AE462
Exercise 2-3

Date
4/02/2024

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Exercise 2

1. Calculation of loads as a function of span

Flight loads using Schrenk's Approximation

$$L_e(y) = \frac{4L}{\pi b}\sqrt{1-\left(\frac{2y}{b}\right)^2}$$

where,

L = total lift = weight (for n=1)
b=wing span
y=distance along span from aircraft centre-line

$$L_t(y) = \frac{2L}{b(1+\lambda)}\left[1-\frac{2y}{b}(1-\lambda)\right]$$

where,

λ=taper ratio of trapezoidal wing

$$L_s(y) = \frac{1}{2}[L_e + L_t]$$

Weight(W)= 612.842 kg=Lift(L)
b(wingspan length)=10880 mm =10.88 m
λ (Taper ratio)=0.6
Wing Area= S = 9.48878 m^2
After putting values we get,

$$L_e(y)=703.556 * \sqrt{1-\frac{y^2}{29.59}}$$

$$L_t(y)=690.714 * \left[1-\frac{y}{13.6}\right]$$

The above equations of Le and Lt gives us Lift distribution per unit length. To better visualize this, we created sections throughout the wingspan. The length of each section along wingspan is 0.1 meter and the Ls used to calculate Lift of that section is average value of Ls at y_1 and y_1+0.1

Multiplying this avg Ls will give us lift in that section.

Since the half wingspan length is 5.44 m, we had total 54 sections. Values of $Lift_{section}$ and $y_{avg(section)}$ are tabulated on next slide

For Lift due to half wing,

integrating along spanwise direction, from root chord(y=0) to tip chord(y=b/2)

L_e= 613.1528 kg

L_t = 612.842 kg

$L_s = 0.5(L_e + L_t)$ = 612.9974 kg

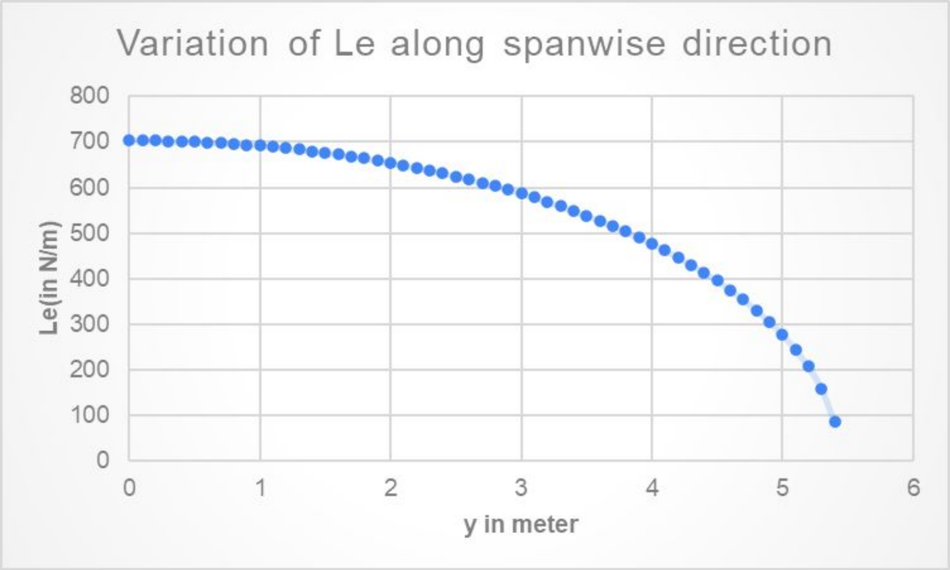
L_s =612.9974 kg ~ 612.842 kg = W

Hence, the total lift adds up to the weight of the aircraft.

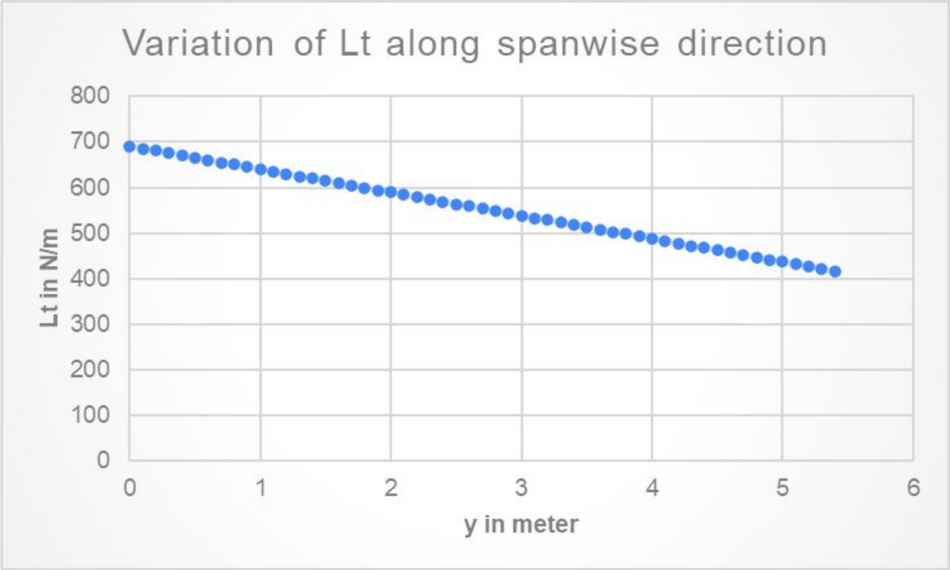
y(in m)	L(elliptic) N/m	L(trapezoidal) N/m	Ls	Section	y(avg for section)	Lift(Section) N
0	703.556	690.714	697.135	1	0.05	69.58355816
0.1	703.4371059	685.6352206	694.5361632	2	0.15	69.31772676
0.2	703.0803029	680.5564412	691.818372	3	0.25	69.03999086
0.3	702.4852284	675.4776618	688.9814451	4	0.35	68.75032622
0.4	701.6512764	670.3988824	686.0250794	5	0.45	68.44869638
0.5	700.5775936	665.3201029	682.9488483	6	0.55	68.1350524
0.6	699.2630759	660.2413235	679.7521997	7	0.65	67.80933263
0.7	697.7063619	655.1625441	676.434453	8	0.75	67.47146243
0.8	695.9058263	650.0837647	672.9947955	9	0.85	67.12135369
0.9	693.8595711	645.0049853	669.4322782	10	0.95	66.75890443
1	691.565415	639.9262059	665.7458104	11	1.05	66.38399823
1.1	689.0208819	634.8474265	661.9341542	12	1.15	65.99650355
continued						
5.1	244.7190071	431.69625	338.2076285	52	5.15	32.73922434
5.2	206.5362459	426.6174706	316.5768582	53	5.25	30.32746193
5.3	158.4060695	421.5386912	289.9723803	54	5.35	27.030432
5.4	84.81260766	416.4599118	250.6362597		5.45	

