Lecture 02: From bits to qubits

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1   Brief Introduction to Quantum Mechanics

1.1   Quantum Computation

A key concept in quantum mechanics is superposition. Unlike classical bits which can only 0 or 1 at any time, qubits (quantum bits) can be 0 and 1 at the same time due to superposition.

But what does "at the same time" mean? This brings us to the representation of a qubit. Each qubit can be represented as follows:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

where:

- ψ is the quantum state

- Complex numbers α and β satisfy the linear combination $|\alpha|^2 + |\beta|^2 = 1$

Intuition tells us that there are an uncountablly infinite number of linear combinations of α and β that satisfy that equation. However, physicists really need "measurements" to figure out what a qubit is. To measure a qubit, physicists read out $|0\rangle$ and $|1\rangle$ with the following probabilities.

$$readout = \begin{cases} |0\rangle \text{ with probability } |\alpha|^2 \\ |1\rangle \text{ with probability } |\beta|^2 \end{cases}$$

Given the probabilitic nature of a qubit, one may wonder how a qubit differs from a random bit. There are two interpretations for this phenomenon. The first one is the Copenhagen interpretation, which says that nature operates in a quantum world, but we live in a classical world. Thus, the only way we get information from the quantum world is measurement. The other is the many-world interpretation, which states that every time one measures a quantum object, the universe branches into two equally real universes.
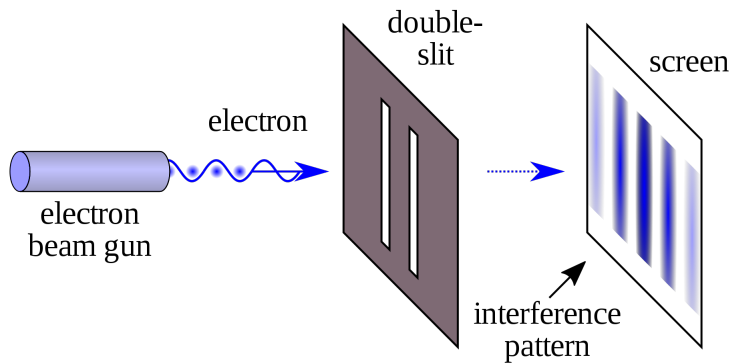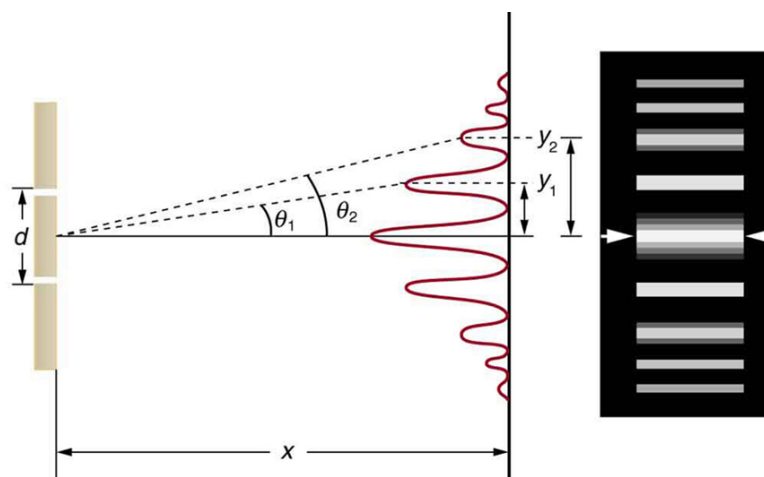
Figure 1: Double Slit Experiment Apparatus [Wik18b]

Figure 2: Double Slit Experiment Result [Lib18]

1.2 Double-Slit Experiment

Conducted by physicist Thomas Young in 1801, the double slit experiment provided experimental proof of particle-wave duality, which states that quantum scale objects can be expressed both as a particle and as a wave, and superposition. In the case of quantum computing, the double-slit experiment provided proof for the fundamental difference between a qubit and a classical random bit.

In the double-slit experiment, a beam gun sends beams through a double-slit wall; the interference pattern of the beam was observed on a screen directly behind the wall (Figure 1). The interference pattern corresponded to the probability of the particle's position.

When only one of the slits S_1 was open, Young found a gaussian distribution of the particle position in front of S_1. Young found a similar result when only slit S_2 open. If the beams were a classical probabilistic object, we would expect to see two gaussian curves next to each other, with both peaks in front of S_1 and S_2. Instead, Young found the pattern in Figure 2, suggesting a quantum interference or a ripple effect due to the wave-like nature

of the beam. This experiment was later tried with 1 photon and still produced the same pattern, giving further experimental evidence for superposition. However, when the photon was measured, it produces the classical probabilistic curve, further supporting the idea that measurement brings information from the quantum world to the classical world.

