Linear-Time Invariant Control of Quadcopter

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***Abstract*—The present work focuses on mathematical modelling and simulation of a quadcopter, aiming to develop an appropriate control strategy. Building upon a previously proposed idea in [1], the objective is to design a linear-time invariant controller for the quadcopter.**

Two control approaches, Linear Quadratic Control (LQR) and LQR with a Gaussian estimator are derived and implemented. These control laws are compared to assess their performance. The LQR approach provides a systematic methodology for designing optimal controllers, while the LQR with a Gaussian estimator incorporates a state estimation component to enhance control performance.

I. Introduction

Multicopters have made a comeback over the past three decades with substantial developments in the miniaturization of mechanical and electronic equipment. The multicopters that came before it was big, complicated, and unstable, and they needed a competent pilot to fly them. Modern multicopters are more manoeuvrable, simpler, smaller, and more reliable [2]. Since multicopters can be remotely operated by a radio transmitter or from a computer ground station, they are categorized as Unmanned Aerial Vehicles (UAVs). [3]

Quadcopters have four rotors, generally in a square pattern. They can manoeuvre in small places and hover with stability. In comparison to other multirotor arrangements, they are also easier to comprehend and construct mechanically. However, they also have significant drawbacks, such as under actuation, which results in less stability than hexacopters and higher power consumption when compared to tricopters. Quadcopters are becoming common because of their adaptability and variety of uses in military and non-combat environments.

Quadcopters are used for a variety of non-combat and military purposes. Aerial photography, search and rescue operations, environmental monitoring, and precision agriculture are a few examples of non-combatant applications. Quadcopters can be used for reconnaissance, surveillance, target acquisition, and battlefield evaluation in a military setting. They are also being investigated for transportation and parcel delivery services in cities. Quadcopters can access regions that are challenging or impossible for other types of aircraft to reach because of their small size and manoeuvrability. To gather data or carry out particular activities, they can also be fitted with a variety of sensors and cameras.

Quadcopters are increasingly being used in more possible industries, which spurs research efforts to better and solve the problems they now face. Obstacle avoidance for autonomous navigation, longer quadrocopter flight times, and enhancing the control system to ignore disturbances and follow reference commands are a few of these difficulties. Quadcopter control is critical for the performance and manoeuvrability of the copter, specifically when there is a presence of wind, turbulence, and dynamic obstacles.

II. Literature Review

A set of control strategies such as PID (Proportional-Integral-Derivative) Control and Model Predictive Control (MPC) are used in quadcopter control.

PID (Proportional-Integral-Derivative) Control is a commonly utilized control approach in several areas, including quadcopter control. It uses proportional, integral, and derivative terms to change the control output in response to the error between the desired and actual states. PID control can perform satisfactorily in some circumstances and is easy to implement [4]. However, it's handling of nonlinear dynamics and disturbances is limited.

Model Predictive Control (MPC) is an approach for optimizing control inputs using a dynamic model to predict the system's future behaviour. It excels at managing restrictions and monitoring reference trajectories. MPC has been used to control quadcopters, and the stability and tracking precision performance is encouraging [5]. However, the computationally demanding optimization procedure needed for MPC may prevent its real-time implementation on platforms with limited resources.

Aiming to modify the control settings in response to the system's shifting dynamics and uncertainties is adaptive control. It is flexible to respond to changes in the quadcopter's settings and outside disturbances. For quadcopters, adaptive control has been investigated, including adaptive backstepping control [6]. Although adaptive control can boost efficiency in many situations, it frequently requires precise knowledge of the system's characteristics and may have stability and convergence problems.

LQG (Linear Quadratic Gaussian) control combines the estimation skills of the Kalman filter with the optimal control method of the LQR (Linear Quadratic Regulator). It offers a framework for a better control design while incorporating system dynamics and measurement noise into account [7]. LQG control is used in many domains, including quadcopter control. LQG control improves stability and performance by defining the control problem as an optimization work and minimizing a cost function that balances control effort and state variances.

LQG control is a good fit for the current study since the mathematical modelling of the quadcopter system can be linearized. Additionally, measurement noise can have a substantial impact on the control performance of the quadcopter. The LQG framework's Kalman filter offers the best estimate of the system state, accounting for the measurement noise. Hence, LQG control enables robust and accurate control of the quadcopter by tweaking the control gains to match particular performance needs. LQG control is an effective control method for the current study, even though PID control is straightforward and MPC offers trajectory tracking capabilities. Adaptive control can also adjust to system fluctuations. It is an efficient option for stabilizing and controlling quadcopters because of its capacity to take into account system dynamics, manage measurement noise, and optimize control inputs.

