J. P. Morgan Quant Mentorship Program 2022

Deadline for submission: June 13th, Monday 2359 Hrs.

General Instructions:

This document contains case study on Monte Carlo Simulations and option pricing with four questions. You are expected to attempt all the questions. It is a team challenge and you're all expected to work in your allocated teams.

This case study introduce the basics of a problem and pose a few questions on them. A question might have multiple sub-parts, all of which need to be answered. The scores for the sub-parts have been indicated against the question. The solution format for each question has been specified alongside the question, In general please provided the code with comments and explain your findings & approach as well.

Please prefer to code in python but in case you're not familiar then use language of your choice. Do not import direct libraries for Monte Carlo & Black Scholes. Limit yourself to use commonly used packages such as Numpy, Pandas and Matplotlib, Seaborn and Plotly.

Please submit in a zip file containing code with file name as the question number and pdf showing your output, plot (if any) along with your thoughts on the result. Please write comments in code wherever necessary. Please use '**Quant Mentorship Case Study 2022 Submission Team <TeamNo.>**' as subject for your submission email. **Only 1 submission per team**

Please note, any other format of submission or any file other than zip will not be evaluated. Only submit in the required format. Keep aside some time before the deadline to be able to successfully submit the case study

You are free to use online resources to improve your understanding of the problem statement. **Plagiarism of any kind will not be tolerated**.

You may reach out to **jpmqrmentorship.mumbai@jpmorgan.com** throughout the duration of the challenge for any kinds of queries/doubts. Please use '**Quant Mentorship Case Study 2022 Doubt**' as subject line for a quick response.
All the best!

Monte Carlo Simulation:

Monte Carlo Simulation (or Method) is a probabilistic numerical technique used to estimate the outcome of a given, uncertain (stochastic) process. This means it's a method for simulating events that cannot be modeled implicitly. This is usually a case when we have random variables in our processes.

But How to perform simulations in real time?

On an imaginary Island where the total population is 50K, we need to compute the average height of the inhabitants. Let's call this variable H; and we know that it follows uniform distribution ~ U [5, 6] units.

This can be solved analytically as we know that the average of uniform variable is equal to the (higher bound + lower bound)/2 which in this case yields to be 5.5.

Simulations can also come handy when dealing with random variables as we will see in this case. When a closed form solutions doesn't exist; which is very common in finance, we usually rely on numerical methods to estimate the target variable.

In the following example, we can simulate the heights of the inhabitants and then take the mean of the heights. Where our input variable is the uniformly distributed variable.

Code:

```
>>> ### The following command, np.random.uniform will generate 50000 uniform random numbers between 5 and 6
... arr = np.random.uniform(low = 5.0, high = 6.0, size = 50000)
>>> arr.mean()
5.4994610108700641
```

Expected Value:

The expected value formula is the probability of an event multiplied by the value of the event (An event can be anything from the number of time it appears to the dollar value associated with it):
E(f(x)) = Summation of (P(x) * f(x)) for all x
e.g. You are playing a dice game, where if you roll a six you win 100 rupees and you lose 10 rupees if anything else happens.
x: be the event that 6 shows up, P(x) = 1/6
y: be the event that any number other than 6 shows up, P(y) = 5/6
So the expected winnings for the game will be:
1/6*100 + 5/6*(-10) = 50/6

Key Points to Note:

1. If you have written the **code**, please **provide us with all the code files.**
2. **Submit all your manual calculations for each problem you attempt in the answers report.**

Q1) Estimate the value of pi using Monte Carlo Simulations. (3 points)
Do it for a number of trials starting ranging from 10 to the number of runs until you finally converge to a single value. Plot the estimated Value vs Number of Runs. Explain your findings with necessary comments supporting your code (5 points)

Q2) Consider a company "XYZ capital" currently trading at 100 rupees(at time = t0) on the National Stock Exchange of Vol-land and it moves up by 1 rupee or moves down by the same amount with equal probability at each minute.

a) What will be the expected value of stock price after: (4 points, 1 points each)
   i) 1 minute
   ii) 10 minutes
   iii) 1 hour
   iv) 1 month

   Are all the values same? Please explain on why they should or should not be same (5 points)

b) What will the probability of the stock price going to 102 before going to 96,
   i) Solve it analytically? (3 points)
   ii) Find the probability using Monte Carlo method, how many runs of a single simulation will get you to the converged value. Plot the estimated Value vs Number of Runs (5 points)
   NOTE: Tolerance limit between analytical and numerical solution is 10^-2
   Solution Format: Please attach your code with comments, describe your analytical solution as well.

Present Value: Present value (PV) is the current value of a future sum of money or stream of cash flows given a specified rate of return

