Control Systems

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Contents

Abstract—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/
    trunk/control/feedback/codes
```

1 Stability

1.1. An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.
Solution:

Parameters	Definition	For given case
Open loop gain	G	$\frac{10^3}{1+\frac{s}{10^4}}$
Feedback factor	H	$\frac{k}{\left(1+\frac{s}{10^4}\right)^2}$
Loop gain	GH	$\frac{10k}{(1+\frac{s}{10^4})^3}$

Fig. 1.1

The given oscillator has a low frequency gain 10^3 and a single-pole rolloff at 10^4 rad/s. So we have a open loop amplifier gain

$$G(s) = \frac{10^3}{1+\frac{s}{10^4}} \tag{1.1.1}$$

Given feedback network transmission gain k and two-pole rolloff at 10^4 rad/s. So, the feedback factor would be

$$H(s) = \frac{k}{\left(1+\frac{s}{10^4}\right)^2} \tag{1.1.2}$$

The resulting loop-gain would be

$$G(s)H(s) = \frac{10^3 k}{\left(1+\frac{s}{10^4}\right)^3} \tag{1.1.3}$$

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TABLE 1.1

The closed loop gain would be

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \tag{1.1.4}$$

Generalized condition for the system to be stable:

$$T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)} \tag{1.1.5}$$

Loop gain, $L(j\omega) = G(j\omega)H(j\omega)$.

$$L(j\omega) = |G(j\omega)H(j\omega)|\, e^{j\phi(\omega)} \tag{1.1.6}$$

Let the frequency at which phase angle $\phi(\omega)$ becomes 180° be ω_{180}. At $\omega = \omega_{180}$, $L(j\omega)$ is a negative real number.

- if $|G(j\omega_{180})H(j\omega_{180})| < 1, T(j\omega_{180}) > 1$ $\implies$ system is stable.
- if $|G(j\omega_{180})H(j\omega_{180})| = 1, T(j\omega_{180}) = \infty$ $\implies$ system is unstable.
- if $|G(j\omega_{180})H(j\omega_{180})| > 1, T(j\omega_{180}) < 1$

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$\implies$ system is unstable.
For the given system :

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} = -3tan^{-1}\left(\frac{\omega}{10^4}\right) \tag{1.1.7}$$

So,

$$180^\circ = -3tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \tag{1.1.8}$$

$$\implies \omega_{180} = -\sqrt{3} \times 10^4 \tag{1.1.9}$$

The Loop gain at ω_{180} is $G(j\omega_{180})H(j\omega_{180})$. The system becomes unstable if

$$G(j\omega_{180})H(j\omega_{180}) \geq 1 \tag{1.1.10}$$

$$\implies \left|\frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3}\right| \geq 1 \tag{1.1.11}$$

$$\left|\frac{10^3 k}{\left(1 - \sqrt{3}j\right)^3}\right| \geq 1 \tag{1.1.12}$$

$$\frac{10^3 k}{\left|\sqrt{1 + \sqrt{3}^2}\right|} \geq 1 \tag{1.1.13}$$

$$\frac{10^3 k}{8} \geq 1 \tag{1.1.14}$$

$$\implies k \geq 0.008 \tag{1.1.15}$$

Hence, the value of k above which the system becomes unstable is 0.008.

1.2. Design the circuit for the given parameters
Solution: Consider the open loop gain,G

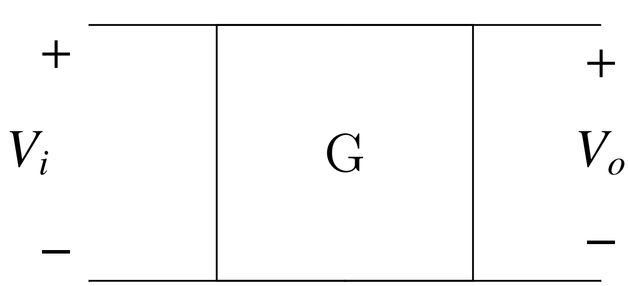

Fig. 1.2

Now,

$$V_i = V_{in} \tag{1.2.1}$$

$$G(s) = \frac{V_0}{V_i - V_-} = \frac{10^3}{1 + \frac{s}{10^4}} \tag{1.2.2}$$

This is of the form $10^3\left(\frac{R}{R+sL}\right)$. This can be realized using the circuit:

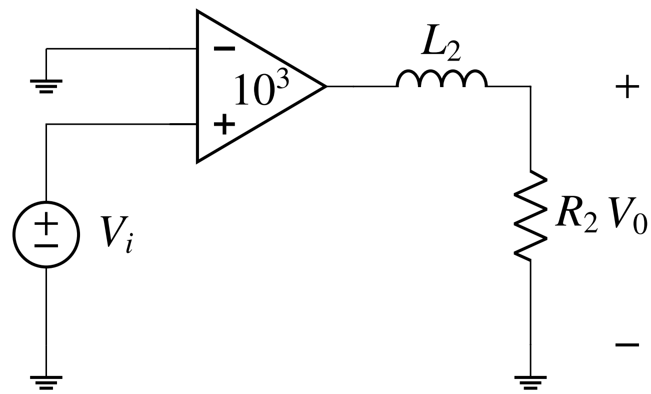

Fig. 1.2

Where the gain of the Op-Amp is 10^3.