1.3 Quantum Entanglement

If there are 2 qubits, their quantum state can be expressed as:

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle \tag{2}$$

where $|\alpha|^2 + \cdots + |\delta|^2 = 1$ Setting the β and γ terms to zero, we get the "bell pair":

$$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \tag{3}$$

If one were to measure the qubits, if one of qubits was measured as zero, the other qubit would also measure as zero, because β and γ have both been set to zero due to quantum entanglement.

How is quantum entanglement different from classical correlation? Consider the "local hidden variable" theory. Assume there are three characters, Alice, Bob and Charlie. Alice and Bob each get a box with a left and right glove inside from Charlie, but Carlie did not tell Alice or Bob which glove they have. Charlie decides which glove goes to which box based on a random coin flip. Afterwards, Alice goes to Mars, while Bob stays on Earth. When Alice opens her box, she finds out she has a left glove, which means that she knows "immediately" that Bob has the right glove. Since Alice and Bob don't communicate and thus information does not travel to Bob, this does not violate special relativity. But is entanglement the same as this classical situation, where Charlie's coin flip is a hidden variable unknown to Alice and Bob, and pre-determines the outcomes? it turns out that entanglement is not just probability correlation, and it can be shown by the CHSH game, which we will not go over today.

2 Modern Classical Computing Systems

2.1 von Neumann Architecture

The von Neumann architecture is a design for a stored program computer and forms the foundation of most modern computers (Figure 3). It consists of four main parts: input devices (e.g. keyboard, mouse, etc), the CPU, memory, and output devices (e.g. screen, printer, etc). Memory is an array of bytes (8 bits), which can represent charge and no charge in a memory cell. The CPU has two parts: the control unit and the arithmetic/logic unit (ALU). The control unit controls processor operations, while the ALU performs the arithmetic and logic. Logic can be expressed using Boolean algebra. Each Boolean operand has a corresponding gate, such as the AND gate (Figure 4) and the OR gate (Figure 5). These gates can be connected to create a circuit, such as the XOR circuit in Figure 6.
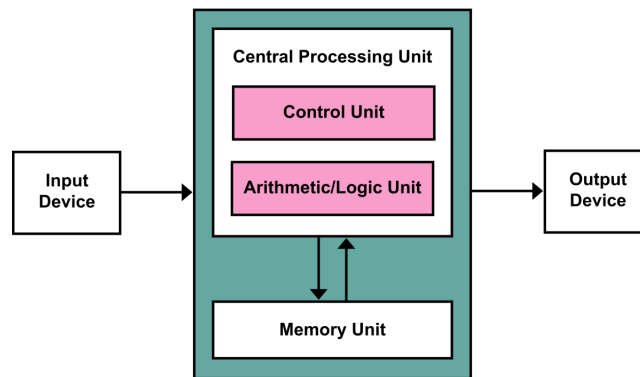
Figure 3: von Neumann Architecture [Wik18d]

Figure 4: AND gate [Wik18a]
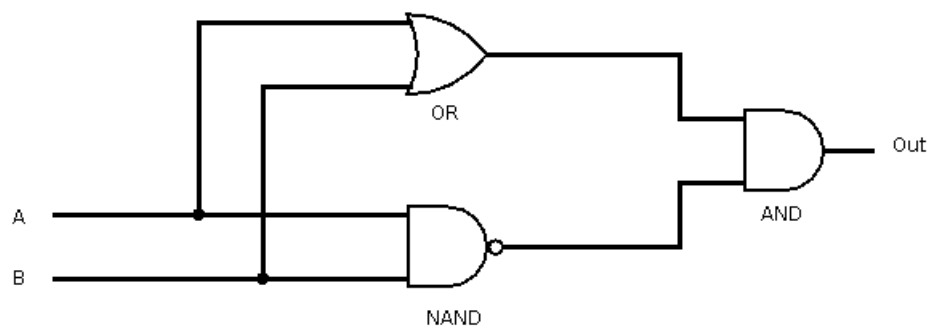

Figure 5: OR gate [Wik18c]

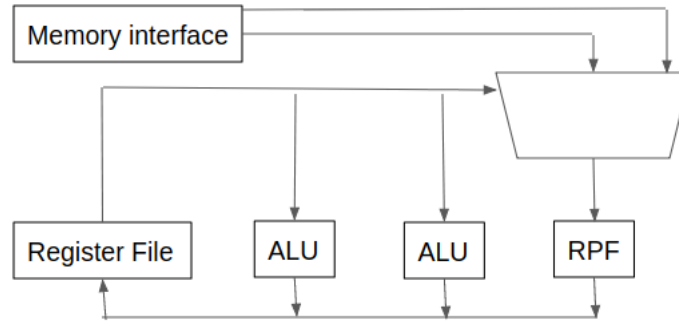Figure 6: XOR circuit made of OR, NAND, and AND gates [NTXGC]

