Maths 1

Lecture 2

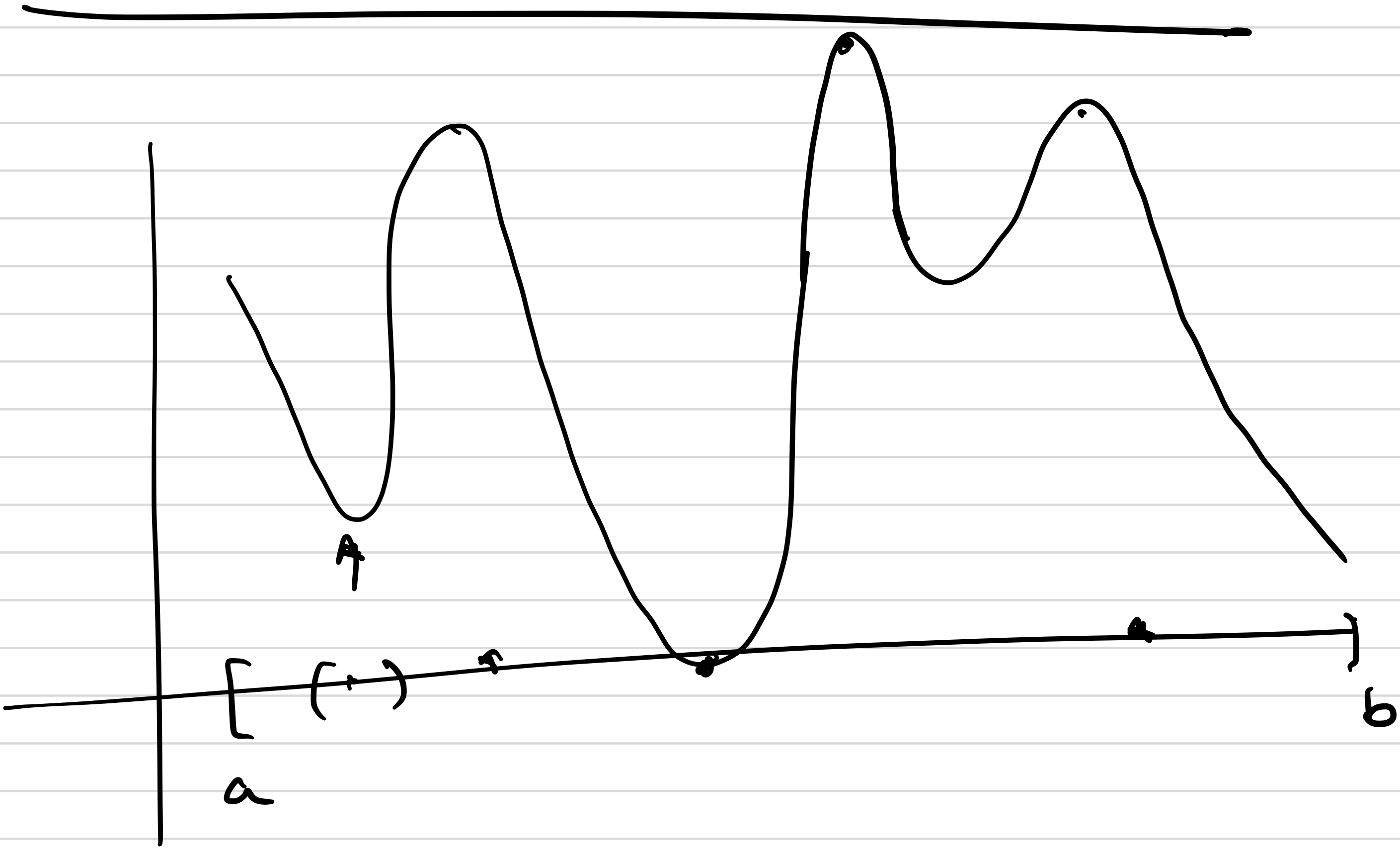
Differentiability.

$f: (a,b) \longrightarrow \mathbb{R}$

$c \in (a,b)$

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Extremum values of a function.