$$\frac{R_2}{L_2} = 10^4 \tag{1.2.3}$$

One set of values that would satisfy this condition would be

$$R_2 = 100\Omega \tag{1.2.4}$$

$$L_2 = 1\mu F \tag{1.2.5}$$

Consider the feedback factor, H

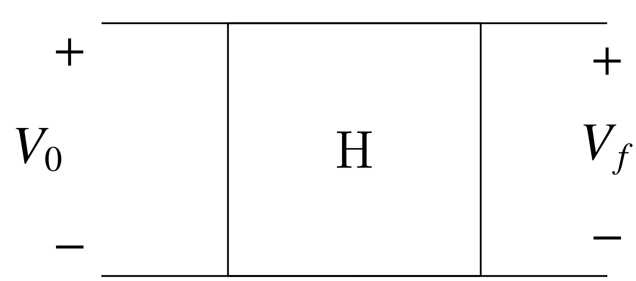

Fig. 1.2

Now,

$$H(s) = \frac{V_f}{V_0} = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} = k\left(\frac{1}{1 + \frac{2s}{10^4} + \frac{s^2}{10^8}}\right) \tag{1.2.6}$$

This is of the form,

$$k\left(\frac{1}{1+sR_1C_1+s^2L_1C_1}\right)=k\left(\frac{\frac{1}{sC_1}}{R_1+sL_1+\frac{1}{sC_1}}\right) \tag{1.2.7}$$

This can be realized using the circuit,

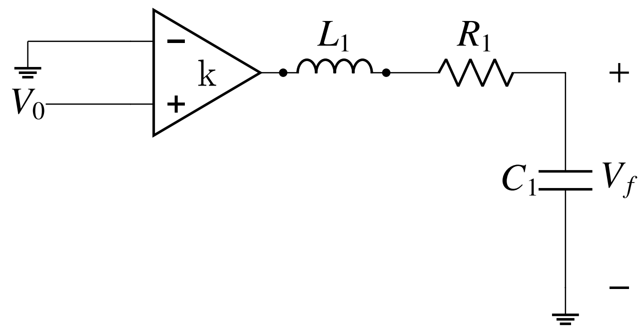

Fig. 1.2

A set of values that satisfy these equations are,

$$R_1 = 200\Omega \tag{1.2.8}$$
$$C_1 = 1\mu F \tag{1.2.9}$$
$$L_1 = 10mH \tag{1.2.10}$$

The final circuit would be,

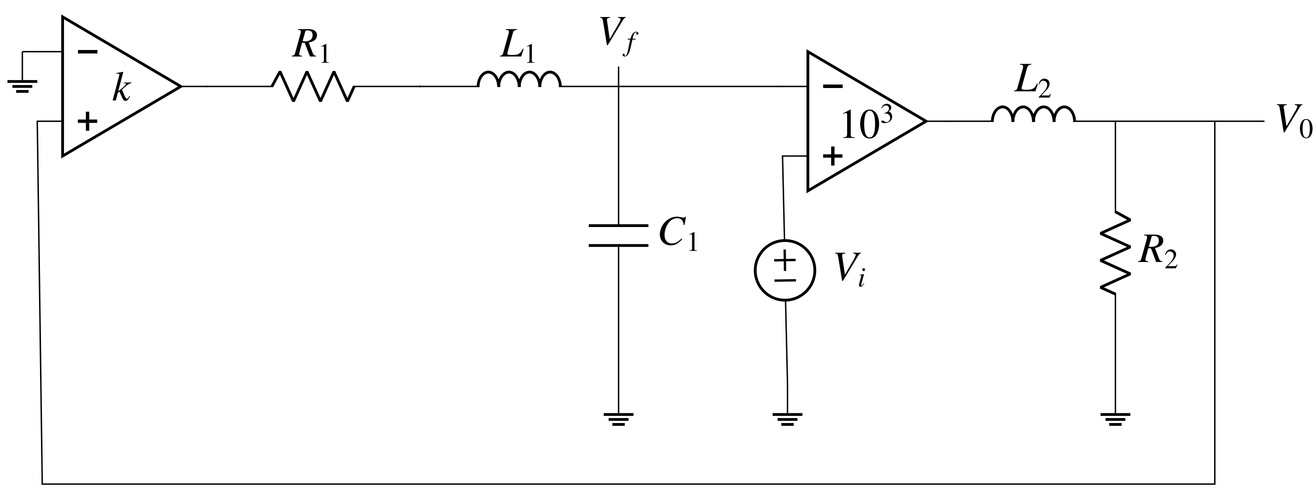

Fig. 1.2