Link for complete sheet

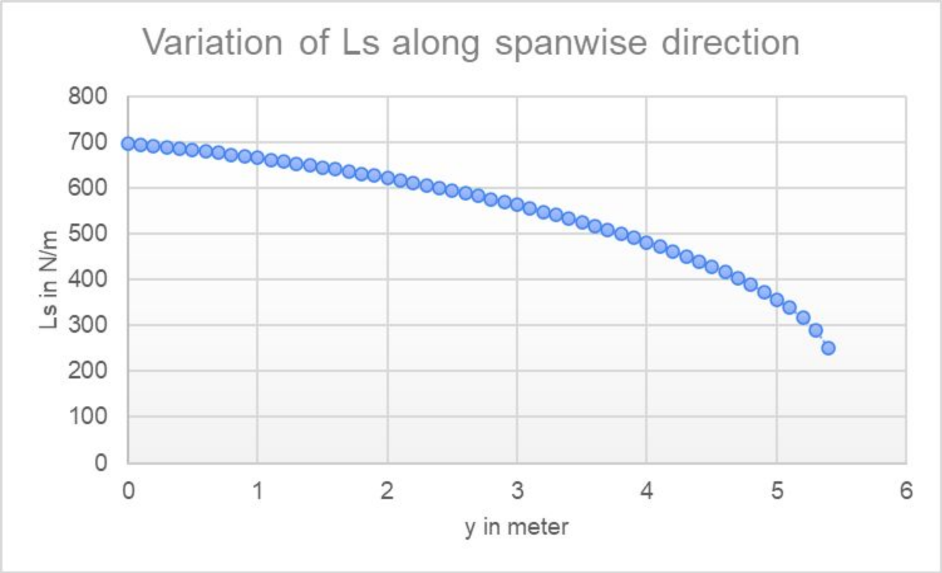
Plots



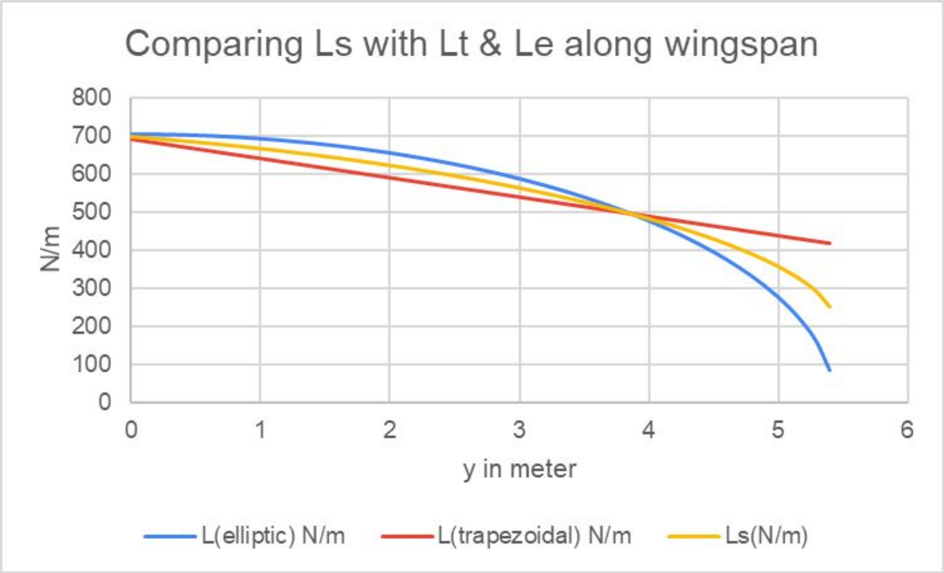
Variation of Le along spanwise direction
800
700
600
500
400
300
200
100
0
Le(in N/m)
0
1
2
3
4
5
6
y in meter



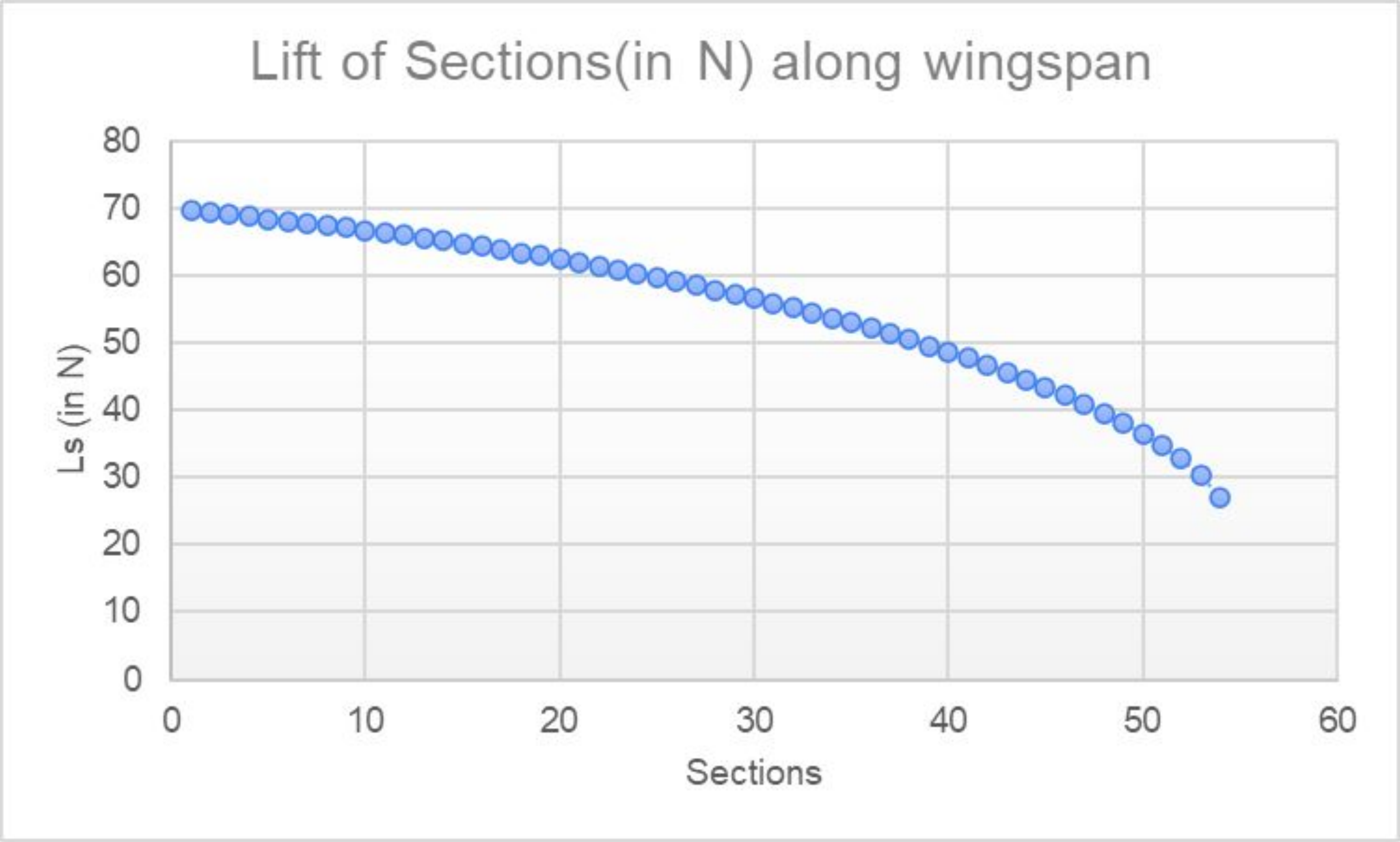
Variation of Lt along spanwise direction
800
700
600
500
400
300
200
100
0
Lt in N/m
0
1
2
3
4
5
6
y in meter