III. Problem Statement

Numerous industries, including aerial surveillance, search and rescue operations, logistics, and photography, have seen a considerable increase in the use of quadcopters. Obstacle avoidance, prolonged flying periods, and robust control in the presence of disturbances are still major issues. An efficient control law could help with steady and accurate quadcopter control. In this analysis a linear-time invariant control strategy is evaluated for quadcopter control. A linearised quadcopter model is used for designing the control system. Two control strategies, Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) are developed.

The LQR control approach seeks to develop a feedback controller that minimizes a quadratic cost function. The LQG control strategy combines the LQR controller with an estimator, like the Kalman filter, to account for system uncertainties and measurement noise. To assess the performance of LQR and LQG control strategies further simulation and analysis are done to understand the advantages and disadvantages of the LQR and LQG strategies. The outcomes will make it possible to choose and apply appropriate quadcopter control systems for uses, including aerial surveillance, search and rescue missions, logistics, and photography.

IV. Results

A. Mathematical Model

The orientation of the quadcopter in a body-fixed frame using ϕ, θ, and ψ which corresponds to the roll, pitch and yaw angles. To transform this rotation into the inertial frame, we can use the rotation matrix R as shown from the body-fixed frame to the earth-fixed frame (The notation of C_x and S_x are the abbreviated versions of the $\cos(x)$ and $\sin(x)$ operators):

$$R = \begin{bmatrix} C\psi C\theta & C\psi S\theta S\varphi - S\psi C\varphi & C\psi S\theta C\varphi + S\psi S\varphi \\ S\psi C\theta & S\psi S\theta S\varphi + C\psi C\varphi & S\psi S\theta C\varphi - C\psi S\varphi \\ -S\theta & C\theta S\varphi & C\theta C\varphi \end{bmatrix}$$

where C_x and S_x represent $\cos(x)$ and $\sin(x)$ respectively. Equations of motion for the system can be given as

$$\ddot{\phi} = \frac{(I_{yy} - I_{zz})\,\dot{\theta}\dot{\psi}}{I_{xx}} + \frac{U_1}{I_{xx}}$$
$$\ddot{\theta} = \frac{(I_{zz} - I_{xx})\,\dot{\phi}\dot{\psi}}{I_{yy}} + \frac{U_2}{I_{yy}}$$
$$\ddot{\psi} = \frac{(I_{xx} - I_{yy})\,\dot{\phi}\dot{\theta}}{I_{zz}} + \frac{U_3}{I_{zz}}$$

As discussed and derived earlier in [8] linearized model for quadcopter could be written as -

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

The system is linearised at hover position and is proved to be observable [9]

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{I_{xx}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix}$$

here, I_{xx}, I_{yy} and I_{zz} represent the moment of inertia of along each of the cartesian axis.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where -

$$x = \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}$$

$$u = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

B. Control Methods

1) LQR Control: The cost function for the LQR controller is typically defined as the sum of quadratic terms of the state and control input:

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

where:

x is the state vector

u is the control input vector

Q is the state weighting matrix

R is the control input weighting matrix In this case, let's choose the state weighting matrix

Q as an identity matrix, which implies equal importance for all state variables. The control input weighting matrix

R determines the relative importance of the control effort.

To compute the LQR gains, appropriate values for the weighting matrices Q and R are chosen. Then solving the algebraic Riccati equation the optimal state feedback gain matrix K is obtained,

K:

$$K = R^{-1} B^T P$$

where

P is the solution to the Riccati equation:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0$$

u can be computed as:

$$u = -Kx$$

2) LQG Controller: The LQG control law consists of two components:

1. The state feedback control law:

$$u = -Kx$$

where K is the state feedback gain matrix.

2. The state estimator:

$$\hat{x} = L(y - \hat{y})$$

where $\hat{x}$ is the estimated state vector, $\hat{y}$ is the estimated output vector, L is the estimator gain matrix, and $(y - \hat{y})$ is the measurement error.

The control law is given by:

$$u = -K\hat{x}$$

where $\hat{x}$ is the estimated state vector obtained from the state estimator.

The LQG control law provides optimal control by minimizing a cost function defined as:

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

where Q and R are positive definite weighting matrices.

The optimal state feedback gain matrix K can be obtained by solving the algebraic Riccati equation:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0$$

where P is the solution to the Riccati equation.

