But $V_1 = V_2$

$\therefore$ from (3) & (4),

$\boxed{AD = BC = 1}$

* Condition for reciprocity

~~The gover~~

$V_1 = AV_2 - BI_1$ —— ①

$I_1 = CV_2 - DI_2$ — ②

which excitation V_1 at i/p port & S.C at o/p port eq^n ① we get

$V_1 = -BI_2 \quad |_{V_2 = 0}$

$$\frac{I_2}{V_1} = -\frac{1}{B}$$

which interchanging of excitation & response the voltage source V_2 is at o/p port while S.C. current I_1 is obtain at i/p port i.e. $V_1 = 0$.

This time the parameter eq^{n}s ① & ② becomes

$0 = AV_2 - BI_2$.

& $I_1 = CV_2 - DI_2$.

These eq^n give

$$I_2 = \frac{AV_2}{B} \quad \& \quad I_1 = CV_2 - \frac{ADV_2}{B}$$

$$V_2 = \left(\frac{BC - AD}{B}\right)$$

$$\frac{I_1}{V_2} = \frac{BC - AD}{B}$$

$$\frac{I_1}{V_2} = \frac{AD - BC}{B} \quad \text{—— ④}$$

Q. 6 Define Y Parameters of two Port ne-twork and derive the condition for reciprocity & symmetry network

Y Parameters (Short circuit admittance Parameters)

In two Port network the i/p current I_1 & I_2 can be expressed in terms of i/p & o/p voltage as V_1 & V_2 resp.

$$[I] = [Y][V]$$

where $[Y]$ is admittance matrix.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2 = 0} \quad \text{and} \quad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2 = 0}.$$

Similarly assuming i/p Port to short circuited i.e. $V_1 = 0$.

$$\therefore Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1 = 0} \quad \& \quad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1 = 0}$$

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2) output is short circuited ($V_2 = 0$)

from ③

$0 = 2I_3 + 2I_2$

$\boxed{I_3 = -I_2}$

from eqn ②

$5(-I_2) + 2I_2 - 2I_1 = 0$

$-3I_2 - 2I_1$

$D = \frac{-I_1}{I_2} = 1.5$

$\boxed{D = 1.5}$ Ans

from eqn ①.

$V_1 = 3I_1 - 2(-I_2)$

$V_1 = 3I_1 + 2I_2$

$V_1 = 3(-1.5I_2) + 2I_2$.

$V_1 = -4.5I_2 + 2I_2$

$V_1 = -2.5I_2$

$B = \frac{-V_1}{I_2} = +2.5$

$\boxed{B = 2.5}$ Ans

This ABCD parameters are

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2.75 & 2.5 \\ 1.25 & 1.5 \end{bmatrix}$$

i) condition for symmetry is $A = D$.

Here $2.75 \neq 1.5$

$\therefore$ N/w is not symmetrical

ii) condition for reciprocity.

$AD - BC = 1$

$(2.75 \times 1.5) - (2.5 \times 1.25) = 1$

Hence N/w is reciprocal

Applying KCL at loop I

$V_1 = I_1 + 2(I_1 - I_3) = 3I_1 - 2I_3$ —— (1)

$I_3 + 2(I_3 + I_2) + 2(I_3 - I_1) = 0$

$5I_3 + 2I_2 - 2I_1 = 0$ —— (2)

$V_2 = 2(I_3 + I_2) = 2I_3 + 2I_2$ —— (3)

i) let output port is O.C. $(I_2 = 0)$

from eqn (3).

$V_2 = 2I_3$

$I_3 = \frac{V_2}{2}$

Substituting value of I_3 in eqn (2)

$5\left[\frac{V_2}{V_1}\right] + 2(0) - 2I_1 = 0$

$2.5V_2 = 2I_1$

$\therefore C = \frac{I_1}{V_2} = \frac{2.5}{2} = 1.25$

$\boxed{C = 1.25}$ Ans

from eqn (1).

$V_1 = 3I_1 - 2\left[\frac{V_2}{2}\right]$

$V_1 = 3I_1 - V_2$

$V_1 = 3(1.25V_2) - V_2$

$V_1 = 3.75V_2 - V_2$

$V_1 = 2.75V_2$

$\therefore A = \frac{V_1}{V_2} = 2.75$

$\boxed{A = 2.75}$ Ans

$B = \frac{1}{4} \angle -180°$

$\boxed{B = +\frac{1}{4}}$

To find C :- C is complex conjugate of B.

