

# Control Systems

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### 1 Stability 1

Abstract—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/
trunk/control/feedback/codes
```

#### 1 Stability

1.1. An op amp having a low-frequency gain of  $10^3$  and a single-pole rolloff at  $10^4$  rad/s is connected in a negative feedback loop via a feedback network having a transmission  $k$  and a two-pole rolloff at  $10^4$  rad/s. Find the value of  $k$  above which the closed-loop amplifier becomes unstable.

Solution:

Parameters	Definition	For given case
Open loop gain	G	$\frac{10^3}{1 + \frac{s}{10^4}}$
Feedback factor	H	$\frac{k}{\left(1 + \frac{s}{10^4}\right)^2}$
Loop gain	GH	$\frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3}$

Fig. 1.1

The given oscillator has a low frequency gain  $10^3$  and a single-pole rolloff at  $10^4$  rad/s. So we have a open loop amplifier gain

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \quad (1.1.1)$$

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Given feedback network transmission gain  $k$  and two-pole rolloff at  $10^4$  rad/s. So, the feedback factor would be

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \quad (1.1.2)$$

The resulting loop-gain would be

$$G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \quad (1.1.3)$$

Parameters	Definition	For given case
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TABLE 1.1

The closed loop gain would be

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (1.1.4)$$

Generalized condition for the system to be stable:

$$T(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)H(j\omega)} \quad (1.1.5)$$

Loop gain,  $L(j\omega) = G(j\omega)H(j\omega)$ .

$$L(j\omega) = |G(j\omega)H(j\omega)| e^{j\phi(\omega)} \quad (1.1.6)$$

Let the frequency at which phase angle  $\phi(\omega)$  becomes  $180^\circ$  be  $\omega_{180}$ . At  $\omega = \omega_{180}$ ,  $L(j\omega)$  is a negative real number.

- if  $|G(j\omega_{180})H(j\omega_{180})| < 1, T(j\omega_{180}) > 1$   
 $\Rightarrow$  system is stable.
- if  $|G(j\omega_{180})H(j\omega_{180})| = 1, T(j\omega_{180}) = \infty$   
 $\Rightarrow$  system is unstable.
- if  $|G(j\omega_{180})H(j\omega_{180})| > 1, T(j\omega_{180}) < 1$

$\Rightarrow$  system is unstable.

For the given system :