Variation of Ls along spanwise direction
Ls in N/m
y in meter



Comparing Ls with Lt & Le along wingspan
N/m
y in meter
L(elliptic) N/m
L(trapezoidal) N/m
Ls(N/m)


Lift of Sections(in N) along wingspan
Ls (in N)
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
Sections

2. Calculation of Gravity loads:

The Gravity load in an aircraft is the force of gravity experienced by the aircraft and its occupants during flight.

W (lbf)= Multiplier * Factor

Values of Multiplier(<u>general aviation in this case</u>) and Factor to be taken from chart below:

TABLE 10.4: Statistical weights of major components on various types of aircraft.

Component	Multiplier			Factor*
	Combat	Transport/ Bomber	General Aviation	
Main Wing	9.0	10.0	2.5	S_W
Horizontal Tail	4.0	5.5	2.0	S_W
Vertical Tail	5.3	5.5	2.0	S_W
Installed Engine	1.3	1.3	1.4	Uninstalled W_{engine}
Landing Gear	0.033 (Navy, 0.045)	0.043	0.057	W_{TO}
	(Navy, 0.045)			W_{TO}
Fuselage	4.8	5.0	1.4	$S_{fuse-wetted}$

***Note:** Areas have units of f^2 and weights have units of pounds.

Further, dead weight of wing structure, fuel/battery etc to be added

For the plane we designed in the course Aircraft Design-I,
Values of required parameters are as follows-

S_W = Wing Area = 9.48878 m^2

$W_{Electric\ Engine(motor)}$ = Weight of Uninstalled Motor= 23 kg

From historical data of E-811 (the first certified electric aircraft motor by EASA on May 18, 2020)

W_{To} = 612.842 kg

$S_{fuse\text{-}wetted}$ = Fuselage Wetted Area (the area which is in contact with the external airflow)= 10410000 mm^2= 10.41 m^2

Total Weight of these components= 368.18 kg
Dead Weight of Wing Structure:
(The wing weight represents about 22-27% of the empty weight)
For our Aircraft, W_{empty}= 356.75 kg
Hence, Dead W_{wing}=27% of 356.75kg= 96.32 kg
Weight of LIB Batteried(calculated in AE461)=142 kg
Total Gravity Loads= 368.2 +96.32 +142 = 606.52 kg
Total takeoff Weight= 612.842 kg

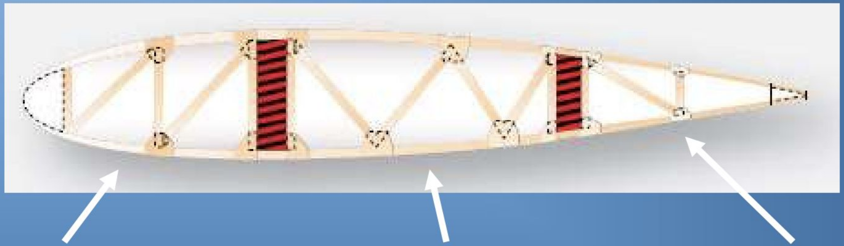
Hence, Total Gravity Loads= 606.52 kg ~ Total Weight

Component	Multiplier	Factor	Factor(corrected*)	Weight(lbf)	Weight(kg)
Main Wing	2.5	9.49 m^2	102.11 f^2	255.275	115.79
Horizontal Tail	2.0	9.49 m^2	102.11 f^2	204.22	92.63
Vertical Tail	2.0	9.49 m^2	102.11 f^2	204.22	92.63
Installed Engine	1.4	23 kg	23 kg	-	32.2
Landing Gear	0.057	612.842 kg	612.842 kg	-	34.93
Fuselage	1.4	10.41 m^2	112.01 f^2	156.814	71.13

*m^2 converted to f^2

Exercise 3: Preliminary design of wing box

- Preliminary Sizing considering only inter-spar box (the main load carrying portion of the wing)



In this exercise first we need to choose a spar cross section, material for spars and ribs and then calculate the variation of wing skin thickness & spar web thickness along wingspan using SF, BM and Torsion

[Excel sheet for Ex3]

Choosing a Material

Material taken: Aluminium Alloy 7075

G = 71.7 GPa

Max stress: 489.52 MPa

Torsional Shear Stress (fts) : 331 MPa

Reason- It can support stresses that are produced during high altitude flights

Choosing spar cross section

As discussed in class, we would be only considering inter-spar box since it carries the main portion of load.

Assuming front spar at 20% of the chord and rear spar at 60% of the chord, we calculated the interspar distance as 0.4*chord

To calculate height of box, we divided the cross section area of box by interspar length.

