

OPAMP Stability

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An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.

1. Find the OPAMP gain $G(s)$

Solution: The given oscillator has a low frequency gain 10^3 and a single-pole rolloff at 10^4 rad/s. So we have a open loop amplifier gain

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \quad (1.1)$$

2. Find the feedback $H(s)$

Solution:

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \quad (2.1)$$

3. Find the loop-gain.

Solution: The loop gain is given by

$$L(s) = G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \quad (3.1)$$

and the various gains summarised in Table 3

4. Find the PM and the condition for stability.

Solution: For stability, $PM > 0$ For the given system :

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} = -3 \tan^{-1} \left(\frac{\omega}{10^4} \right) \quad (4.1)$$

Parameters	Definition	For given case
Open loop gain	G	$\frac{10^3}{1 + \frac{s}{10^4}}$
Feedback factor	H	$\frac{k}{\left(1 + \frac{s}{10^4}\right)^2}$
Loop gain	GH	$\frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3}$
Transfer Function	$\frac{G}{1+GH}$	$\frac{\frac{10^3}{1 + \frac{s}{10^4}}}{1 + \frac{1}{\left(1 + \frac{s}{10^4}\right)^3}}$

TABLE 3

So,

$$180^\circ = -3 \tan^{-1} \left(\frac{\omega_{180}}{10^4} \right) \quad (4.2)$$

$$\Rightarrow \omega_{180} = -\sqrt{3} \times 10^4 \quad (4.3)$$

The Loop gain at ω_{180} is $G(j\omega_{180})H(j\omega_{180})$.

The system becomes unstable if

$$G(j\omega_{180})H(j\omega_{180}) \geq 1 \quad (4.4)$$

$$\Rightarrow \left| \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} \right| \geq 1 \quad (4.5)$$

$$\left| \frac{10^3 k}{(1 - \sqrt{3}j)^3} \right| \geq 1 \quad (4.6)$$

$$\frac{10^3 k}{\left| \sqrt{1 + \sqrt{3}^2} \right|} \geq 1 \quad (4.7)$$

$$\frac{10^3 k}{8} \geq 1 \quad (4.8)$$

$$\Rightarrow k \geq 0.008 \quad (4.9)$$

Hence, the value of k above which the system becomes unstable is 0.008.

5. Design the feedback circuit H .

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Solution:

$$H(s) = \frac{V_f}{V_0} = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} = k \left(\frac{1}{1 + \frac{2s}{10^4} + \frac{s^2}{10^8}} \right) \quad (5.1)$$

This is of the form,

$$k \left(\frac{1}{1 + sR_1C_1 + s^2L_1C_1} \right) = k \left(\frac{\frac{1}{sC_1}}{R_1 + sL_1 + \frac{1}{sC_1}} \right) \quad (5.2)$$

This can be realized using the circuit,

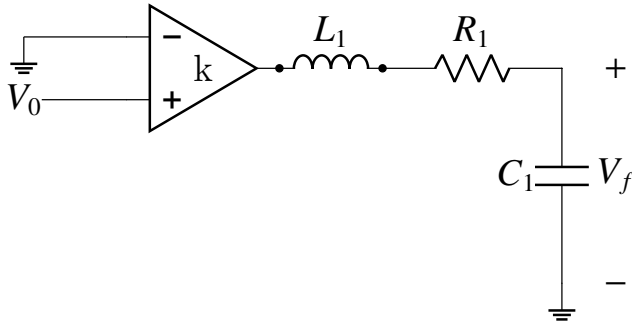


Fig. 5

A set of values that satisfy these equations are,

$$R_1 = 200\Omega \quad (5.3)$$

$$C_1 = 1\mu F \quad (5.4)$$

$$L_1 = 10mH \quad (5.5)$$

6. Design the closed loop circuit. You may choose a suitable value of k such that the system is stable.

Solution: Let $k=0.001$. The closed loop gain is $T(s)$.

$$T(s) = \frac{\frac{10^3}{1 + \frac{s}{10^4}}}{1 + \frac{1}{\left(1 + \frac{s}{10^4}\right)^3}} \quad (6.1)$$

$$= \frac{10^7 s^2 + 2 \times 10^{11} s + 10^{15}}{s^3 + 3 \times 10^4 s^2 + 3 \times 10^8 s + 2 \times 10^{12}} \quad (6.2)$$

The final circuit would be:

