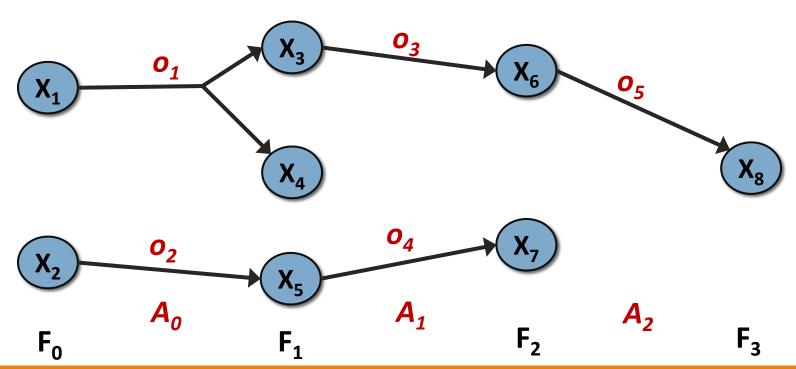
o Is h<sub>add</sub> admissible?



- o Is h<sub>add</sub> admissible? No
  - Given goals  $x_3$ ,  $x_4$ ,  $x_8$ ,  $h_{add} > h^*$



- o Given goals  $x_3$ ,  $x_4$ ,  $x_7$ ,  $x_8$ , calculate  $h_{max}$  and  $h_{add}$ ?
- $\circ$  Given goals  $x_6$ ,  $x_7$ , calculate  $h_{max}$  and  $h_{add}$ ?



- o Given goals  $x_3$ ,  $x_4$ ,  $x_7$ ,  $x_8$ , calculate  $h_{max}$  and  $h_{add}$ ?  $h_{max} = 3$ ,  $h_{add} = 7$
- o Given goals  $x_6$ ,  $x_7$ , calculate  $h_{max}$  and  $h_{add}$ ?
- Concise representation using facts (F) and actions (A)

$$^{\circ} F_0 = x_1, x_2$$

$$\circ$$
 A<sub>0</sub> = O<sub>1</sub>, O<sub>2</sub>

$$\circ$$
  $F_1 = x_1, x_2, x_3, x_4, x_5$ 

$$\circ$$
 A<sub>1</sub> = O<sub>3</sub>, O<sub>4</sub>

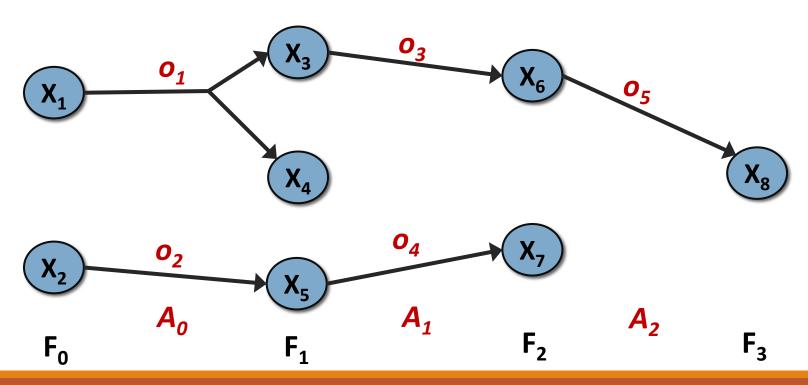
$$\circ$$
  $F_2 = x_1, x_2, x_3, x_4, x_5, x_6, x_7$ 

$$^{\circ}$$
 A<sub>2</sub> = o<sub>5</sub>

$$F_3 = X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$$

- o Given goals  $x_3$ ,  $x_4$ ,  $x_7$ ,  $x_8$ , calculate  $h_{max}$  and  $h_{add}$ ?  $h_{max} = 3$ ,  $h_{add} = 7$

• Given goals  $x_6$ ,  $x_7$ , calculate  $h_{max}$  and  $h_{add}$ ?