The Kalman filter is an estimator that estimates the system state based on measurements from sensors. It provides an optimal estimate of the system state by combining the prediction from the dynamic model and the measurement update. The Kalman filter equations are as follows:

Prediction Step:

$$\hat{x}^+ = A\hat{x}^- + Bu$$

$$P^+ = AP^- A^T + Q$$

Measurement Update Step:

$$K = P^+ C^T (CP^+ C^T + R)^{-1}$$

$$\hat{x} = \hat{x}^+ + K(y - C\hat{x}^+)$$

$$P = (I - KC)P^+$$

where $\hat{x}^+$ is the predicted state, $\hat{x}$ is the estimated state, P^+ is the predicted error covariance matrix, P is the updated error covariance matrix, C is the measurement matrix, Q is the process noise covariance, R is the measurement noise covariance, and y is the measured output.

The combination of the LQR controller and the Kalman filter forms the LQG control system. The LQG controller utilizes the estimated state from the Kalman filter to compute the optimal control input, taking into account both the system dynamics and the available measurements. This integration of estimation and control enables robust and optimal performance in the presence of uncertainties and disturbances

V. NUMERICAL RESULTS/EXPERIMENTS

The system parameters are modelled as -

TABLE I: Drone Properties

Property	Value
Inertia (I_{xx}, I_{yy}, I_{zz})	(1, 1, 1)
Length	0.1

A. LQR Control

An initial state is defined for the quadcopter representing the initial values of roll, roll rate, pitch, pitch rate, yaw, and yaw rate. Simulation is run for 20 seconds, and a LQR controller function is defined. It takes the current time t and the current state x as inputs and returns the control input based on the LQR control law. The ode45 function is used to numerically integrate the system dynamics over the defined time span using the LQR controller.

The results are as figure 1 -

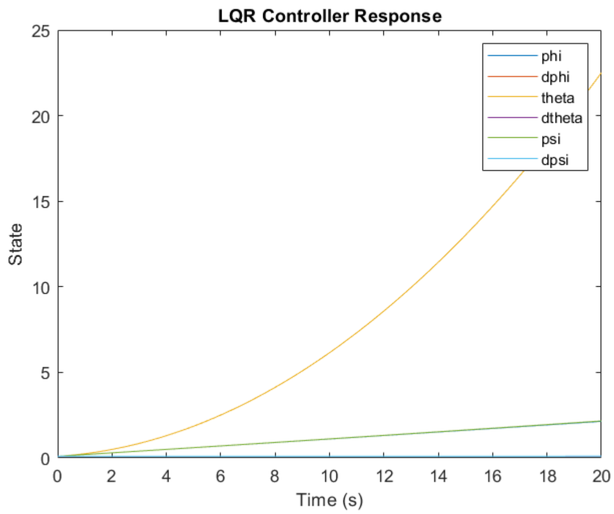

Fig. 1: LQR Control Design

B. LQG Control

The system matrices A, B, C, and D define the state-space representation of the quadcopter system. The ss function is used to construct the state-space model sys using the system matrices. The lqry function is used to design the LQG controller based on the state-space model sys, the weighting matrices Q, R, and N. The resulting controller gain matrix K is computed. The ode45 function is used to numerically integrate the system dynamics over the defined time span using the LQG controller.

The results are as shown in figure 2

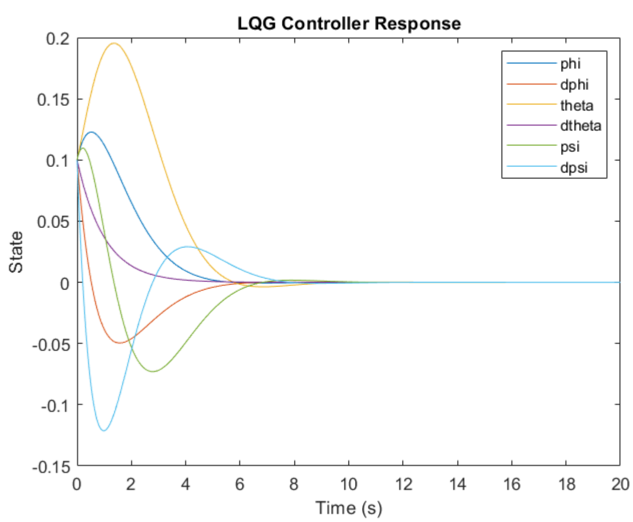

Fig. 2: LQG Control Design

A animation representing the quadcopter is as figure 3-

The error and control input plots are as for control inputs in figure 4 and for control error in figure 5 -