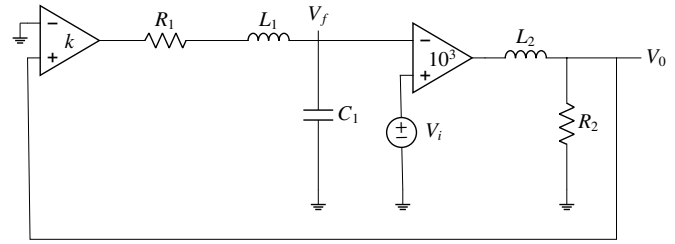


Fig. 6

$$R_1 = 200\Omega \quad (6.3)$$

$$C_1 = 1\mu F \quad (6.4)$$

$$L_1 = 10mH \quad (6.5)$$

$$L_2 = 1\mu F \quad (6.6)$$

$$R_2 = 100\Omega \quad (6.7)$$

$$k = 10^{-3} \quad (6.8)$$

7. Sketch the Bode plot of the closed loop system Solution: The following code gives the Bode plot of the closed loop system

```
codes/ee18btech11006/ee18btech11006_1.
py
```

Bode Plot:

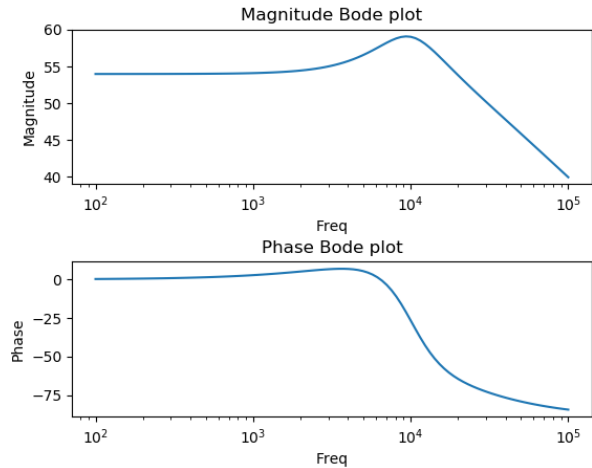


Fig. 7

8. Find the output of the circuit for an appropriate input using spice.

Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

```
codes/ee18btech11006/spice/README
```

The following netlist simulates the given circuit.

```
codes/ee18btech11006/spice/
ee18btech11006.net
```

The following code plots the output from the spice simulation which is shown in Fig. ??.

```
codes/ee18btech11006/spice/
ee18btech11006_spice.py
```

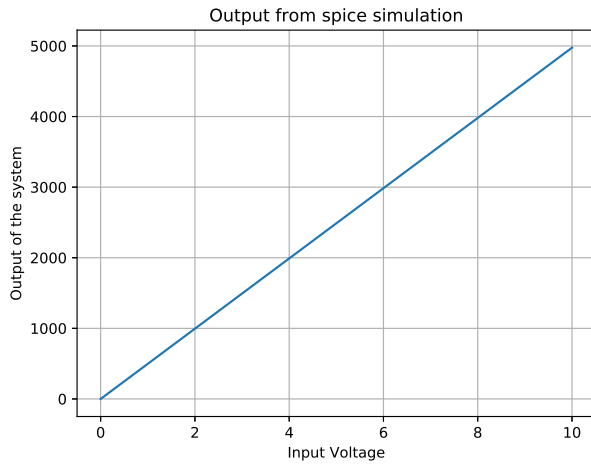


Fig. 8.4

Verification: The Output of the system would be :

$$Y(s) = T(s)X(s) \quad (8.1)$$

$$X(s) = \frac{A}{s} \quad (8.2)$$

$$Y(s) = A \frac{10^7 s^2 + 2 \times 10^{11} s + 10^{15}}{s^4 + 3 \times 10^4 s^3 + 3 \times 10^8 s^2 + 2 \times 10^{12} s} \quad (8.3)$$

The following python codes plot the inverse Laplace of $Y(s)$ giving the time domain output for different values of A .

```
codes/ee18btech11006/spice/
ee18btech11006_2.py
```

On plotting, we obtain the given figure.

Hence verified that the designed circuit does represent the given system.

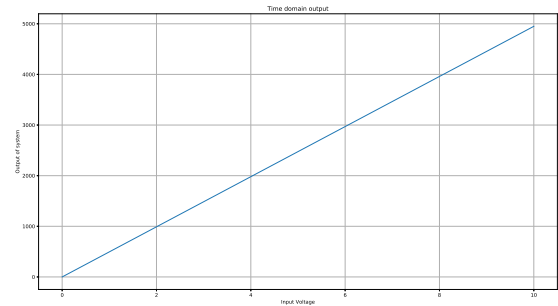


Fig. 8.5