We assumed rib-spacing as 50 cm or 19.68 inches

y(in m)	chord length(m)	Interspar distance(w) (in m)	height of wing box (h) (in m)
0	1.1288365	0.4515346	0.1245406553
0.1	1.1205095	0.4482038	0.123621966
0.2	1.1121825	0.444873	0.1227032767
0.3	1.1038555	0.4415422	0.1217845874
0.4	1.0955285	0.4382114	0.1208658981
0.5	1.0872015	0.4348806	0.1199472087
0.6	1.0788745	0.4315498	0.1190285194
0.7	1.0705475	0.428219	0.1181098301
0.8	1.0622205	0.4248882	0.1171911408
0.9	1.0538935	0.4215574	0.1162724515
1	1.0455665	0.4182266	0.1153537621
1.1	1.0372395	0.4148958	0.1144350728
continued			
5.1	0.7124865	0.2849946	0.07860618932
5.2	0.7041595	0.2816638	0.0776875
5.3	0.6958325	0.278333	0.07676881068
5.4	0.6875055	0.2750022	0.07585012136

Calculation of Limit load distribution

We will calculate the limit load in each section,
For a section
$q(y)=L_s(\text{section})-W_t(\text{section})$

$L_s(\text{section})$= lift of section calculated using Schrenk's approximation
$W_t(\text{section})$=Weight of wing calculated considering trapezoidal wing

$$W_t(y)=\frac{2*W}{b(1+\lambda)} * [1-\frac{2y}{b}(1-\lambda)]$$

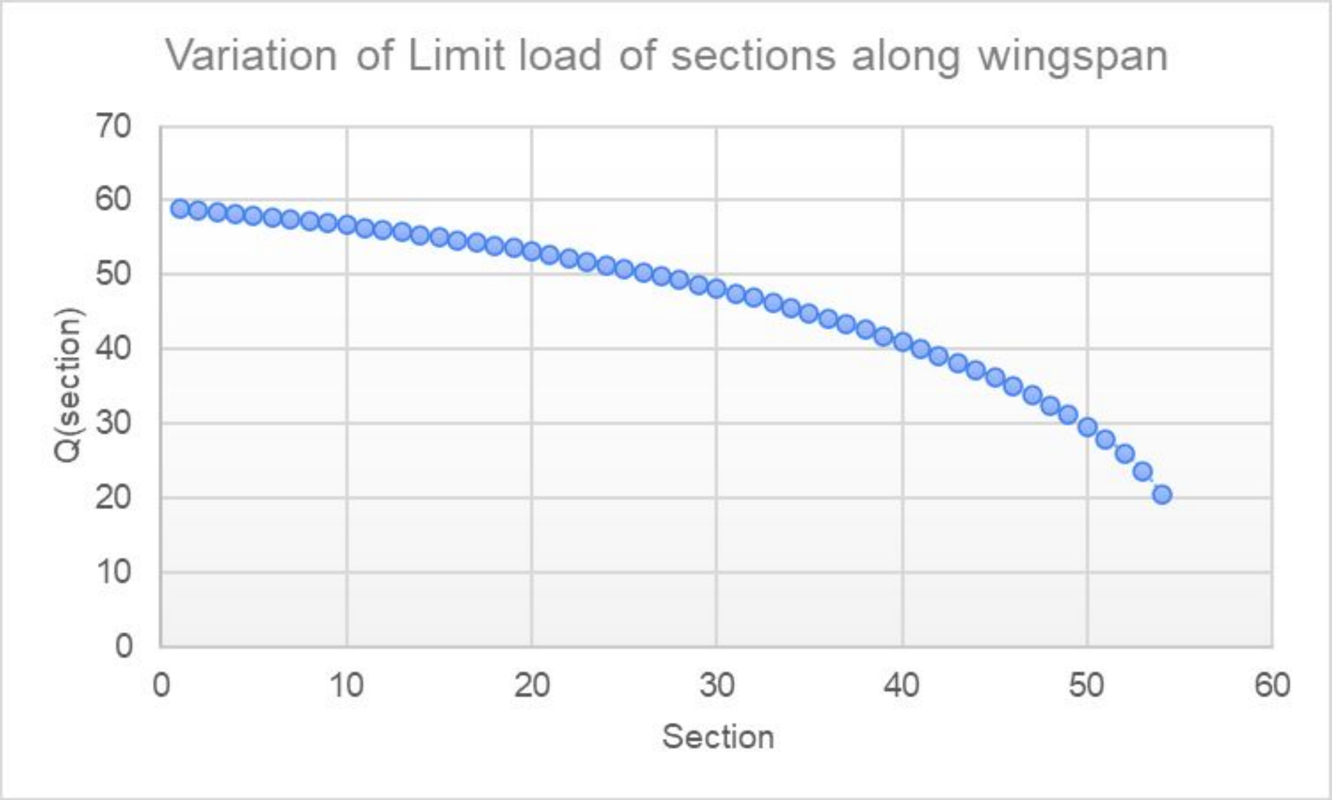
W=96 kg
b=10.88
λ=0.6

$W_t(y)=108.198 * [1- y/13.6]$
First we calculated $W_t(y)$, then we calculated W_t for each section by taking avg of W_t at two extreme y of that section
$W_t(\text{section})=0.5*(W_t(y_1)+W_t(y_1+0.1))$

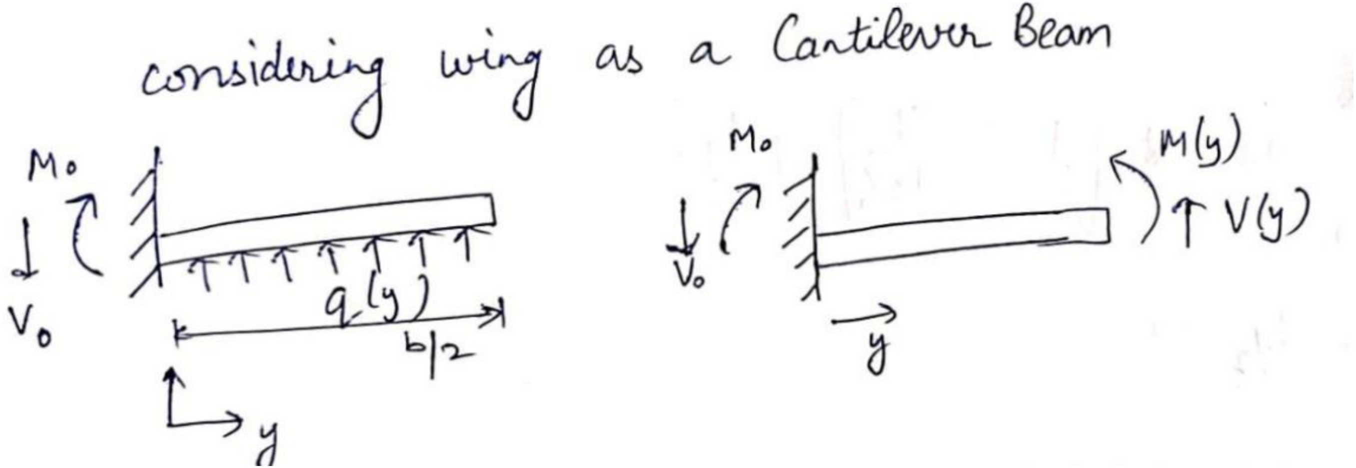
In Excel we have calculated limit load for all sections