$\boxed{C = \frac{-1}{4}}$

Substituting value of A, B, C in eq^n ①

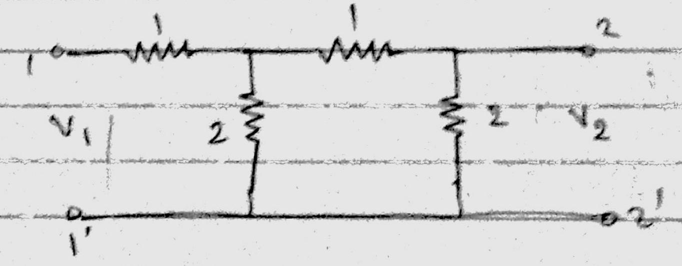
$$I(s) = \frac{-10}{2}\left[\frac{1}{s+5}\right] + \frac{10}{4}\left[\frac{1}{s+3+2j}\right] - \frac{10}{4}\left[\frac{1}{s+3-2j}\right]$$

$$I(s) = -5\left[\frac{1}{s+5}\right] + 2.5\left[\frac{1}{s+3+2j}\right] - 2.5\left[\frac{1}{s+3-2j}\right]$$

Taking laplace inverse of I(s)

$$i(t) = -5e^{-5t} + 2.5\,e^{(-3-2j)t} - 2.5\,e^{(-3+2j)t}$$

Q.7 Obtain ABCD Parameters of network shown and find wheather circuit is reciprocal and symmetrical.



Q.5 For the network given as

$I(s) = \frac{10(s+1)}{(s+5)(s^2+6s+13)}$ place the poles and the zeros on s-plane and hence find $i(t)$

$$I(s) = \frac{10(s+1)}{(s+5)(s^2+6s+13)}$$

$$I(s) = \frac{10(s+1)}{(s+5)(s+3+2j)(s+3-2j)}$$

Zeros $- s = -1$

Poles $- \quad s = -5, \; s = -3+2j, \; -3-2j$

By Partial fraction.

$$I(s) = \left[\frac{A}{s+5} + \frac{B}{s+3+2j} + \frac{C}{s+3-2j}\right] 10 \quad \text{--- (1)}$$

* To find A.

$$A = \frac{4\angle 180^\circ}{\sqrt{8}\angle 135 \cdot \sqrt{8}\angle -135}$$

$$\frac{4}{8}\angle 180 = \frac{-1}{2}.$$

$$\therefore \boxed{A = \frac{-1}{2}}$$

* To find B

$$B = \frac{\sqrt{8}\angle -135}{(\sqrt{8}\angle 135)(4\angle -90)}$$

solving eq^n i, ② ③, ④ & ⑤ .

$V_3 = V_2 + I_3 s = V_2 + sV_2 s = V_2 + s^2 V$ — ⑥

$V_3 = V_2(1+s^2)$

$I_4 = sV_3 = sV_2(1+s^2)$

$I_1 = I_3 + I_4 = sV_2 + sV_2(1+s^2)$ — ⑦

$I_1 = sV_2[1+1+s^2] = sV_2(s^2+2)$

Substituting eq^n ⑥ & ⑦ in eq^n ⑤

$V_1 = V_2(1+s^2) + sV_2(s^2+2)s$

$\quad = V_2(1+s^2) + s^2 V_2(s^2+2)$

$V_1 = V_2[1+s^2+s^4\ 2s^2]$

$$\boxed{\therefore G_{12}(s) = \frac{V_2}{V_1} = \frac{1}{s^4+3s^2+1}}$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{Z_{12}}{G_{12}} = \frac{V_2/I_1}{V_2/V_1} = \frac{V_1}{I_1}$$

$$\boxed{Z_{11} = \frac{s^4+3s^2+1}{s^3+2s}}$$

$$Z_{12} = \cancel{\frac{V_1}{I_1}} = \cancel{\frac{Z_{12}}{G_{12}}} = \cancel{V_2/I_1}\ \ \frac{V_2}{I_1}$$

$$\boxed{Z_{12} = \frac{1}{s^3+2s}}$$

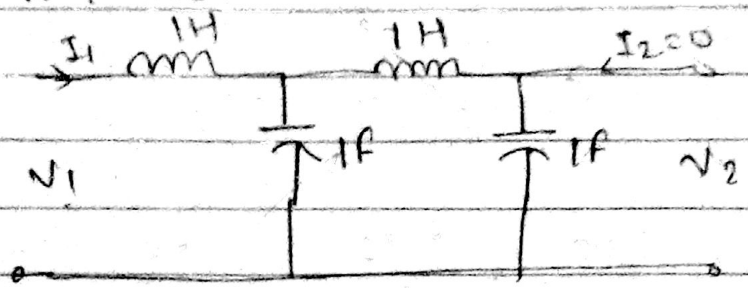
$$R_2 \frac{V}{L} + L \frac{d^2 i_2(t)}{dt^2} = R_1\left[\frac{V}{CR_1^2}\right]$$

