Maths I

Lecture 9

Limits :

one variable case :

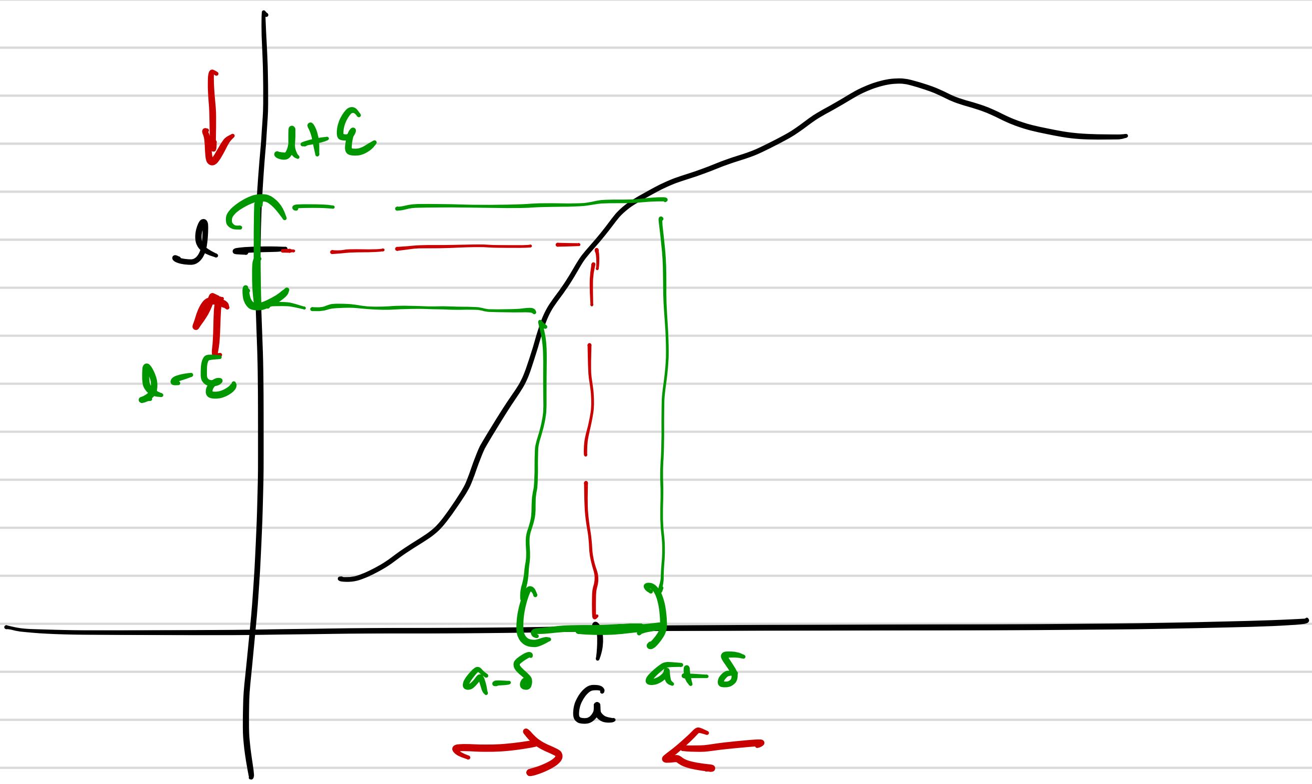
$f : \mathbb{R} \to \mathbb{R}$, let $a \in \mathbb{R}$