$$PV = FV \frac{1}{(1+r)^n}$$

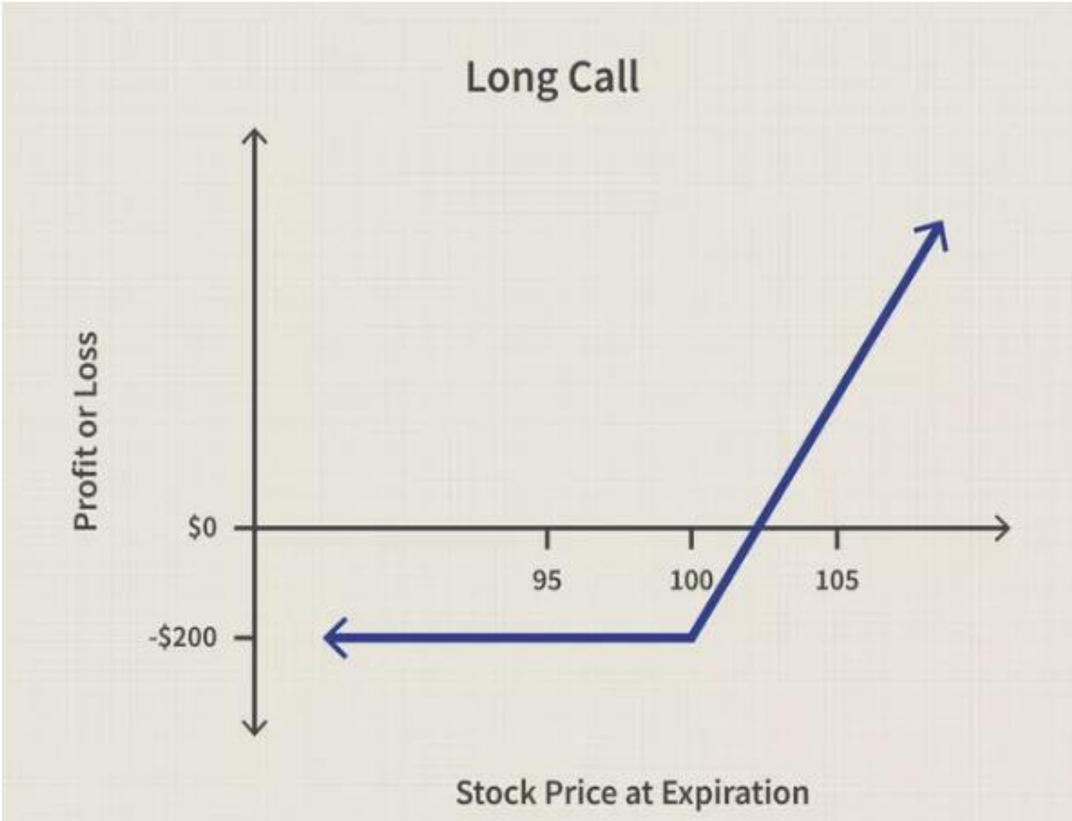
PV : Present Value
FV : Future Value
r : rate of return
n : number of periods

Option: An option is a contract which conveys to its owner, the holder, the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified price(known as strike price) on or before a specified date(known as expiration date)

For this exercise let’s focus on a call option (already discussed in derivatives session)

Call Option: Call options are financial contracts that give the option buyer the right but not the obligation to buy a stock, bond, commodity, or other asset or instrument at a specified price within a specific time period.

Pay-off of a call option: Max(0, spot price - strike price)



Pricing an Option:

Theoretically the price of an option should be the present value of expected payoff of the option at expiry.
There are several ways to price an option

a) Using closed form analytical solutions, such as Black-Scholes model
b) Using numerical techniques such as Monte-Carlo simulations, Binomial tree pricing, finite differencing.

For this exercise we will be looking at Black-Scholes and Monte Carlo simulations.

To define a unique option contract we need the following things:

1) Strike Price: K
2) Spot Price: S(t) at time t
3) Implied Vol: V (for this exercise we will assume it to be constant)
4) Interest Rate: rf (risk free rate)
5) Time To expiration: T

Pricing of a call option

Here is the Black-Scholes Formula:

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-rT}$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$C(S,t)$	(call option price)
$N(\,)$	(cumulative distribution function)
$T = (T_1 - t)$	(time left til maturity (in years))
S	(stock price)
K	(strike price)
r	(risk free rate)
σ	(volatility)

Q3) Consider a company “XYZ capital” currently trading at 200 rupees (at time = t0) on the National Stock Exchange of Vol-land. We have an option contract with the following parameters:

1. Strike: 180
2. Time To expiration: 30 days (1/12th of a year)
3. Implied Vol: 15%
4. Interest Rate: 2%
5. Current Stock Price: 200

Compute the value of the above described option contract using Black Scholes formula. Note that you can rely on stats tools & packages for this exercise but are not allowed to directly import the black Scholes library if available on the open source platform. (15 points)

Q4) as we discussed above the price of an option should be the present value of expected payoff of the option. Now consider a case where we know how the underlying asset moves with time and we also know how to compute the payoff of an option.

Here is the equation on how the underlying asset moves with time. Please compute the value (don’t forget to compute the present value) of option with the same parameters as question 4. Also plot the value of the option vs number of runs. (15+10 points)

Equation:-

S(t) = S(t-1)*exp((u-0.5*sigma**2)*dt + sigma*N(0,1)*sqrt(dt))

Where dt = t – (t-1)
Sigma = Implied Vol
N(0,1) = Standard Normal variable

PFA the equation snippet as well:

$$S(\Delta t) = S(0)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \left(\sigma\sqrt{\Delta t}\right)\varepsilon\right]$$

Where Epsilon = Standard Normal Variable same as N(0, 1)

NOTE: You will have to define another variable time step (T- (T-1)) to discretize the process and simulate your paths. Each path will consist of list of stock prices, ideally we should evaluate the stock price continuously but in numerical techniques, we have to perform discretization, so please take 240 data points (time steps) for a single path. Tolerance limit for the difference between analytical solution (bs model) and numerical (monte carlo) is 10^-2