Section	Lift(Section) N	Wt (section) N	Q (section)(in N)
1	69.58355816	10.78002132	58.80353684
2	69.31772676	10.70046397	58.61726279
3	69.03999086	10.62090662	58.41908424
4	68.75032622	10.54134926	58.20897696
5	68.44869638	10.46179191	57.98690447
6	68.1350524	10.38223456	57.75281784
7	67.80933263	10.30267721	57.50665543
8	67.47146243	10.22311985	57.24834257
9	67.12135369	10.1435625	56.97779119
10	66.75890443	10.06400515	56.69489928
11	66.38399823	9.984447794	56.39955044
12	65.99650355	9.904890441	56.0916131
continued			
52	32.73922434	6.722596324	26.01662801
53	30.32746193	6.643038971	23.68442296
54	27.030432	6.563481618	20.46695039

Plots


Variation of Limit load of sections along wingspan
Q(section)
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
Section

Calculation of Shear force distribution



We are idealising the wing as a cantilever beam for all calculations in this report

Calculation of Shear force distribution

Shear force $V(y) = V_O - \int_0^y q(y)\,dy$ where, $V_O = \int_0^{b/2} q(y)\,dy$

We have calculated the expression for q(y) earlier

$$q(y) = 351.778 * \sqrt{1 - \frac{y^2}{29.59}} + 236.439\left(1 - \frac{y}{13.6}\right)$$

Integrating it from 0 to b/2, we get V_O

Hence V_O=2528 N

Now putting that value in V(y) expression we can get shear force distribution

Calculations are done in excel.(Integrated function is written as formula)

$$\int_0^y q(y)\,dy = \left(5.44 * \sin^{-1}\left(\frac{y}{5.44}\right) + y * \sqrt{\frac{29.59 - y^2}{29.59}}\right) * 175.889 + \left(y - \frac{y^2}{27.2}\right) * 236.439$$

OR(using sections)

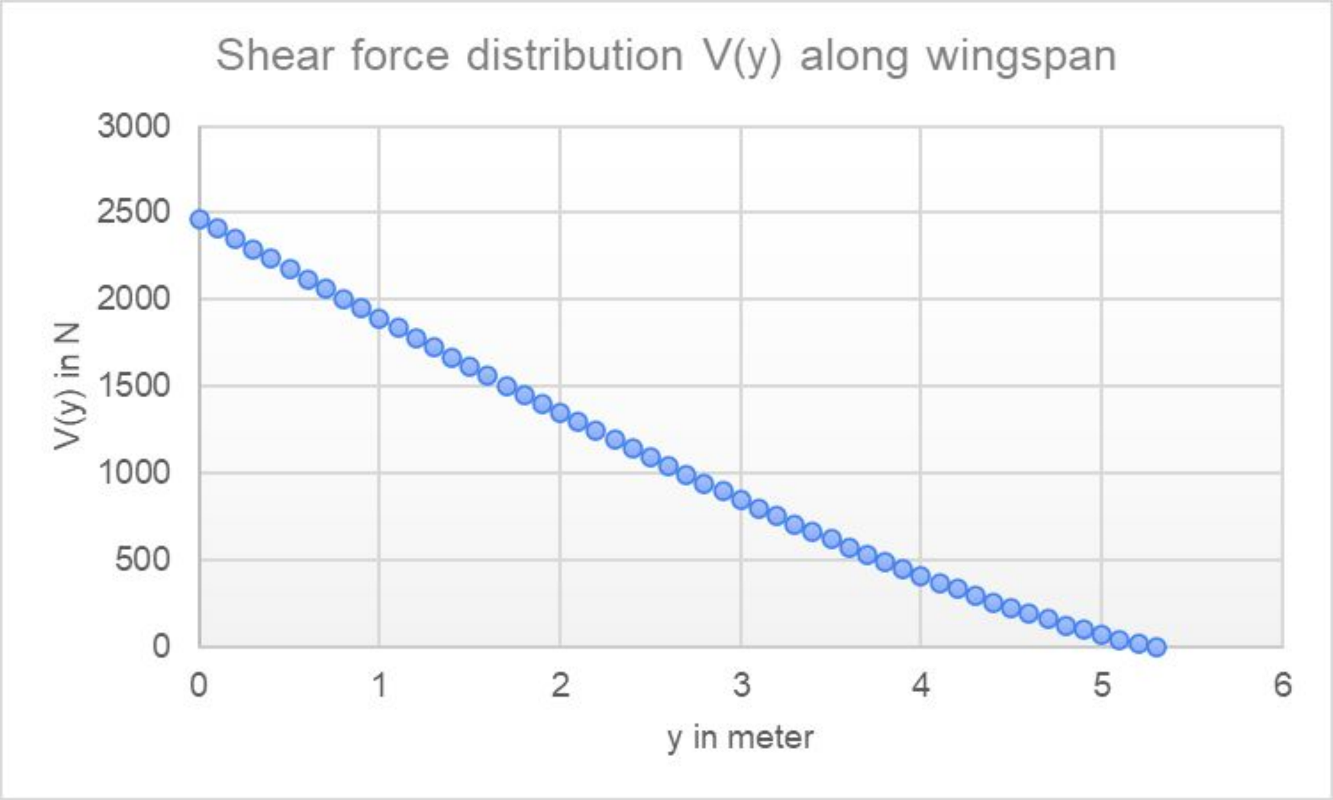
To calculate the V(y) at distance y from root chord, we can simply add the q(sections) of all the sections left to the y and then subtract that value from VO

We have done calculations using both methods and found that V(y) comes out to be the same.

