OPAMP Stability

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An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.

1. Find the OPAMP gain $G(s)$
Solution: The given oscillator has a low frequency gain 10^3 and a single-pole rolloff at 10^4 rad/s. So we have a open loop amplifier gain

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \tag{1.1}$$

2. Find the feedback $H(s)$
Solution:

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \tag{2.1}$$

3. Find the loop-gain.
Solution: The loop gain is given by

$$L(s) = G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \tag{3.1}$$

and the various gains summarised in Table 3

Parameters	**Definition**	**For given case**
Open loop gain	G	$\frac{10^3}{1+\frac{s}{10^4}}$
Feedback factor	H	$\frac{k}{\left(1+\frac{s}{10^4}\right)^2}$
Loop gain	GH	$\frac{10^3 k}{(1+\frac{s}{10^4})^3}$
Transfer Function	$\frac{G}{1+GH}$	$\frac{\frac{10^3}{1+\frac{s}{10^4}}}{1+\frac{1}{\left(1+\frac{s}{10^4}\right)^3}}$

TABLE 3

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4. Find the condition for stability.
Solution: For stability, $(GM)_{dB}$ & $PM > 0$.
For the given system :

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} = -3tan^{-1}\left(\frac{\omega}{10^4}\right) \tag{4.1}$$

So the phase crossover frequency is,

$$180^\circ = \left|-3tan^{-1}\left(\frac{\omega_{180}}{10^4}\right)\right| \tag{4.2}$$

$$\implies \omega_{180} = \sqrt{3} \times 10^4 \tag{4.3}$$

The Loop gain at ω_{180} is $|G(j\omega_{180})H(j\omega_{180})|$.
(i) $GM_{dB} > 0$

$$\implies |G(j\omega_{180})H(j\omega_{180})| < 1 \tag{4.4}$$

$$\implies \left|\frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3}\right| < 1 \tag{4.5}$$

$$\left|\frac{10^3 k}{\left(1 - \sqrt{3}j\right)^3}\right| < 1 \tag{4.6}$$

$$\frac{10^3 |k|}{\left|\sqrt{1 + \sqrt{3}^2}\right|^3} < 1 \tag{4.7}$$

$$\frac{10^3 |k|}{8} < 1 \tag{4.8}$$

$$\implies |k| < 0.008 \tag{4.9}$$

(ii) $PM = \phi_m = 180^\circ + \angle L\left(j\omega_{gc}\right) > 0$ i.e.. $180^\circ - 3tan^{-1}\left(\frac{\omega_{gc}}{10^4}\right) > 0$.
Gain crossover frequency ω_{gc},

$$\left|G(j\omega_{gc})H(j\omega_{gc})\right| = 1 \tag{4.10}$$

$$\left|\frac{10^3 k}{\sqrt{1 + \frac{\omega_{gc}{}^2}{10^8}}^3}\right| = 1 \tag{4.11}$$

$$\implies \omega_{gc} = \sqrt{10^8\left(10^2 k^{\frac{2}{3}} - 1\right)} \tag{4.12}$$

Now,

$$\phi_m = 180^\circ - 3tan^{-1}\left(\frac{\sqrt{10^8\left(10^2k^{\frac{2}{3}}-1\right)}}{10^4}\right) > 0 \tag{4.13}$$

$$\sqrt{10^8\left(10^2k^{\frac{2}{3}}-1\right)} < \sqrt{3}\times 10^4 \tag{4.14}$$

$$10^2k^{\frac{2}{3}} < 4 \tag{4.15}$$

$$\implies |k| < 0.008 \tag{4.16}$$

From (i) and (ii) the value of k above which the system becomes unstable is 0.008.

5. Design the feedback circuit H.
Solution:

$$H(s) = \frac{V_f}{V_0} = \frac{k}{\left(1+\frac{s}{10^4}\right)^2} = k\left(\frac{1}{1+\frac{2s}{10^4}+\frac{s^2}{10^8}}\right) \tag{5.1}$$

This is of the form,

$$k\left(\frac{1}{1+sR_1C_1+s^2L_1C_1}\right) = k\left(\frac{\frac{1}{sC_1}}{R_1+sL_1+\frac{1}{sC_1}}\right) \tag{5.2}$$

This can be realized using the circuit,

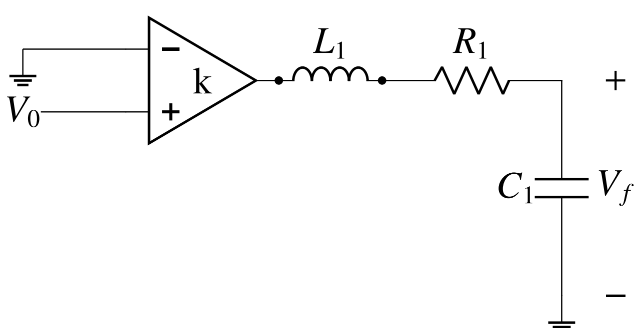

Fig. 5

A set of values that satisfy these equations are,

$$R_1 = 200\Omega \tag{5.3}$$
$$C_1 = 1\mu F \tag{5.4}$$
$$L_1 = 10mH \tag{5.5}$$