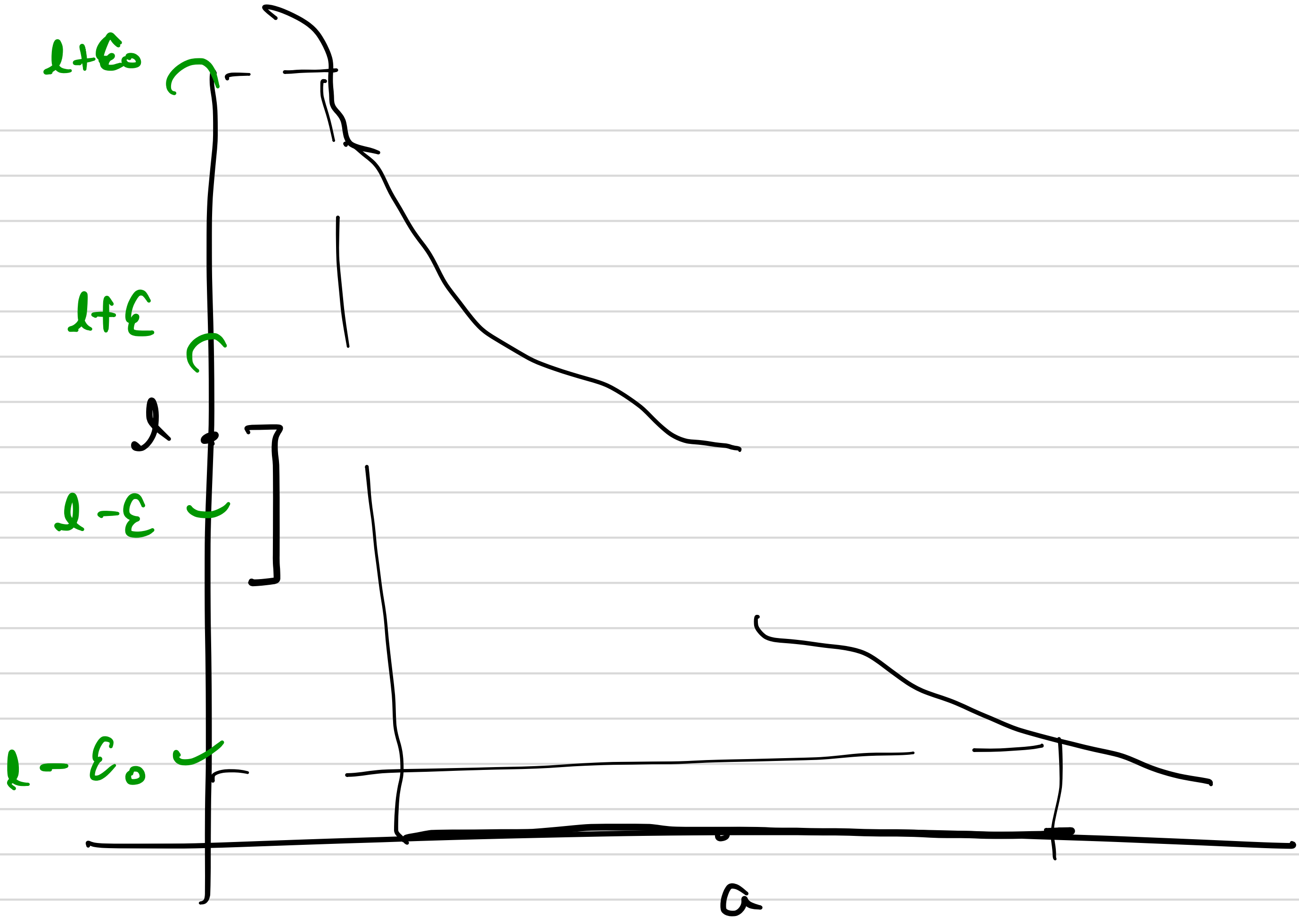
$$\lim_{x \to a} f(x) = l$$

We know: $\lim_{x \to a} f(x) = l$ is same as

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$$LHL = RHL$$





Definition: We say that a $f(x,y)$ has a limit L as $(x,y) \to (x_0, y_0)$ and write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

if for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every (x,y) in the domain of f,

$|f(x,y) - L| < \varepsilon$ whenever

$$0 < \sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta$$

Case of 1-variable.