Two important properties.

i) If a function $f : [a, b] \longrightarrow \mathbb{R}$ is continuous, then it attains its maximum and minimum.

ii) $f : [a,b] \longrightarrow \mathbb{R}$ is such that f is differentiable on (a,b) & continuous on $[a,b]$, then for every extremum of f, $f'(c) = 0$

<u>Rolle's theorem</u>:

Suppose $y = f(x)$ is continuous over the closed interval $[a,b]$ and differentiable on the open interval (a,b) and $f(a) = f(b)$. Then there exists a point $c \in (a,b)$ such that $f'(c) = 0$

Proof :

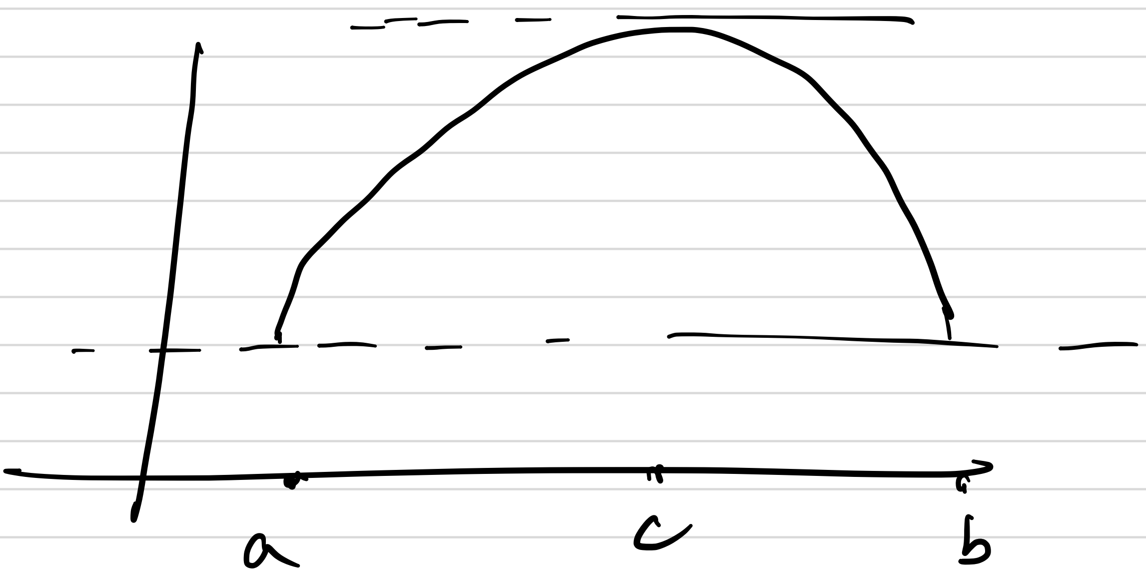
Since f is continuous function on $[a,b]$, f attains its maximum and minimum values on $[a,b]$.

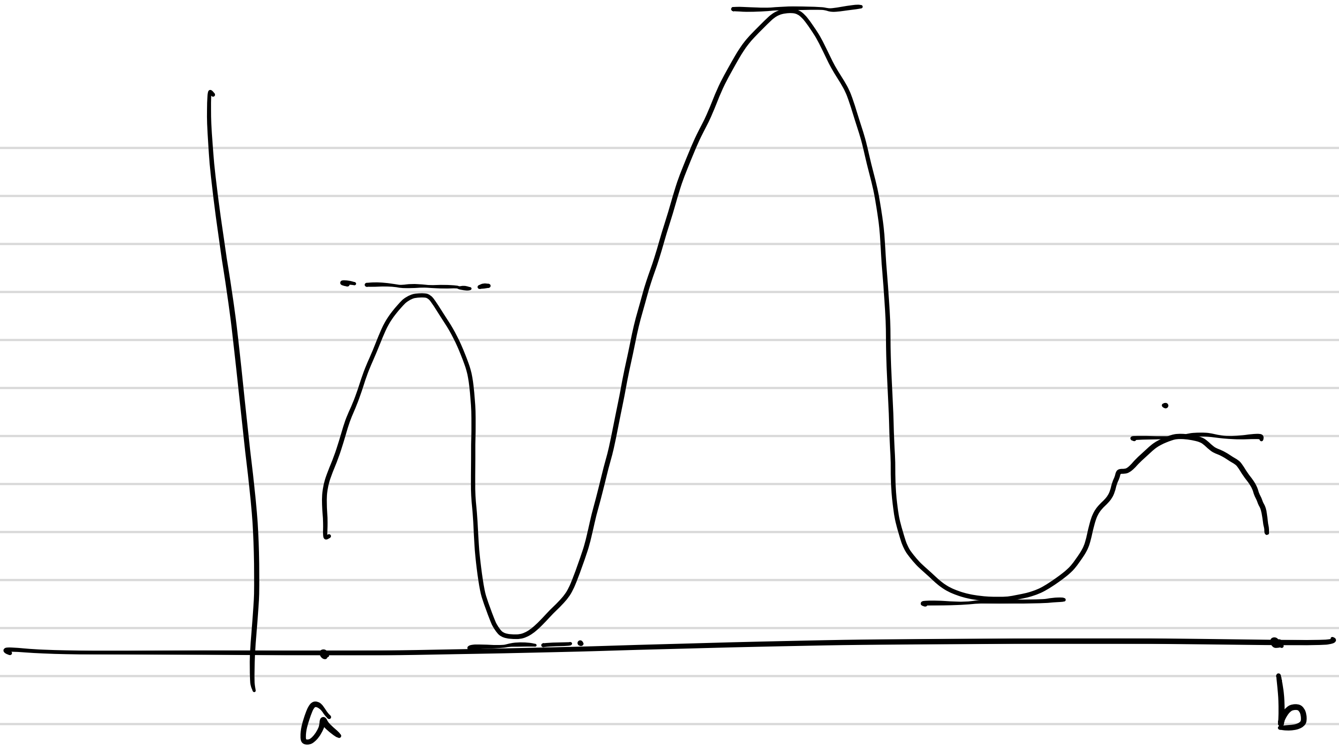
This can happen as follows:

1) The point(s) of maximum and minimum are interior points and f' is zero.

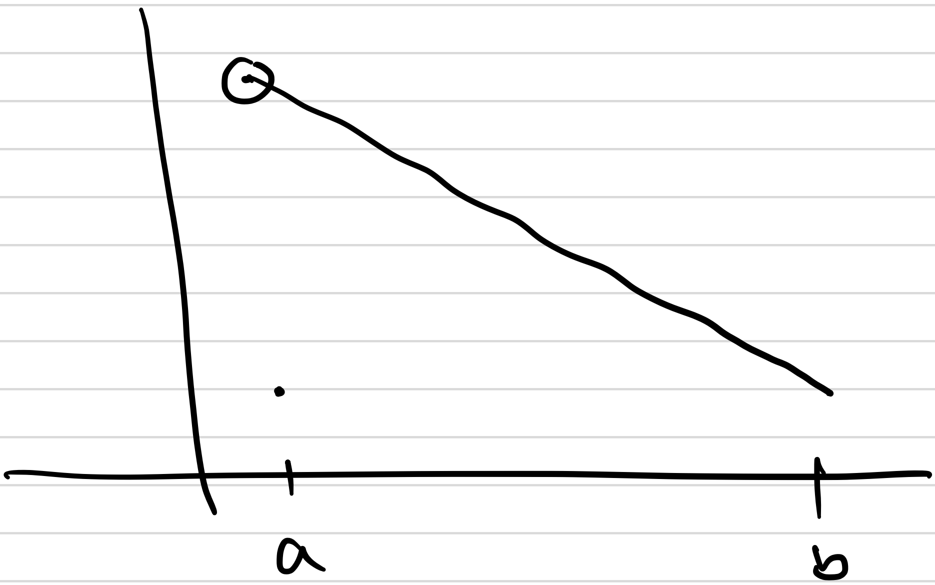
2) The points of maximum & minimum coincide with a & b.

$\blacksquare$





case (i)



case (ii)

$f(x) = x^3 + 3x + 1$

Example :

Show that $x^3 + 3x + 1 = 0$

has exactly one real solution.

Soln :

$f(-1) = -3 < 0$

$f(0) = 1 > 0$

$\Rightarrow$ Between -1 & 0, $\exists$ at least one point say x_1 $(x_1 \in (-1,0))$ such that $f(x_1) = 0$

Let there be another real number x_2 s.t. $f(x_2) = 0$

Then observe $[x_1, x_2]$ f satisfies all the conditions of Rolle's theorem.

$\therefore \exists\ c \in (x_1, x_2)$ s.t. $f'(c) = 0$

But $f'(x) = 3(x^2+1) > 0$ $\boxed{\text{a contradiction}}$