$$\therefore L\frac{d^2 i_2}{dt^2} = -\frac{R_2 V}{L} + \frac{V}{CR_1}$$

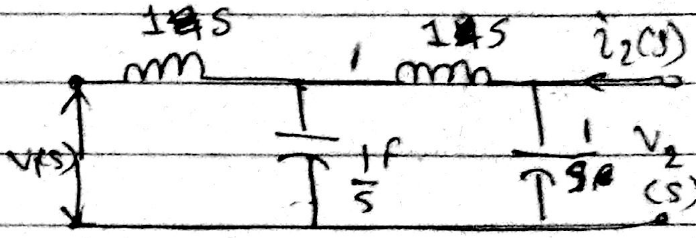
$$\therefore \frac{d^2 i_2}{dt^2} = -\frac{R_2 V}{L^2} + \frac{V}{CLR_1}$$

$$\boxed{\frac{d^2 i_2}{dt^2} = \frac{V}{L}\left[\frac{1}{CR_1} - \frac{R_2}{L}\right]}$$

Q.4 For ladder network shown in figure Z_{11}, Z_{12} and G_{12} assume $I_2 = 0$.



convert in 's' domain.
from n/w



$$I_3 = \frac{V_2}{1/s} = \underline{sV_2} \quad -(1)$$

$$V_3 = sI_3 + V_2 \quad -(2)$$

$$I_4 = \frac{V_3}{1/s} = sV_3 \quad -(3)$$

$$I_1 = I_3 + I_4 \quad -(4)$$

$$V_1 = V_3 + I_1 s \quad -(5)$$

from eqⁿ ① & ②

$$R_2 i(t) + L\frac{di_2}{dt} + R_1\left[-\frac{v}{R_1}\right] = 0.$$

$$R_2 i_2(t) + L\frac{di_2}{dt} - v = 0.$$

$$R_2 i_2(t) + L\frac{di_2}{dt} = v.$$

But $i_2(t) = 0$

$$\therefore L\frac{di_2}{dt} = v.$$

$$\boxed{\frac{di_2}{dt} = \frac{v}{L}.}$$

differentiating eqⁿ ② w.r.t 't'.

$$R_2\frac{di_2(t)}{dt} + L\frac{d^2i}{dt^2} + R_1\left[\frac{di_2(t)}{dt} - \frac{di_1(t)}{dt}\right] = 0$$

Differentiating eqⁿ ① w.r.t 't'.

$$\frac{1}{C} i(t) + R_1\left[\frac{di_1(t)}{dt} - \frac{di_2(t)}{dt}\right] = 0.$$

But $i_1(t) = \frac{v}{R_1}$

$$\frac{1}{C}\frac{v}{R_1} + R_1\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0.$$

$$\frac{di_2}{dt} - \frac{di_1}{dt} = \frac{v}{CR_1^2}$$

Substitute value of $\frac{di_2}{dt}$

$$\boxed{\frac{di_1}{dt} = \frac{v}{L} - \frac{v}{R_1^2 C}}$$

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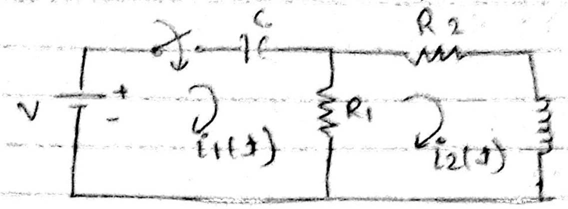
Q. 3 In the given network assuming all initial condition as zero, find $i_1, i_2, di_1/dt, di_2/dt, d^2i_1/dt^2, \& d^2i_2/dt^2$ at $t=0^+$



When switch is closed at $t=0$.
Apply KVL in loop 1.

$$-V + \frac{1}{C}\int i_1\,dt + R_1[i_1(t) - i_2(t)] = 0.$$

$$\therefore \frac{1}{C}\int i_1\,dt + R_1[i_1(t) - i_2(t)] = V \quad —① $$

Apply KVL to loop 2

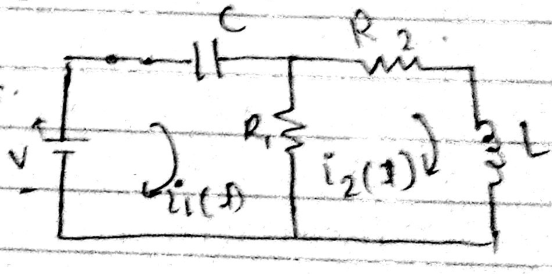
$$R_2 i_2(t) + L\frac{di_2(t)}{dt} + R_1[i(t) - i(t)] = 0 \quad —②$$

at $t=0$ capacitor → short circuit.
Inductor $= 0$.

$$\therefore \frac{1}{C}\int i_1(t)\,dt = 0.$$

$\boxed{\therefore i_2(t) = 0}$ Ans.