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- o Given goals  $x_3$ ,  $x_4$ ,  $x_7$ ,  $x_8$ , calculate  $h_{max}$  and  $h_{add}$ ?  $h_{max} = 3$ ,  $h_{add} = 7$
- o Given goals  $x_6$ ,  $x_7$ , calculate  $h_{max}$  and  $h_{add}$ ?  $h_{max} = 2$ ,  $h_{add} = 4$
- Concise representation using facts (F) and actions (A)
  - $^{\circ} F_0 = x_1, x_2$
  - $\circ$  A<sub>0</sub> = O<sub>1</sub>, O<sub>2</sub>
  - $\circ$   $F_1 = X_1, X_2, X_3, X_4, X_5$
  - $^{\circ}$  A<sub>1</sub> = 0<sub>3</sub>, 0<sub>4</sub>
  - $\cdot F_2 = X_1, X_2, X_3, X_4, X_5, X_6, X_7$
  - $\circ$   $A_2 = o_5$
  - $\circ$   $F_3 = X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$

## h<sub>max</sub> versus h<sub>add</sub>

- h<sub>max</sub> is too optimistic tends to underestimate true cost
  - Only considers the longest path of actions (assumes that all goals are on that path)
  - Ignores that there can be partially non-overlapping or independent paths of actions leading to multiple goals.
- h<sub>add</sub> is too pessimistic tends to overestimate true cost
  - Assumes that there are only independent paths leading to each goal
  - However, multiple goals or facts can exist on a single path of action
- In reality, paths of actions can be partially independent, and partially overlapping/shared



### Summary

- Discussed planning problem formulation using STRIPS
- Discussed a relaxation to the planning problem by removing negations
- Discussed various heuristics:
  - h+ is the optimal plan to a delete-relaxed problem
  - h<sub>max</sub> is admissible, but often too loose, too low.
  - h<sub>add</sub> is inadmissible, sometimes way too large compared to h+ which we want to approximate
  - h<sub>FF</sub> is inadmissible, but in practice useful



o Given this problem definition:

```
• Variables: x_1, x_2, x_3, x_4, x_5, x_6
```

Initial State: x<sub>1</sub>

• Goal:  $x_2, x_5, x_6$ 

• Actions:  $o_1$ : precond:  $x_1$ , postcond:  $x_2$ 

 $o_2$ : precond:  $x_2$ , postcond:  $-x_1$ ,  $x_3$ 

 $o_3$ : precond:  $x_3$ , postcond:  $-x_2$ ,  $x_1$ ,  $x_4$ 

 $o_4$ : precond:  $x_2$ ,  $x_4$ , postcond:  $x_5$ 

 $o_5$ : precond:  $x_1$ ,  $x_5$ , postcond:  $\neg x_5$ ,  $x_6$ 

- Task: Make this a delete-relaxed problem
- Task: Determine the level where all goal facts are achieved
- Task: Calculate the h<sub>max</sub> and h<sub>add</sub> heuristics



o Given this problem definition:

```
• Variables: x_1, x_2, x_3, x_4, x_5, x_6
```

Initial State: x<sub>1</sub>

• Goal:  $x_2, x_5, x_6$ 

• Actions:  $o_1$ : precond:  $x_1$ , postcond:  $x_2$ 

 $o_2$ : precond:  $x_2$ , postcond:  $-x_1$ ,  $x_3$ 

 $o_3$ : precond:  $x_3$ , postcond:  $-x_2$ ,  $x_1$ ,  $x_4$ 

 $o_4$ : precond:  $x_2$ ,  $x_4$ , postcond:  $x_5$ 

 $o_5$ : precond:  $x_1$ ,  $x_5$ , postcond:  $-x_5$ ,  $x_6$ 

Task: Make this a delete-relaxed problem



o Given this problem definition:

```
• Variables: x_1, x_2, x_3, x_4, x_5, x_6
```

Initial State: x<sub>1</sub>

• Goal:  $x_2, x_5, x_6$ 

• Actions:  $o_1$ : precond:  $x_1$ , postcond:  $x_2$ 

 $o_2$ : precond:  $x_2$ , postcond:  $-x_1$ ,  $x_3$ 

 $o_3$ : precond:  $x_3$ , postcond:  $-x_2$ ,  $x_1$ ,  $x_4$ 

 $o_4$ : precond:  $x_2$ ,  $x_4$ , postcond:  $x_5$ 

 $o_5$ : precond:  $x_1$ ,  $x_5$ , postcond:  $-x_5$ ,  $x_6$ 

Task: Determine the level where all goal facts are achieved

Concise representation using facts (F) and actions (A)