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} = -3 \tan^{-1} \left( \frac{\omega}{10^4} \right) \quad (1.1.7)$$

So,

$$180^\circ = -3 \tan^{-1} \left( \frac{\omega_{180}}{10^4} \right) \quad (1.1.8)$$

$$\Rightarrow \omega_{180} = -\sqrt{3} \times 10^4 \quad (1.1.9)$$

The Loop gain at  $\omega_{180}$  is  $G(j\omega_{180})H(j\omega_{180})$ . The system becomes unstable if

$$G(j\omega_{180})H(j\omega_{180}) \geq 1 \quad (1.1.10)$$

$$\Rightarrow \left| \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} \right| \geq 1 \quad (1.1.11)$$

$$\left| \frac{10^3 k}{(1 - \sqrt{3}j)^3} \right| \geq 1 \quad (1.1.12)$$

$$\frac{10^3 k}{\left| \sqrt{1 + \sqrt{3}^2} \right|} \geq 1 \quad (1.1.13)$$

$$\frac{10^3 k}{8} \geq 1 \quad (1.1.14)$$

$$\Rightarrow k \geq 0.008 \quad (1.1.15)$$

Hence, the value of k above which the system becomes unstable is 0.008.

1.2. Design the circuit for the given parameters

Solution: Consider the open loop gain, G

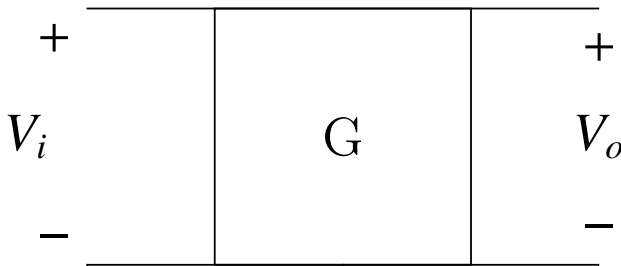


Fig. 1.2

Now,

$$V_i = V_{in} - V_f \quad (1.2.1)$$

$$G(s) = \frac{V_o}{V_i} = \frac{10^3}{1 + \frac{s}{10^4}} \quad (1.2.2)$$

This is of the form  $10^3 \left( \frac{R}{R+sL} \right)$ . This can be realized using the circuit:

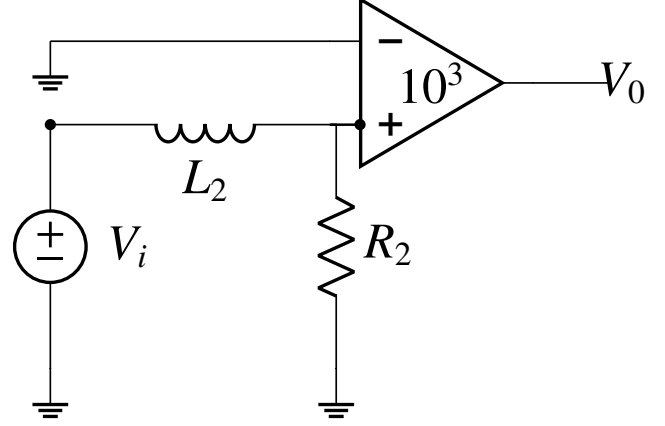


Fig. 1.2

Where the gain of the Op-Amp is  $10^3$ .

$$\frac{R_2}{L_2} = 10^4 \quad (1.2.3)$$

One set of values that would satisfy this condition would be

$$R_2 = 100\Omega \quad (1.2.4)$$

$$L_2 = 1\mu F \quad (1.2.5)$$

Consider the feedback factor, H

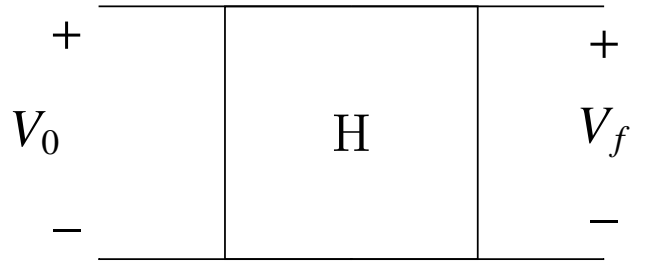


Fig. 1.2

Now,

$$H = \frac{V_f}{V_o} H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} = k \left( \frac{1}{1 + \frac{2s}{10^4} + \frac{s^2}{10^8}} \right) \quad (1.2.6)$$

This is of the form,

$$k \left( \frac{1}{1 + sR_1C_1 + s^2L_1C_1} \right) = k \left( \frac{\frac{1}{sC_1}}{R_1 + sL_1 + \frac{1}{sC_1}} \right) \quad (1.2.7)$$

This can be realized using the circuit,

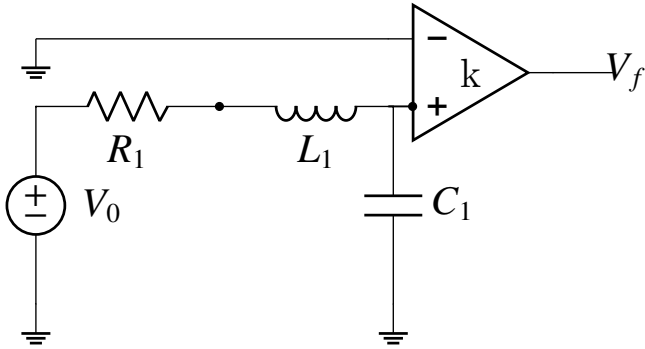


Fig. 1.2

A set of values that satisfy these equations are,

$$R_1 = 200\Omega \quad (1.2.8)$$

$$C_1 = 1\mu F \quad (1.2.9)$$

$$L_1 = 10mH \quad (1.2.10)$$

$$R_2 = 100\Omega \quad (1.2.11)$$

$$L_2 = 1\mu F \quad (1.2.12)$$

The final circuit would be,

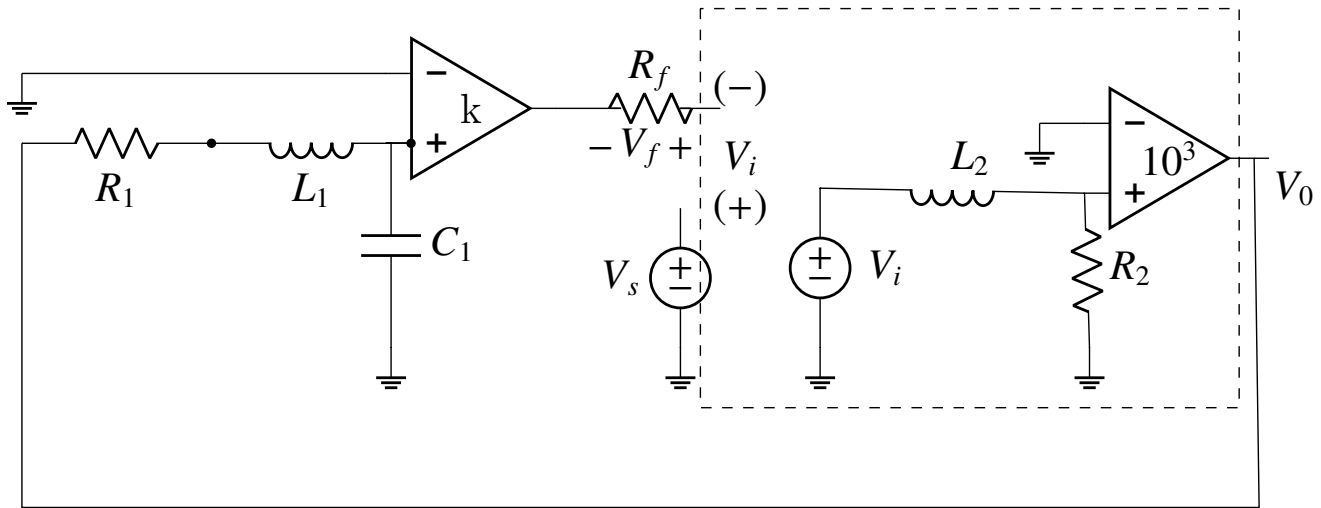


Fig. 1.2