Presenting: 'Cryptographic primitives based on hard learning problems: Blum, Furst, Kearns, Lipton'

Vishvas Vasuki

April 19, 2009

Outline

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- A new definition for hardness of learning.

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- A pseudorandom bit generator using hard to learn class of functions.

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- Computer wins if it succeeds with good probability.

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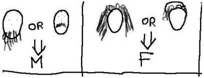
- Unknown classification function (concept) $c : \{0, 1\}^n \rightarrow \{M, F\}$. Eg: 

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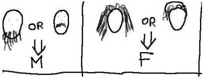
- Unknown classification function (concept) $c : \{0,1\}^n \rightarrow \{M, F\}$. Eg: 

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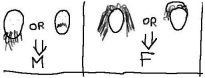
- Unknown classification function (concept) $c : \{0,1\}^n \rightarrow \{M, F\}$. Eg: 

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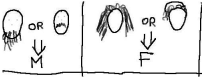
- Unknown classification function (concept) $c : \{0,1\}^n \rightarrow \{M, F\}$. Eg: 

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- Now, classify test set: $\{s'_1, s'_2, ..\}$. 

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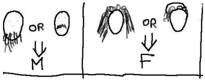
- Fix some $c \in C_n$. 

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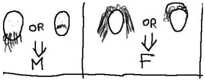
- Fix some $c \in C_n$. 

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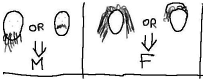
- Fix some $c \in C_n$. 

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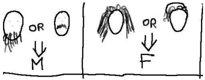
- Fix some $c \in C_n$. 

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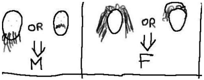

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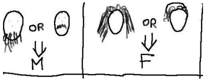
- Fix some $c \in C_n$. 

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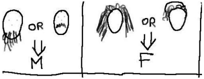

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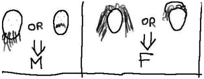

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- Take (P_n, D_n). Pick classifier c using P_n. Pick many examples using D_n. Your alg cannot match c(x) with non negligible probability. 

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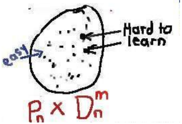

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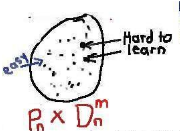

- If 'Learning C is hard' were a cryptographic assumption, any proof of security built on this assumption would be **worthless**. L is strong enough to break this assumption, by cryptographic standards.

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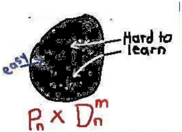
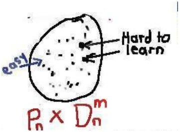
The new defintion, pictorially

'Hard to learn': 

'Learnable': 

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- Now, construct a PRBG.

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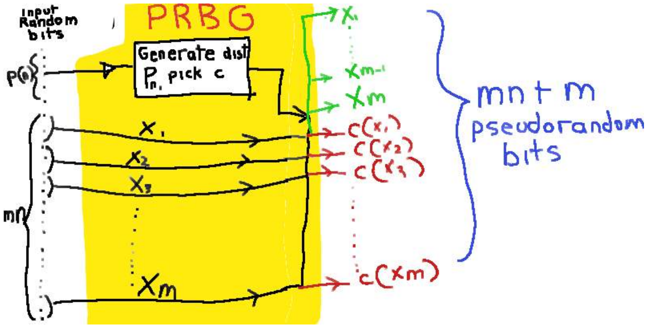

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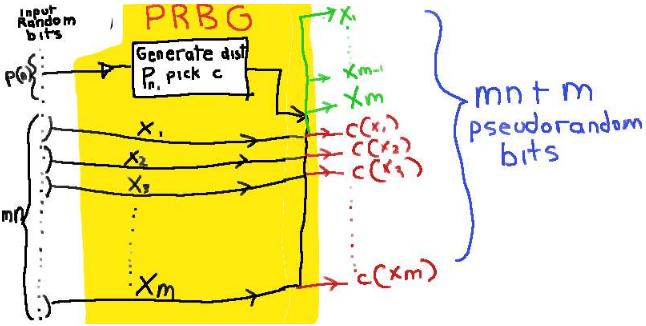

- Proof by contradiction: If you could break this PRBG, C_n not hard to learn wrt $(P_n, U(\{0,1\}^n)$.
- 11100111001111000001011001... *'Can I predict the next bit?'*

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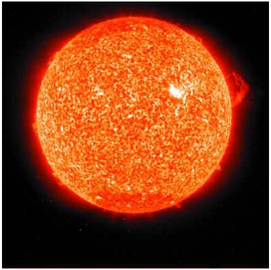
- They take more pains to relate the circuit size and depth required to evaluate functions in hard to learn C_n with the circuit depth and size of the primitives generated.

The take home message



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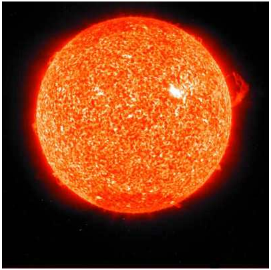

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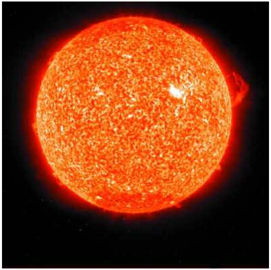

- Can use hardness of learning, properly defined, as a cryptographic assumption.
- Can generically make pseudorandom bit generator from hard to learn but easy to evaluate classes of functions.

Bye!