$\boxed{i_1(t) = \frac{V}{R_1}}$



eqn ① ⇒ $0 + R_1[i_1(t) - i_2(t)] = V.$

$$i_1(t) - i_2(t) = \frac{V}{R_1} \quad —③$$

Differentiate eq^n (1) w.r.t. 't'.

$$0 = R_1\left[\frac{di_1}{dt} - \frac{di}{dt}\right] + \frac{1}{c}(i_1 - i)$$

$$0 = R_1\left[\frac{di_1}{dt} - \frac{V_0}{L}\right] + \frac{1}{c}\left[\frac{V_0}{R_1} - 0\right]$$

$$\frac{R_1 di_1}{dt} = \frac{V_0 R_1}{L} - \frac{V_0}{R_1 C} = V_0\left[\frac{R_1}{L} - \frac{1}{R_1 C}\right]$$

$$\therefore \frac{di_1}{dt} = \frac{V_0}{L} - \frac{V_0}{R_1^2 C}$$

Differentiate eq^n (2) w.r.t 't'.

$$0 = L\frac{d^2 i}{dt^2} + R_2\frac{di}{dt} + \frac{1}{c}(i - i_1) + R_1\left[\frac{di}{dt} - \frac{di_1}{dt}\right]$$

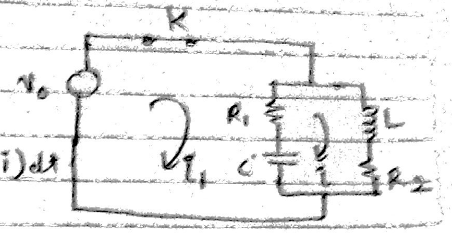
$$0 = L\frac{d^2 i}{dt^2} + R_2\frac{V_0}{L} + \frac{1}{c}\left(0 - \frac{V_0}{R_1}\right) + R_1\left[\frac{V_0}{L} - \frac{V_0}{L} + \frac{V_0}{R_1^2 C}\right]$$

$$0 = L\frac{d^2 i}{dt^2} + \frac{R^2}{L}V_0 - \frac{V_0}{R_1 C} + \frac{V_0}{R_1 C}$$

$$L\frac{d^2 i}{dt^2} + \frac{R_2 V_0}{L}$$

$$\boxed{\frac{d^2 i}{dt^2} = -\frac{R_2 V_0}{L^2}} \Rightarrow \text{Ans.}$$

2) When switch is closed at $t=0$:



Apply KVL to loop 1

$$V_0 = R_1(i_1 - i) + \frac{1}{C}\int (i_1 - i)\,dt \quad \text{— ①}$$

Apply KVL to loop 2

$$0 = L\frac{di}{dt} + R_2 i + \frac{1}{C}\int (i - i_1)\,dt + R_1(i - i_1) \quad \text{— ②}$$

At $t = 0^+$:-

capacitor acts as s.c & inductor → open ckt

$\therefore \frac{1}{C}\int i_1 - i = \frac{1}{C}\int i - i_1 = 0$.

Similarly current through L is zero,

$\therefore i = 0$

$\therefore$ ① $\Rightarrow V_0 = R_1(i_1 - 0) + 0$

$$\boxed{i_1 = \frac{V_0}{R_1}}$$

from ②.