Calculation of Shear force distribution

y(in m)	V(y) N
0	2468.964674
0.1	2410.347411
0.2	2351.928327
0.3	2293.71935
0.4	2235.732446
0.5	2177.979628
0.6	2120.472972
0.7	2063.22463
0.8	2006.246839
0.9	1949.551939
1	1893.152389
1.1	1837.060776
continued	
5.1	70.16800172
5.2	44.15137371
5.3	20.46695075
5.4	0.000000365469532

Plots



Shear force distribution V(y) along wingspan
3000
2500
2000
1500
1000
500
0
V(y) in N
0
1
2
3
4
5
6
y in meter

Calculation of Bending Moment

$$M(y) = M_O - \int_0^y q(y)*y\,dy, \text{ where } M_O = \int_0^{b/2} q(y)*y\,dy$$

$$M_O = \int_0^{b/2} 351.778*y*\sqrt{1 - \frac{y^2}{29.59}} + 236.439*y*\left(1 - \frac{y}{13.6}\right)$$

Integrating it from 0 to b/2, we get M_O

Hence M_O=6000 N

Now putting that value in M(y) expression we can get moment distribution

Calculations are done in excel.(Integrated function is written as formula)

$$\int_0^y q(y)*y\,dy = -3435*\left(1 - \frac{y^2}{29.59}\right)^{1.5} + 236.439*\left(\frac{y^2}{2} - \frac{y^3}{40.8}\right)$$

OR(using sections)

Similar approach to V(y), here to calculate moment of a section, we take cumulative sum of

$\sum_0^s q(s)*y(\text{section\_avg})$ of all the sections left to the y, and subtract it from M_O

Both Methods give the same result, can be seen in excel.

Calculation of Bending Moment

Since we have chosen the rib spacing as 50 cm, there will be 5 sections between two consecutive ribs.
In total there will be 10 ribs for half wingspan.

We need to know M(y) only at the cross sections of these ribs.

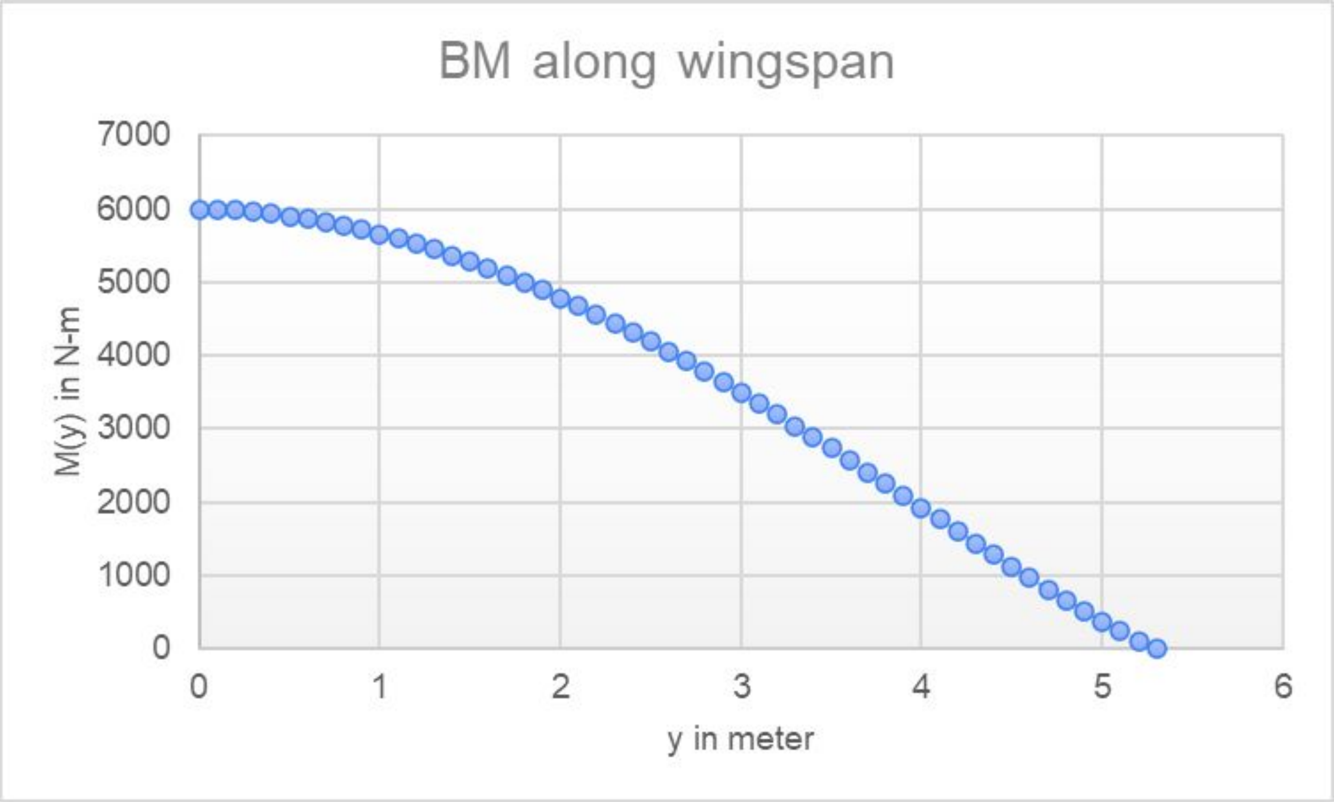
We have done calculation of M(y) at rib using section method which has been discussed on previous slides

Calculation of Bending Moment

y(in m)	M(y) in N-m
0	6001.481063
0.1	5992.688474
0.2	5978.083703
0.3	5957.710561
0.4	5931.616454
0.5	5899.852404
0.6	5862.473078
0.7	5819.536821
0.8	5771.105698
0.9	5717.245544
1	5658.026016
1.1	5593.520661
continued	
5.1	367.8270382
5.2	233.8414039
5.3	109.4981834
5.4	0

Ribs	Rib(location)in m	M(ribs) in N-m
1	0.5	5931.616454
2	1	5717.245544
3	1.5	5369.085009
4	2	4897.363147
5	2.5	4315.05845
6	3	3638.411546
7	3.5	2887.807307
8	4	2089.397641
9	4.5	1278.513371
10	5	508.999989

Plots



BM along wingspan
M(y) in N-m
y in meter
7000
6000
5000
4000
3000
2000
1000
0
0
1
2
3
4
5
6