6. Design the closed loop circuit. You may choose a suitable vale of k such that the system is stable.
Solution: Let k=0.001. The closed loop gain is T(s).

$$T(s) = \frac{\frac{10^3}{1+\frac{s}{10^4}}}{1+\frac{1}{\left(1+\frac{s}{10^4}\right)^3}} \tag{6.1}$$

$$= \frac{10^7s^2+2\times10^{11}s+10^{15}}{s^3+3\times10^4s^2+3\times10^8s+2\times10^{12}} \tag{6.2}$$

The final circuit would be:

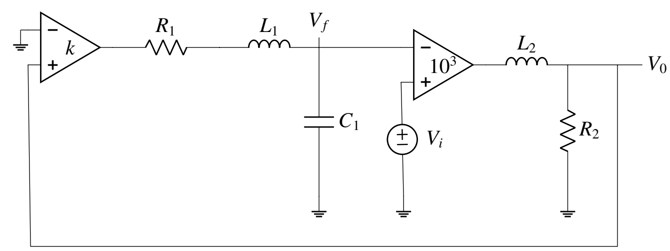

Fig. 6

$$R_1 = 200\Omega \tag{6.3}$$
$$C_1 = 1\mu F \tag{6.4}$$
$$L_1 = 10mH \tag{6.5}$$
$$L_2 = 1\mu F \tag{6.6}$$
$$R_2 = 100\Omega \tag{6.7}$$
$$k = 10^{-3} \tag{6.8}$$

7. Sketch the Bode plot of the closed loop system
Solution: The following code gives the Bode plot of the closed loop system

```
codes/ee18btech11006/ee18btech11006_1.py
```

Bode Plot:

8. Find the output of the circuit for an appropriate input using spice.
Solution: The following readme file provides necessary instructions to simulate the circuit in spice.

```
codes/ee18btech11006/spice/README
```

The following netlist simulates the given circuit.

```
codes/ee18btech11006/spice/ee18btech11006.
    net
```

The following code plots the output from the spice simulation which is shown in Fig. 8.4.

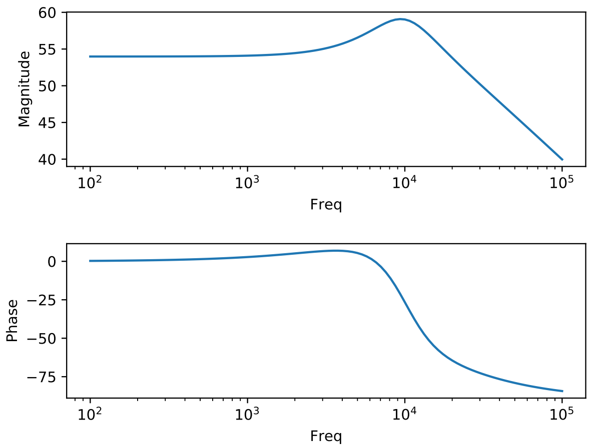

Fig. 7

codes/ee18btech11006/spice/
    ee18btech11006_spice.py

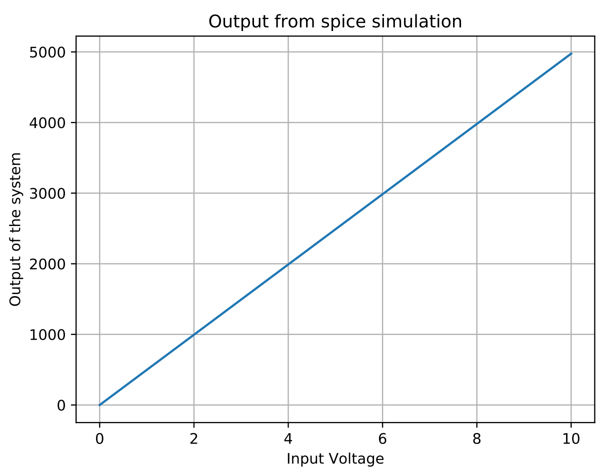

Fig. 8.4

Verification: The Output of the system would be :

$$Y(s) = T(s)X(s) \tag{8.1}$$

$$X(s) = \frac{A}{s} \tag{8.2}$$

$$Y(s) = A\frac{10^7 s^2 + 2\times 10^{11} s + 10^{15}}{s^4 + 3\times 10^4 s^3 + 3\times 10^8 s^2 + 2\times 10^{12} s} \tag{8.3}$$

The following python codes plot the inverse Laplace of Y(s) giving the time domain output for different values of A.

codes/ee18btech11006/spice/
    ee18btech11006_2.py

On plotting, we obtain the given figure.
 Hence verified that the designed circuit does

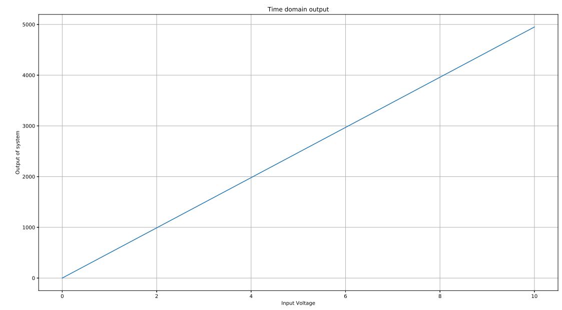

Fig. 8.5

represent the given system.