$$^{\circ}$$
  $F_0 = x_1$ 

$$^{\circ}$$
 A<sub>0</sub> = o<sub>1</sub>,

$$F_1 = X_1, X_2$$

$$\circ$$
 A<sub>1</sub> = O<sub>2</sub>

$$F_2 = X_1, X_2, X_3$$

$$^{\circ}$$
 A<sub>2</sub> = O<sub>3</sub>

$$F_3 = X_1, X_2, X_3, X_4$$

$$\circ$$
 A<sub>4</sub> = O<sub>4</sub>

$$F_4 = X_1, X_2, X_3, X_4, X_5$$

$$\circ$$
 A<sub>5</sub> = o<sub>5</sub>

$$\circ$$
  $F_5 = x_1, x_2, x_3, x_4, x_5, x_6$ 

```
Variables: x_1, x_2, x_3, x_4, x_5, x_6

Initial State: x_1

Goal: x_2, x_5, x_6

Actions: o_1: pre: x_1, post: x_2

o_2: pre: x_2, post: -x_1, x_3

o_3: pre: x_3, post: -x_2, x_1, x_4

o_4: pre: x_2, x_4, post: x_5

o_5: pre: x_1, x_2, post: -x_3, x_4
```

Concise representation using facts (F) and actions (A)

```
^{\circ} F_0 = x_1
```

$$^{\circ}$$
 A<sub>0</sub> = o<sub>1</sub>,

$$^{\circ} F_1 = x_1, x_2$$

$$\circ$$
 A<sub>1</sub> = O<sub>2</sub>

$$F_2 = X_1, X_2, X_3$$

$$\circ$$
  $A_2 = o_3$ 

$$\circ$$
  $F_3 = X_1, X_2, X_3, X_4$ 

$$\circ$$
 A<sub>4</sub> = O<sub>4</sub>

$$F_4 = X_1, X_2, X_3, X_4, X_5$$

$$\circ$$
 A<sub>5</sub> = O<sub>5</sub>

$$\circ$$
  $F_5 = x_1, x_2, x_3, x_4, x_5, x_6$ 

```
Variables: x_1, x_2, x_3, x_4, x_5, x_6

Initial State: x_1

Goal: x_2, x_5, x_6

Actions: o_1: pre: x_1, post: x_2

o_2: pre: x_2, post: -x_1, x_3

o_3: pre: x_3, post: -x_2, x_1, x_4

o_4: pre: x_2, x_4, post: x_5

o_5: pre: x_1, x_5, post: -x_5, x_6
```

Task: Calculate the  $h_{max}$  and  $h_{add}$  heuristics

#### Concise representation using facts (F) and actions (A)

$$_{0}$$
  $F_{0}$  =  $x_{1}$ 

$$^{\circ}$$
 A<sub>0</sub> = o<sub>1</sub>,

$$F_1 = X_1, X_2$$

$$^{\circ}$$
 A<sub>1</sub> = 0<sub>2</sub>

$$F_2 = X_1, X_2, X_3$$

$$^{\circ}$$
 A<sub>2</sub> = o<sub>3</sub>

$$\circ$$
  $F_3 = X_1, X_2, X_3, X_4$ 

$$\circ$$
 A<sub>4</sub> = O<sub>4</sub>

$$F_4 = X_1, X_2, X_3, X_4, X_5$$

$$^{\circ}$$
 A<sub>5</sub> = o<sub>5</sub>

$$\circ$$
 F<sub>5</sub> = x<sub>1</sub>, x<sub>2</sub>, x<sub>3</sub>, x<sub>4</sub>, x<sub>5</sub>, x<sub>6</sub>

Variables: 
$$x_1, x_2, x_3, x_4, x_5, x_6$$
  
Initial State:  $x_1$   
Goal:  $x_2, x_5, x_6$   
Actions:  $o_1$ : pre:  $o_1$ : pre:  $o_2$ : pre:  $o_2$ : pre:  $o_3$ 

#### Task: Calculate the $h_{max}$ and $h_{add}$ heuristics

• 
$$h_{max} = max(1, 4, 5) = 5$$

• 
$$h_{add} = 1 + 4 + 5 = 10$$



### Overall Summary

- Able to describe planning problems in a simple formalism that is finding a sequence of actions.
- Able to formulate a planning problem as a STRIPS instance
- Understand the PDDL planning language
- Develop relaxed versions of planning problems and understand the various heuristics that can be used