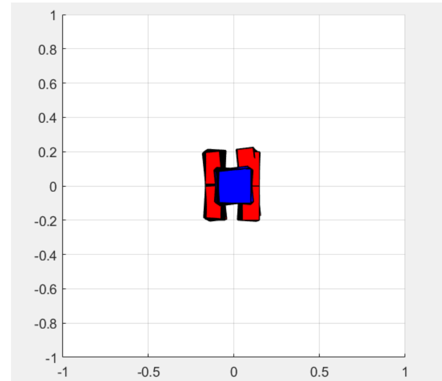

Fig. 3: Animation - Quadcopter

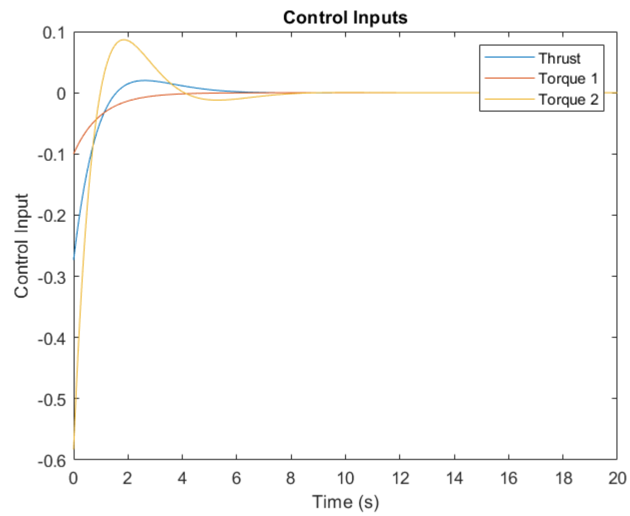

Fig. 4: Control Inputs

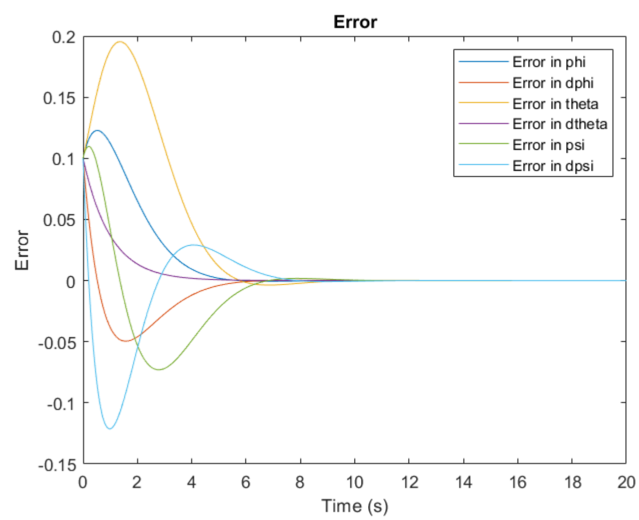

Fig. 5: Control Error

Clearly, here LQG controller is able to efficiently stabilize the system as compared to LQR controller, hence it is used for present analysis. The implementation in MATLAB refer to [10]

VI. Conclusion and future work

In this study, we presented a design and implementation of LQG controller for a quadrotor system. The LQG controller was designed based on the system matrices and weigting matrices using the LQR and Kalman filtering techniques. The performance of the controller was evaluated through simulations, considering the response of the system's states and the corresponding control inputs. The results demonstrated the effectiveness of the LQG controller in stabilizing the quadrotor system. The controller successfully regulated the states, including roll angle, pitch angle, yaw angle, and their corresponding derivatives, to the desired values. Additionally, the control inputs, including thrust and torques, were computed to drive the system towards the desired states.

However, there are several avenues for future work and improvement. First, the current controller design assumed perfect knowledge of the system matrices and noise statistics. In practice, these parameters are often uncertain and subject to variations. Therefore, developing robust control strategies that can handle model uncertainties and disturbances would be beneficial. Second, the presented LQG controller focused on state regulation and tracking. Future research can explore the inclusion of additional objectives, such as optimizing energy consumption or minimizing control effort. This could involve incorporating additional terms in the weighting matrices or utilizing advanced control techniques like model predictive control.

Furthermore, real-world experiments can be conducted to validate the controller's performance and robustness in practical scenarios. The implementation of the controller on an actual quadrotor platform would provide valuable insights into its limitations and enable further refinement.

In conclusion, the LQG controller demonstrated promising results in stabilizing and tracking the quadrotor system. By addressing the challenges of model uncertainties, incorporating additional objectives, and conducting real-world experiments, the controller's performance can be enhanced, making it more applicable to real-world applications.

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