Confidence Interval

Part (a): Confidence Interval Using Sample Standard Deviation

When the population standard deviation is unknown, we use the sample standard deviation and the t-distribution to construct the confidence interval. Here are the steps to follow:

1. **Calculate the Sample Mean and Sample Standard Deviation:**
    - **Sample Mean (x¯\bar{x}x¯)**:

$$\bar{x} = \frac{\sum x_i}{n}$$

where xi are the sample observations and n is the sample size.

- **Sample Standard Deviation (s)**:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

Data: 1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

- **Mean Calculation**:

x¯=1.13+1.55+1.43+0.92+1.25+1.36+1.32+0.85+1.07+1.48+1.20+1.33+1.18+1.22+1.29/15

x¯≈18.85/15 ≈ 1.257

Mean =1.257

- **Standard Deviation Calculation**: First, calculate each squared deviation from the mean, sum them, and then divide by n−1 and take the square root.

s≈∑(xi−1.257)2/14 ≈ 0.233s

Standard deviation = 0.233s

Determine the t-Score for a 99% Confidence Interval:

For a 99% confidence level with n−1=14 degrees of freedom, use a t-table or calculator to find the critical t-value (t*)

- For α=0.01 and df=14 , t∗≈2.977

2. **Construct the Confidence Interval:**

The formula for the confidence interval is:

$$\mathrm{CI} = \bar{x} \pm t^* \left(\frac{s}{\sqrt{n}}\right)$$

where x¯ is the sample mean, s is the sample standard deviation, and n is the sample size.

- **Margin of Error Calculation**:

$$\mathrm{ME} = t^* \left(\frac{s}{\sqrt{n}}\right) = 2.977 \times \left(\frac{0.233}{\sqrt{15}}\right) \approx 0.122$$

- **Confidence Interval**:

$$\mathrm{CI} = 1.257 \pm 0.122$$

$$\mathrm{CI} \approx [1.135, 1.379]$$

Part (b): Confidence Interval Using Known Population Standard Deviation

When the population standard deviation is known, we use the normal distribution (Z-distribution) instead of the t-distribution. Here's how to do it:

1. **Calculate the Sample Mean:**

This remains the same as in Part (a):

x¯≈1.257

2. **Use the Known Population Standard Deviation (σ\sigmaσ)**:
   - Given σ=0.2 million characters.
3. **Determine the Z-Score for a 99% Confidence Interval:**

For a 99% confidence level, use the Z-table to find the critical Z-value (Z*).

   - For α=0.01, Z∗≈2.576Z.

4. **Construct the Confidence Interval:**

The formula for the confidence interval is:

$$\mathrm{CI} = \bar{x} \pm Z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

- **Margin of Error Calculation**:

$$\mathrm{ME} = Z^* \left(\frac{\sigma}{\sqrt{n}}\right) = 2.576 \times \left(\frac{0.2}{\sqrt{15}}\right) \approx 0.133$$

- **Confidence Interval**:

$$\mathrm{CI} \approx [1.124, 1.390]$$

Summary:

- **Using Sample Standard Deviation**: The 99% confidence interval for the mean durability is approximately [1.135, 1.379] million characters.
- **Using Known Population Standard Deviation**: The 99% confidence interval for the mean durability is approximately [1.124, 1.390] million characters.

The slight difference is due to the fact that the t-distribution accounts for additional uncertainty by being wider than the Z-distribution, reflecting the additional variability when the population standard deviation is not known.