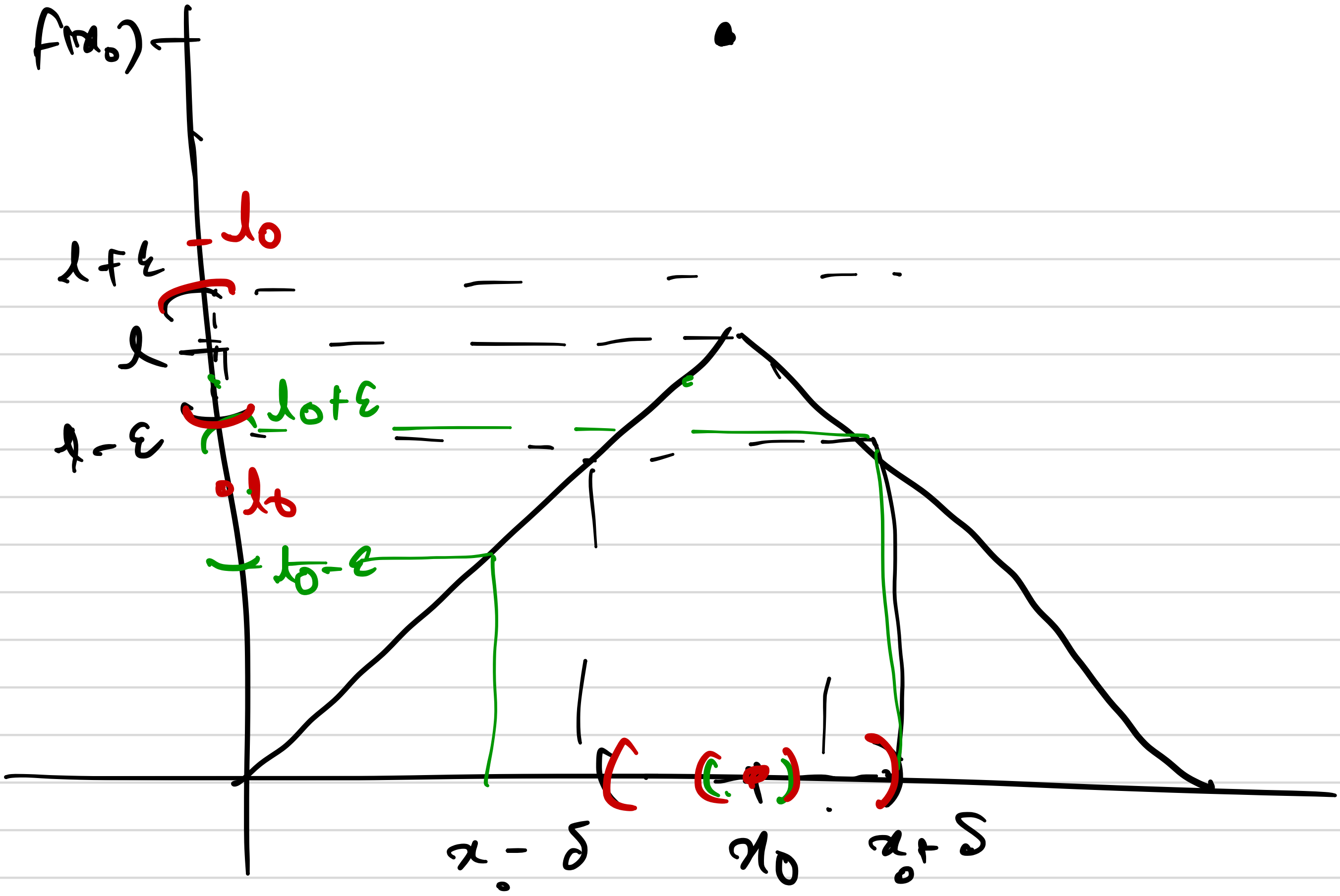
We say that $f(x)$ has a limit L as x approaches x_0 and write