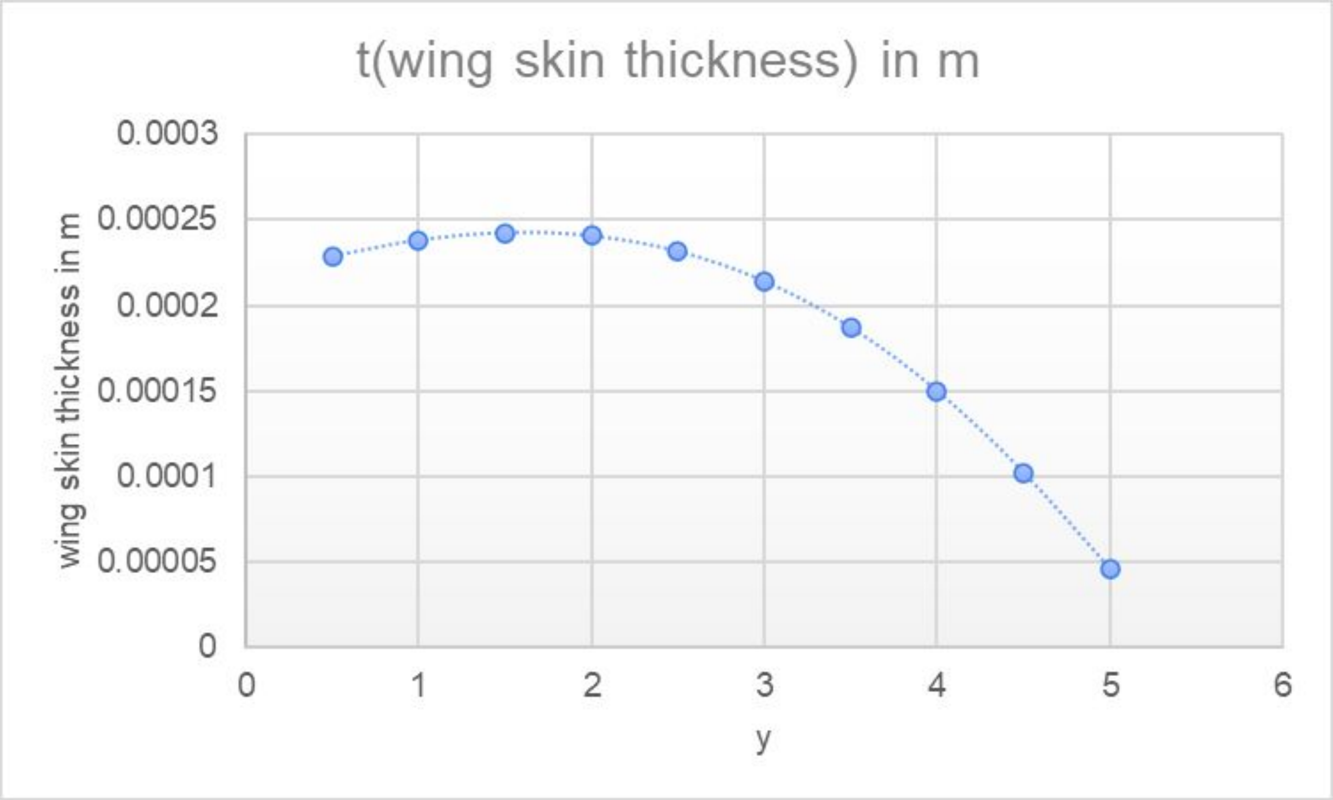
Calculation of Wing Skin thickness

$$t = \frac{M}{w*h*\sigma(tu)}$$

Where
t=wing skin thickness(in m)
M=Bending Moment at rig cross section
w=width of spar(in m)
h=height of spar(in m)
σ_{tu}=ultimate tensile strength of material =489.52 MPa

Rib(location)in m	t (wing skin thickness) in m
0.5	0.0002287784779
1	0.000238277403
1.5	0.0002425531287
2	0.0002406302212
2.5	0.0002314490873
3	0.0002138986055
3.5	0.0001868985855
4	0.0001495919642
4.5	0.000101806079
5	0.00004534834213

Plots



t(wing skin thickness) in m
wing skin thickness in m
0.0003
0.00025
0.0002
0.00015
0.0001
0.00005
0
0
1
2
3
4
5
6
y

Calculation of Torsion

Since the weight is acting on CG but the lift is acting at c/4, torque will be generated.
First we calculated the lift force acting at a cross section at distance y, and then multiplied it with the distance between cg and AC.
That Lead us to,

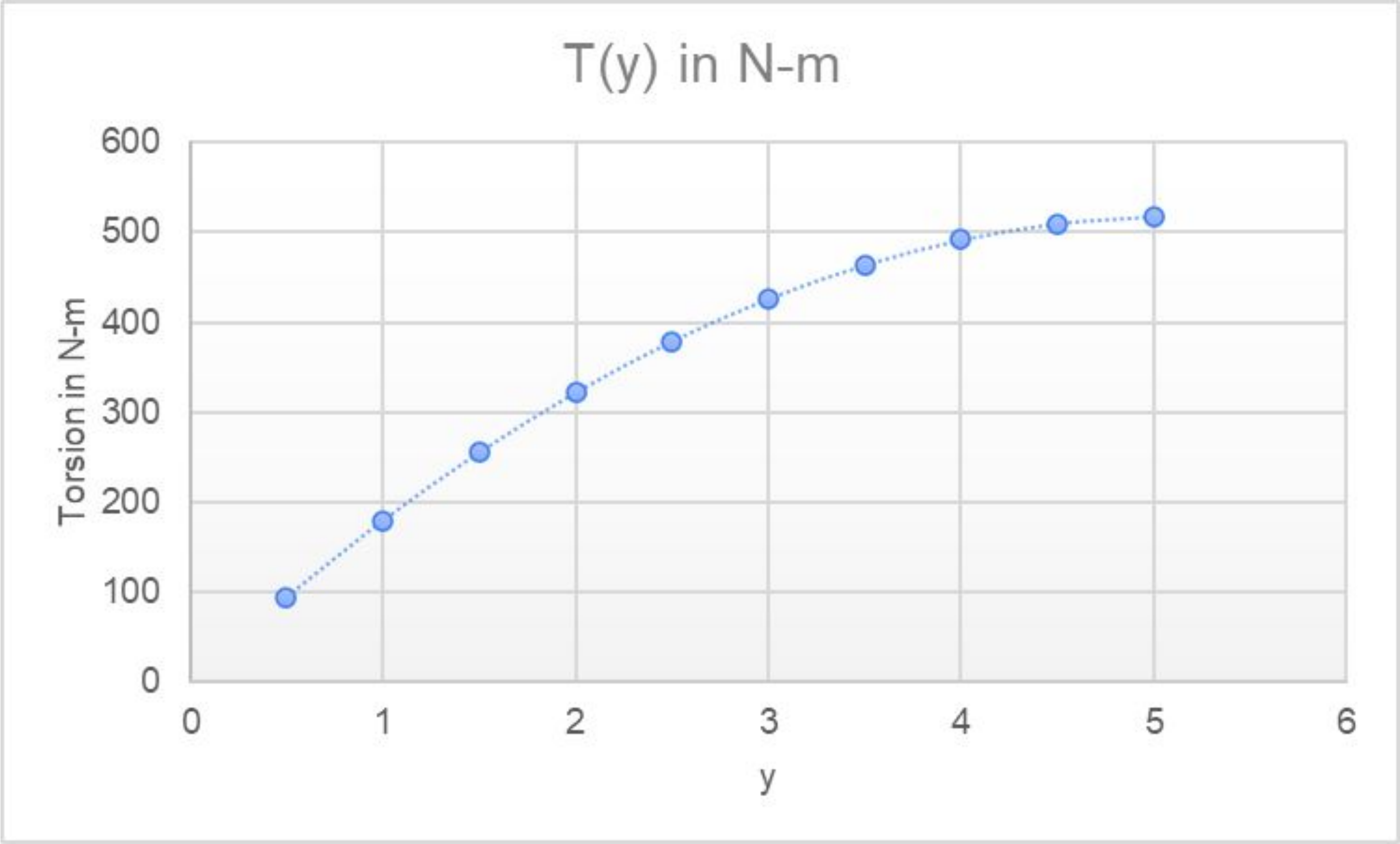
We are calculating Torsion in the cross section of a rig,
For a rig cross section, the weight is acting at CG of cross section but the lift is acting at AC(here c/4) Hence Torsion will be generated.