$$L\frac{di}{dt} + R_2(0) + 0 + R_1\left(0 - \frac{V_0}{R_1}\right) = 0$$

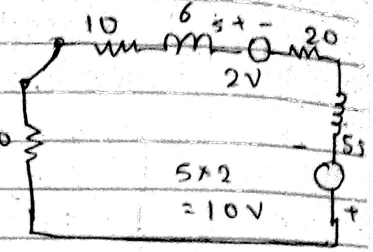
$$L\frac{di}{dt} - V_0 = 0$$

$$\boxed{\frac{di}{dt} = \frac{V_0}{L}} \Rightarrow \text{Ans.}$$

$(40+6s)\, I(s) = 12$

$I(s) = \dfrac{12}{6s+40} = \dfrac{12}{6(s+40/6)} = \dfrac{2}{s+\frac{40}{6}}$

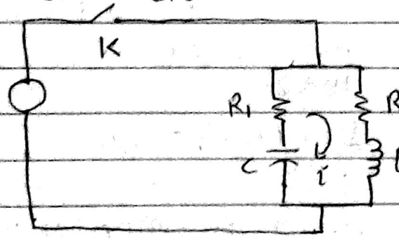
$I(s) = \dfrac{2}{s+40/60}$



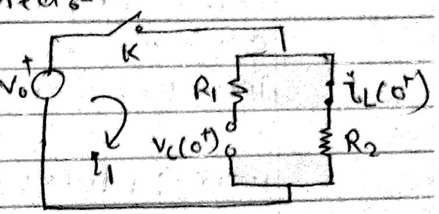
Taking laplace inverse

$\therefore\ i(t) = 2e^{-\frac{40}{6}t}$

Q.2 For the network shown in fig. switch K is closed at instant $t=0$, connecting to battery to an unenergised network find $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$



1) When switch is opened:-



Inductor → short circuit

Capacitor → open circuit

$i_L(0^+) = 0$ $\boxed{\therefore\ i=0}$

$V_c(0^+) = 0$

$\boxed{i=0}$

Assignment NO-2.

Shruti Ghodke roll no- 'B-15'
Section -B

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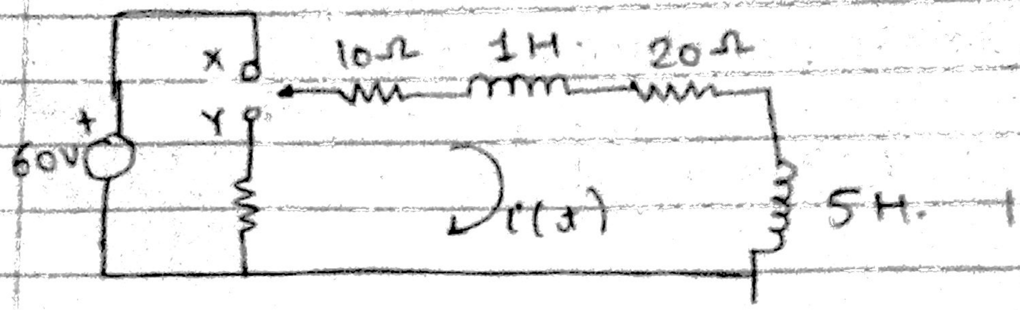
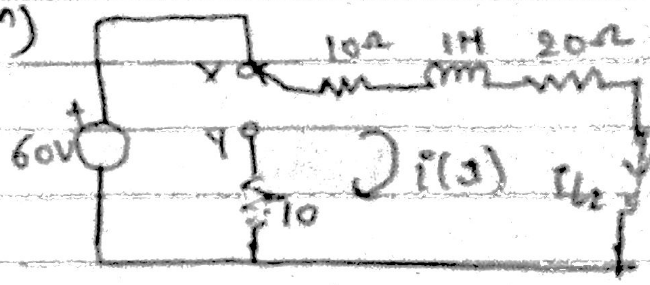
Q.1 For the network shown, find particular solution for $i(t)$ when switch K is moved from position X to Y at time $t=0$, and the steady state current having previously established in the circuit



When switch is at 'X'
(steady state condⁿ)



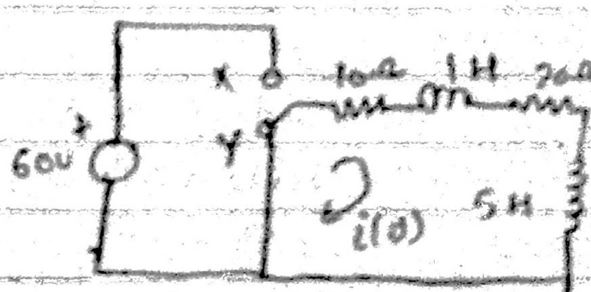
Inductor acts as a S.C

$$i(L_1) = i_{L_2} = \frac{60}{10+20} = \frac{60}{30}$$

$$i_{L_1}(0+) = I_{L_2}(0+) = 2A$$

When switch is at position 'y':-

→ converting the above network into 's' domain



Apply KVL to loop

$$10I(s) + 10(s) + SI(s) - 2 + 20I(s) + 5SI(s) - 10 = 0$$