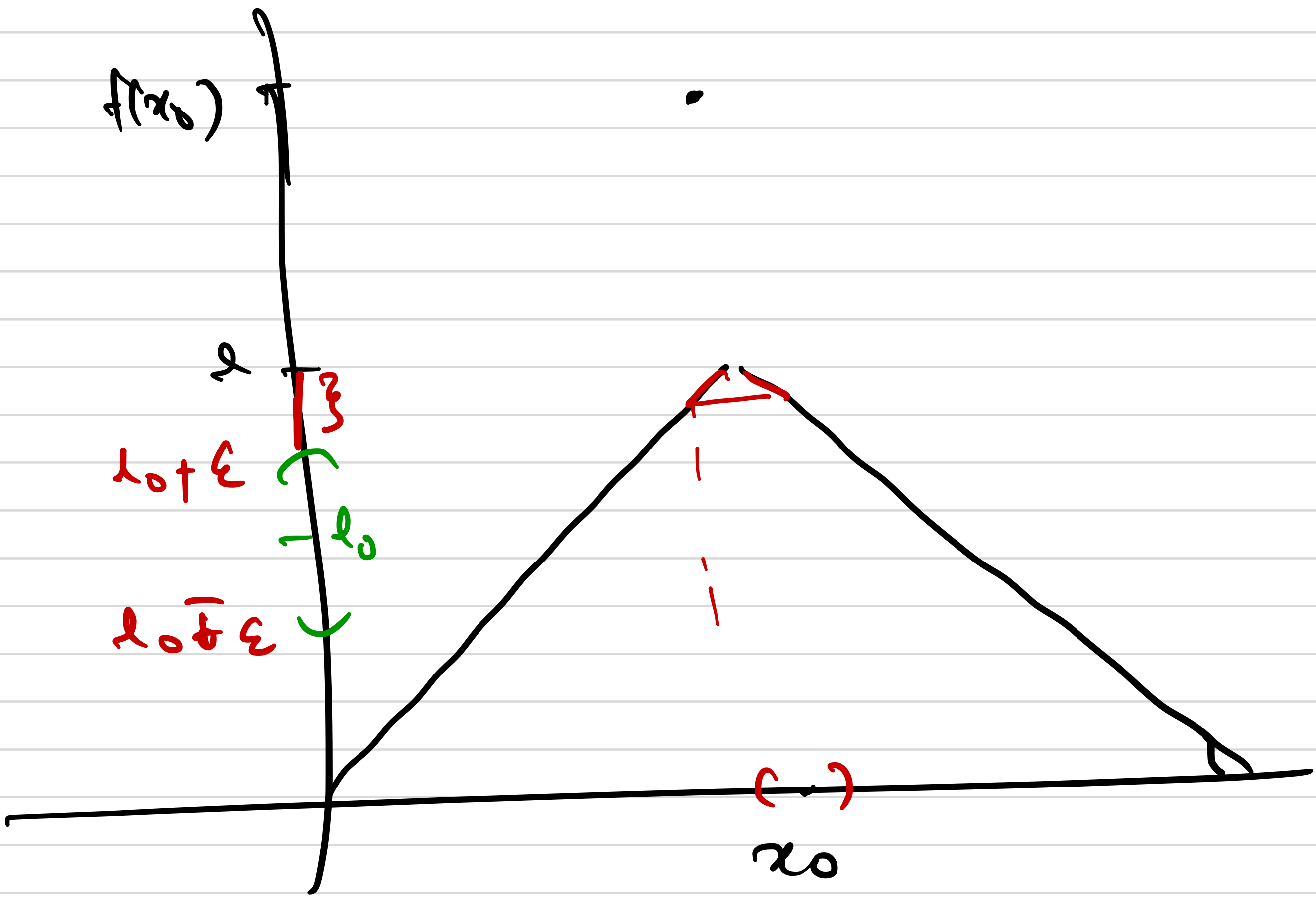
$$\lim_{x \to x_0} f(x) = L$$

if for every $\varepsilon > 0$, $\exists$ a $\delta > 0$ such that for all $x \in$ domain f

whenever $0 < |x - x_0| < \delta$,

$$|f(x) - L| < \varepsilon$$





Claim $\lim_{x \to x_0} f(x) = l_0$