We will take the leading edge of airfoil as our reference to measure the CG and AC of cross section.
Torque about CG,
$T=L*(x_{CG}- x_{AC})$

Here L=Lift in Newton generated by all the sections left to the y $_{\text{(cross section of rig)}}$

Calculation for all rigs is has been done in excel

Plots



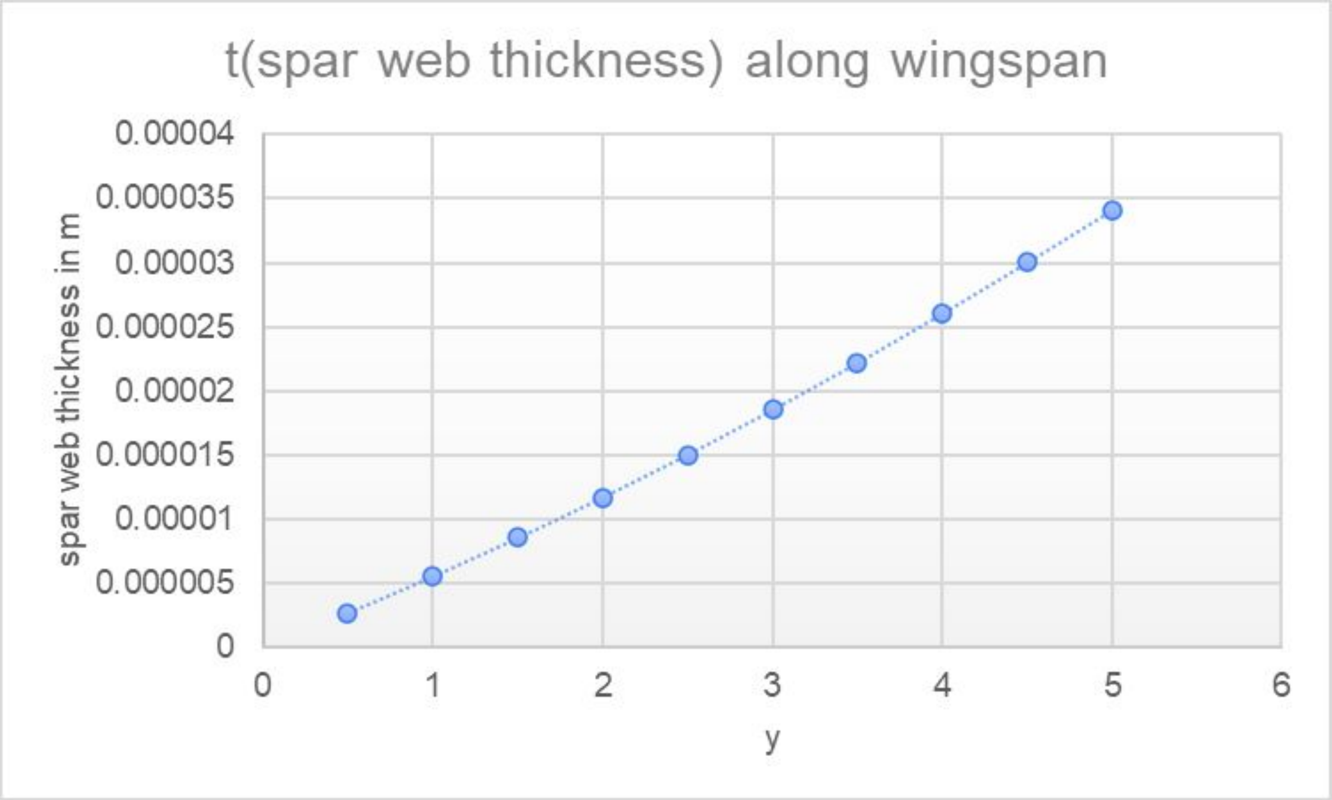
Calculation of Spar web thickness

$$f = \frac{T}{2a*b*t}$$

Where
t=Spar web thickness(in m)
T=Torsion at rig cross section
b=width of spar(in m)
a=height of spar(in m)
f=Min(Tensile strength, compressive strength)=331 MPa

Rib(location)in m	T(y) in N-m	t(spar web thickness) in m
0.5	94.52775834	0.000002695962921
1	179.8038226	0.000005541245411
1.5	255.6846667	0.000008541295277
2	322.0702632	0.00001170175337
2.5	378.8951008	0.00001502796691
3	426.1172236	0.00001852412366
3.5	463.7018823	0.0000219162542
4	491.5917106	0.000260258047
4.5	509.6391972	0.00003000846013
5	517.3978541	0.00003408636699

Plots



t(spar web thickness) along wingspan
spar web thickness in m
0.00004
0.000035
0.00003
0.000025
0.00002
0.000015
0.00001
0.000005
0
0
1
2
3
4
5
6
y