Tutorial 4

1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$

$a \geq 1$, $b \geq 1$

On comparing $a=3, b=2$ $f(n)=n^2$

Now,

$C = \log_b a = \log_2 3$

$= 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^2$

$\boxed{T(n) = \theta(n^2)}$

2) $T(n) = 4T(n/2) + n^2$

$a \geq 1 \quad b > 1$

$a = 4, b = 2, f(n) = n^2$

$C = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore \boxed{T(n) = \theta(n^2 \log_2 n)}$

3) $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$f(n) = 2^n$

$C = \log_b a = \log_2 C = 0$

$n^c = n^b = 1$

$f(n) > n$

$\boxed{T(n) = \theta(2^n)}$

4) $T(n) = 2^n T(n/2) + n^n$

$a = 2^n$

$b = 2, \quad f(n) = n^2$

$C = \log_b a = \log_2 2^m$

$= n$

$n^c + n^n$

$f(n) = n^c$

$\boxed{T(n) = \theta(n^2 \log_2 n)}$

5) $T(n) = 16T(n/4) + n$

$a = 16, \quad b = 4$

$f(n) = n$

$C = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4$

$= 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore \boxed{T(n) = \theta(n^2)}$

6) $T(n) = 2T(n/2) + n \log n$

$a = 2, \quad b = 2$

$f(n) = n \log n$

$C = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$\boxed{T(n) = \theta(n \log n)}$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{n\log n} < n$

$\therefore f(n) < n^c$

$\therefore \boxed{T(n) = \theta(n)}$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4\ f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore \boxed{T(n) = \theta(n^{0.51})}$

9) $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2$

$a \geq 1$ but here $a < 0.5$

so we can't apply master's method

10) $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

as $n! > n^2$

$\therefore \boxed{T(n) = \theta(n!)}$

11) $4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\because \log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$\boxed{T(n) = \theta(n^2)}$

12) $T(n) = \text{sqrt}(n)\,T(n/2) + \log n$

$a = \sqrt{n},\ b = 2$

$c = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log n$

$\therefore f(n) > n^c$

$\boxed{T(n) = \theta(\log(n))}$

13) $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2$

$c = \log_2 3 = 1.584$

$n^c = n^{1.58}$

$n < n^{1.58} \Rightarrow f(n) = n^c$

$\boxed{T(n) = \theta(n^{1.58})}$

14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3\ \ b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$\text{sqrt}(n) < n$

$n^c > f(n)$

$\boxed{T(n) = \theta(n)}$

$T(n) = 4T(n/2) + n$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$\boxed{T(n) = \theta(n^2)}$

16) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3, \quad b = 4$

$c = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^c < f(n)$

$\boxed{T(n) = \theta(n\log n)}$

17) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3 \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$n^c > f(n)$

$\boxed{T(n) = \theta(n)}$

18) $T(n) = 6T\left(\frac{n}{3}\right) + n^2\log n$

$a = 6, \quad b = 3$

$c = \log_3 6 = 1.6309$

$n^c < n^2\log n$

$\boxed{T(n) = \theta(n^2\log n)}$

19) $T(n) = 4T(n/2) + n/\log n$

$a = 4 \; \& \; b = 2 \quad f(n) = \frac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\boxed{T(n) = \theta(n^2)}$

20) $T(n) = 64T\left(\frac{n}{8}\right) - n^2\log n$

$a = 64, \quad b = 8$

$c = \log_8 64 = 2$

$n^c = n^2$

$n^2\log n > n^2$

$\boxed{T(n) = \theta(n^2\log n)}$

21) $T(n) = 7T(n/3) + n^2$

$a = 7, \quad b = 3$

$c = \log_3 7 = 1.7712$

$n^c = n^{1.77}$

$n^c < f(n)$

$\boxed{T(n) = \theta(n^2)}$

22) $T(n) = T(n/2) + n/2 - \cos n$

$a = 1, \quad b = 2$

$c = \log_2 1 = 0$

$n^c = 1$

$n(2 - \cos n) > n^c$

$\boxed{T(n) = \theta(n(2 - \cos n))}$