EE3025-Assignment 1

C Shruti - EE18BTECH11006

Download all python codes from

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https://github.com/shruti−chepuri/EE3025/blob/
    main/Assignment_1/codes
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and latex-tikz codes from

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https://github.com/shruti−chepuri/EE3025/tree/
    main/Assignment_1
```

(5.3) The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{0.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 Solution

Given system:

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.1}$$

$$y(n) = 0 \text{ for } y < 0 \tag{1.0.2}$$

Applying Z-transform:

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{1.0.3}$$

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{1.0.4}$$

Finding H(z):

$$H(z) = \frac{Y(z)}{X(z)} \tag{1.0.5}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{1.0.6}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{1.0.7}$$

Impulse response of this system(h(n)):

$$h(n) = Z^{-1}\left(\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right) \tag{1.0.8}$$

$$h(n) = \left(\frac{-1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^{n-2} u(n-2) \tag{1.0.9}$$

Given stability criteria :

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.10}$$

Substituting h(n) we have,

$$\sum_{n=-\infty}^{\infty} \left|\left(\frac{-1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^{n-2} u(n-2)\right| < \infty \tag{1.0.11}$$

$$\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-2} u(n-2)\right| < \infty \tag{1.0.12}$$

$$2\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n)\right| < \infty \tag{1.0.13}$$

$$2\left(\frac{1}{1-\frac{1}{2}}\right) = 4 < \infty \tag{1.0.14}$$

So, the system is stable.
Verification in Z plane:
When h(n) is bounded i.e

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.15}$$

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \infty \tag{1.0.16}$$

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \left|\sum_{n=-\infty}^{\infty} h(n)z^{-n}\right|_{|z|=1} \tag{1.0.17}$$

$$\implies |H(z)|_{|z|=1} < \infty \tag{1.0.18}$$

from triangle inequality. This shows us that the unit circle should lie in the ROC for the system to be stable. Now,

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{1.0.19}$$

$$Poles = 0, -\frac{1}{2} \tag{1.0.20}$$

$$Zeros = +1j, -1j \tag{1.0.21}$$

Hence from the Z plane plot the system is stable.
Verification:
BIBO Stability : For a given system and for every

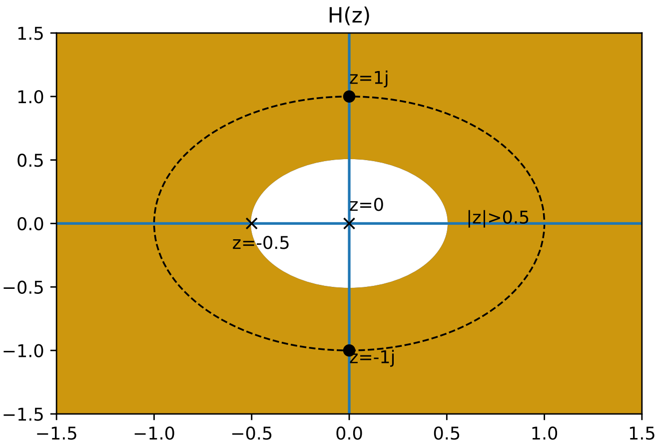

Fig. 0: Unit circle lies in ROC

bounded input, if the output is bounded then it is stable. For a system to be stable the output should be bounded for every bounded input.(BIBO stability). Consider a bounded input sequence x(n)

$$x(n) < B_x(finite) \tag{1.0.22}$$

Then by using the convolution property,

$$|y(n)| = |\sum_{-\infty}^{\infty} h(k)x(n-k)| \tag{1.0.23}$$

$$|y(n)| \leq B_x \sum_{-\infty}^{\infty} |h(k)| \tag{1.0.24}$$

This holds only if

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \tag{1.0.25}$$

$$\text{then we have} |y(n)| \leq B_y < \infty \tag{1.0.26}$$

Thus the output is bounded for bounded input if the impulse response is absolutely summable.
Verification:- Given bounded input x(n).,

$$x(n) = \{1, 1, 2, 4, 3, 1\} \tag{1.0.27}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.28}$$

x(n) is bounded.

The system returns bounded output for the given bounded input. Implies, the system is stable.

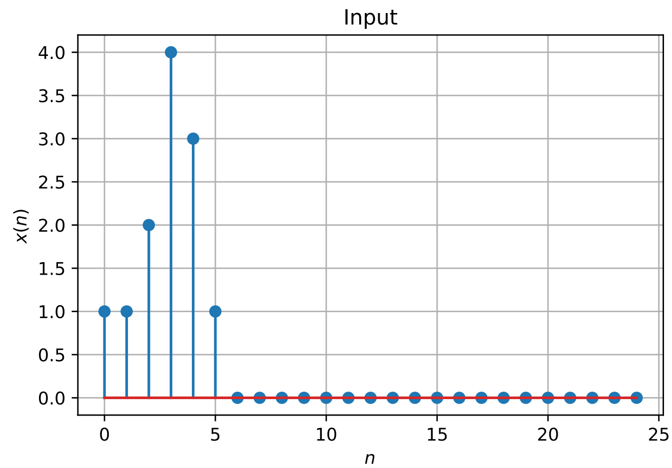

Fig. 0: Given input

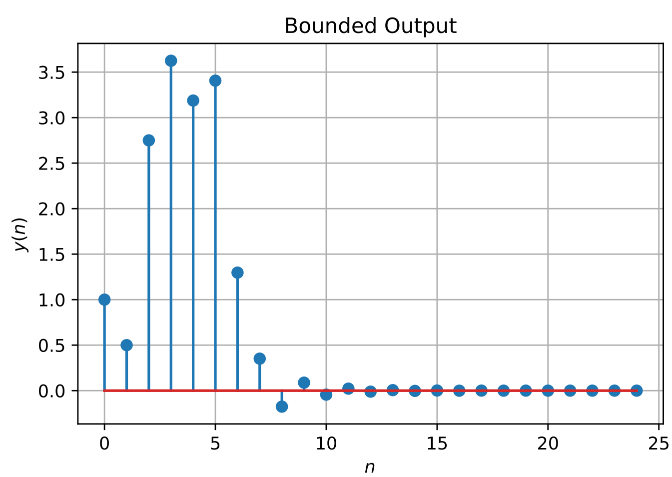

Fig. 0: Bounded output