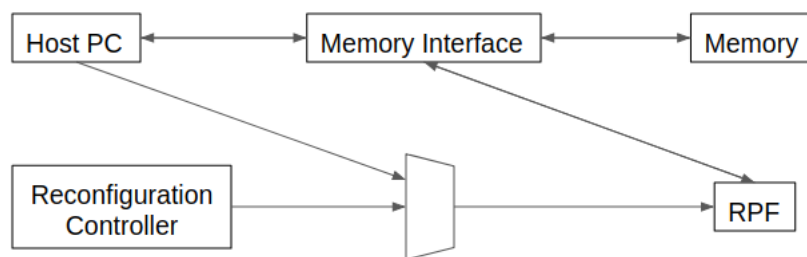Figure 7: CPU in von Neumann Architecture (Processor + RPF Architecture)

Figure 8: Independent Coprocessor Architecture

2.2   Reconfigurable Computing Architecture

Field programmable gate arrays (FPGA) are integrated circuits designed to be configured by a customer or a designer after manufacturing hence "field-programmable". A user can change what circuit do at compile and run time. Another example is the reconfigurable Processing Fabric (RPF) which comprises of logic elements or LUTs (lookup tables) that imlements Boolean functions, and interconnects. RPFs can be static, which means they can be configured between applications, or dynamic, which means they can be configured during applications.

RPFs are commonly integrated in various architectures. For example, in the CPU of the von Neumann Architecture (Figure 7), also known as the Processor and RPF architecture, data from memory and the register files flow into a MUX, whose output is processed in the RPF. Together with the outputs from the ALU, the output of the RPF flows into the register file. In coprocessor architecture (Figure 8), outputs from the MUX and the memory interface flow into the RPF, and the output of the RPF is written into memory.
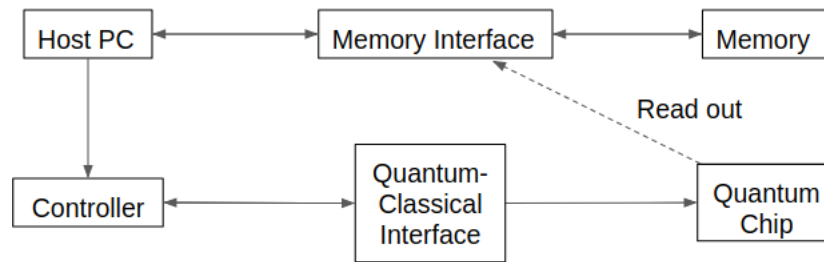
Figure 9: Quantum Computer Architecture

3 Quantum Computers

3.1 Quantum Computer Architecture

Quantum computer architecture (Figure 9) is similar to the independent coprocessor architecture, except that instead of a MUX, there is a quantum-classical interface, which translates between the quantum chip and the rest of the classical components. Data from the quantum chip can also be read out and written to the memory interface.

3.2 Quantum Circuit

Since quantum gates needs to be reversible, quantum gates must have the same number of inputs and outputs, unlike classical gates. Below are some examples on quantum gates:

- NOT(X) gate

$|a\rangle$ —[X]— $|Not(a)\rangle$

- CNOT(X) gate

$|a\rangle$ —•— $|a\rangle$
$|b\rangle$ —⊕— $|a \oplus b\rangle$

- Toffoli (CCNOT) gate

$|a\rangle$ —•— $|a\rangle$
$|b\rangle$ —•— $|b\rangle$
$|c\rangle$ —•— $|(a \wedge b) \oplus c\rangle$

Below is an example circuit. Qubits $|a\rangle, |b\rangle$, and $|c\rangle$ are input, while $|d\rangle$, and $|e\rangle$ are ancillae, or extra qubit. The first two outputs are valid, where the last three are garbage. While wires in classical circuits are metal, quantum wires represent time, with the leftmost part of the wire being earliest in time and the rightmost being the latest in time.

$|a\rangle$
$|b\rangle$
$|c\rangle$
$|d\rangle$
$|e\rangle$
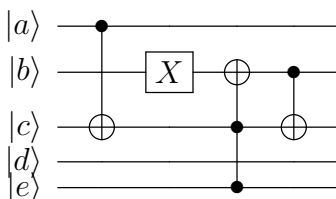
3.3 Performance Comparison: Quantum vs Classical

Let's compare two computers – one classical and one quantum.

The classical computer in our comparison is Intel Xeon Phi Processor, also known as "Knight's Corner" [OPD⁺17]. It is a commonly used processor in High Performance Computing (HPC), such as supercomputing clusters. This processor clocks at 1 GHz. Oliveira et. al. collected in 2017 the "soft error rate" (SER) which are radiation induced errors which can flip states. Its "soft error rate" was found to be around 100 FIT (failure in time), where 1 FIT is 1 error per billion device hours (114, 077 years).

Compare Knight's Corner to the the IBM Q16 Rueschlikon, a quantum computer. It clocks at 5.26 GHz. The statistics are obtained on the IBM calibration data page in October 2018. Its gate error was $\approx 2.21 \times 10^{-3}$, its read out error $\approx 5.76 \times 10^{-2}$ (5 errors for every 100 measurement), and its multi-qubit gate error $\approx 4.4 \times 10^{-2}$, all of which are orders of magnitude more frequent than the "soft error rate" of the Knight's Corner.

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