EE3025-Assignment 1

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and latex-tikz codes from

(5.3) The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \qquad (0.0.1)$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 SOLUTION

Given system:

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \qquad (1.0.1)$$

$$y(n) = 0 \text{ for } y < 0 \qquad (1.0.2)$$

Applying Z-transform:

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \qquad (1.0.3)$$

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \qquad (1.0.4)$$

Finding H(z):

$$H(z) = \frac{Y(z)}{X(z)} \qquad (1.0.5)$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \qquad (1.0.6)$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \qquad (1.0.7)$$

Impulse response of this system(h(n)):

$$h(n) = Z^{-1}\left(\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right) \qquad (1.0.8)$$

$$h(n) = \left(\frac{-1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^{n-2} u(n-2) \qquad (1.0.9)$$

Given stability criteria :

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \qquad (1.0.10)$$

Substituting h(n) we have,

$$\sum_{n=-\infty}^{\infty} \left|\left(\frac{-1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^{n-2}u(n-2)\right| < \infty \qquad (1.0.11)$$

$$\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-2}u(n-2)\right| < \infty \qquad (1.0.12)$$

$$2\sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n)\right| < \infty \qquad (1.0.13)$$

$$2\left(\frac{1}{1-\frac{1}{2}}\right) = 4 < \infty \qquad (1.0.14)$$

So, the system is stable.
Verification:
BIBO Stability : For a given system and for every bounded input, if the output is bounded then it is stable. For a system to be stable the output should be bounded for every bounded input.(BIBO stability). Consider a bounded input sequence x(n)

$$x(n) < B_x(finite) \qquad (1.0.15)$$

Then by using the convolution property,

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \qquad (1.0.16)$$

$$|y(n)| \le B_x \sum_{-\infty}^{\infty} |h(k)| \qquad (1.0.17)$$

This holds only if

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \qquad (1.0.18)$$

$$\text{then we have} |y(n)| \le B_y < \infty \qquad (1.0.19)$$

Thus the output is bounded for bounded input if the impulse response is absolutely summable.

Verification:- Given bounded input x(n).,

$$x(n) = \{1, 1, 2, 4, 3, 1\} \qquad (1.0.20)$$

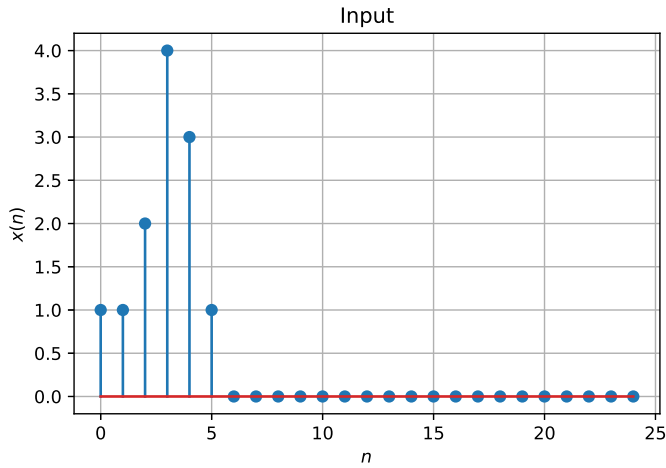$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \qquad (1.0.21)$$

x(n) is bounded.

Fig. 0: Given input

From the python code,y(n) vanishes to zero as n tends to infinity. So the output is bounded.
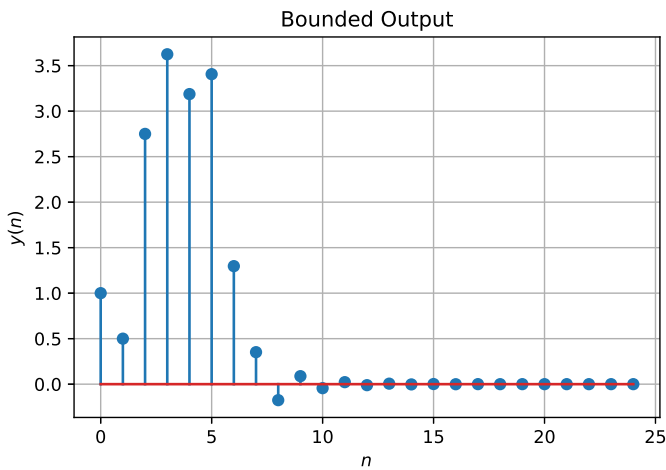

Fig. 0: Bounded output

The system returns bounded output for the given bounded input. Implies, the system is stable.
Hence, verified.