Price Competition in a Channel Structure when an Extra Effort is put in Production

(Supervised by Prof. Vinay Ramani)
Shruti Agarwal, 200957

1 Introduction

This paper discusses the effect of an extra effort (e.g., Greening Effort) put by the manufacturer in the production that leads to an increase in the manufacturing cost which in turn leads to an increase in the price of the product. The channel structure consists of one manufacturer and one retailer who sells only that manufacturer's product. This paper studies two noncooperative games of different power structures between the manufacturer and the retailer, i.e., two Stackelberg. This paper considers three possible power balance scenarios for which the rules of the game are given as follows:

Manufacturer Stackelberg (MS) - The manufacturer chooses the wholesale price and effort using the response function of the retailer. The retailer determines the demand of the product so as to maximize profit given the wholesale price and the effort.

Retailer Stackelberg (RS) - The retailer chooses the margin using the reaction function of the manufacturer. The manufacturer determines the wholesale price and effort given the margin of the retailer.

The first scenario represents a market in which there are a few large manufacturers with many relatively small retailers (such as processed foods and over-the-counter drugs). The market is controlled by the manufacturer who can play the role of Stackelberg leaders with respect to the retailer by taking the retailer's reaction function into consideration for his wholesale price decisions.

The second scenario assumes that the retailer has a considerable influence on the market compared to the manufacturer because of its size (e.g., large department stores and speciality stores such as apparels market). In this market, manufacturers are mostly concerned with receiving orders from the retail giants. The retailer uses the reaction function of the manufacturer for its retail pricing decision.

We also discuss the Nash bargaining between the manufacturer and the retailer where they both cooperate to increase their profits.

2 The Base Model

This section describes the basic framework of the analysis and derives several important equilibrium quantities using a linear demand function. As in many previous studies, the market under consideration has a two-level channel structure, i.e., the manufacturer and retailer levels. The retailer is constrained to sell only one manufacturer's brand.

For simplicity, we assume that the effort put by the manufacturer is θ and it costs θ^2 to the manufacturer which leads to an increase in the retail price of the product by an amount of θ.

Our base model uses the following linear monopoly inverse demand function

$$p = a - bq + \theta, \tag{2.1}$$

where q is the demand for the product at price p and θ is the effort put by the manufacturer. The parameters are assumed to satisfy $b > 0$.

Let w denote the manufacturer's wholesale price, m the retailer's margin on the product and c manufacturer's variable cost of producing the product. The manufacturer's cost function is given as

$$\mathrm{C} = cq + \theta^2, \tag{2.2}$$

the manufacturer's profit function is given as

$$\Pi_M = (w - c)q - \theta^2, \tag{2.3}$$

and the retailer's profit is given as

$$\Pi_R = mq, \tag{2.4}$$

where $m = p - w$, the retail margin on the product. As in many previous studies, each member of the distribution channel is assumed to seek to maximize its own profit and no cooperation is assumed between the members. This is the most common (and legal) institutional arrangement of the channel structure under consideration. We now derive analytical equilibrium solutions for this base model under each power structure scenario.

2.1 Manufacturer Stackelberg

Under the assumption MS, the manufacturer takes the retailer's reaction function into consideration for his price decision. The retailer's reaction function given wholesale price w and effort θ can be derived from the first order condition of (2.4):

$$\frac{\partial \Pi_R}{\partial q} = -2bq + \theta - w + a = 0, \tag{2.1.1}$$

for which the second order derivative is

$$\frac{\partial^2 \Pi_R}{\partial q^2} = -2b, \tag{2.1.2}$$

which is negative since $b > 0$, satisfying the second order condition for a maximum.

From (2.1.1) , the retailer's reaction function can be derived:

$$q = \frac{a + \theta - w}{2b}, \tag{2.1.3}$$

which is linear in wholesale price and effort. Using the reaction function (2.1.3), the manufacturer's equilibrium wholesale price and effort can be derived from the following first order conditions of the manufacturer's profit maximization problem:

$$\begin{aligned} \frac{\partial}{\partial w}\Pi_M(q(w,\theta), w, \theta) &= \frac{-2w + a + \theta + c}{2b} = 0, \\ \frac{\partial}{\partial \theta}\Pi_M(q(w,\theta), w, \theta) &= \frac{-4b\theta + w - c}{2b} = 0. \end{aligned} \tag{2.1.4}$$

for which the Hessian matrix is

$$\begin{bmatrix} \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial \theta} \\ \frac{\partial^2 \Pi_M}{\partial \theta \partial w} & \frac{\partial^2 \Pi_M}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{b} & \frac{1}{2b} \\ \frac{1}{2b} & -2 \end{bmatrix} \tag{2.1.5}$$

which satisfies the second order condition when $b > \frac{1}{8}$.

Solving (2.1.4) results in the following wholesale price and effort

$$w = \frac{4b(a + c) - c}{8b - 1}, \qquad \theta = \frac{a + c}{8b - 1}, \tag{2.1.6}$$

and the corresponding demand can be obtained from (2.1.3):

$$q = \frac{2(a - c)}{8b - 1}, \tag{2.1.7}$$

which always has a nonnegative value when $a \geq c$ since b $> \frac{1}{8}$. It is straightforward to derive other related quantities summarised in Table 1.

It should be noted that both the manufacturer's and retailer's profits are always positive regardless of the price and cost levels. The contribution margin for manufacturer can be derived as:

$$w - c = \frac{4b(a - c)}{8b - 1}. \tag{2.1.8}$$

For a manufacturer to be profitable, the contribution margin has to be non negative. Therefore, we have the following condition as an upper bound of production cost:

$$c \leq a. \tag{2.1.9}$$

Also, notice that this is the same condition as the one for nonnegative demand. That is, when the contribution margin is negative, demand is also negative, resulting in fraudulent positive profits. As will be seen, this condition also applies to the next two types of power structure. Therefore, for the rest of the paper, we assume that this condition is satisfied.

2.2 Retailer Stackelberg

Under the assumption RS, the retailer becomes the leader and the manufacturer the follower. In this market, the leader takes the follower's reaction functions into account for its own retail price decisions. Manufacturer conditions its wolesale price and the effort on the retail margin m. The manufacturer's reaction functions can be derived from the following first order conditions:

$$\frac{\partial \Pi_M}{\partial w} = \frac{-2w + \theta - m + a + c}{b} = 0,$$

$$\frac{\partial \Pi_M}{\partial \theta} = \frac{-2b\theta + w - c}{b} = 0. \tag{2.2.1}$$

for which the Hessian matrix is

$$\begin{bmatrix} \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial \theta} \\ \frac{\partial^2 \Pi_M}{\partial \theta \partial w} & \frac{\partial^2 \Pi_M}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -\frac{2}{b} & \frac{1}{b} \\ \frac{1}{b} & -2 \end{bmatrix} \tag{2.2.2}$$

which satisfies the second order condition when $b > \frac{1}{4}$.

The resulting reaction functions are

$$w = \frac{2b(a + c - m) - c}{4b - 1}, \qquad \theta = \frac{2a - c - m}{4b - 1}. \tag{2.2.3}$$

That is, the wholesale price is positively related to the production cost and negatively related to the retailer's margin. The effort is negatively related to both.

The retailer exploits these reaction functions by setting optimal price in its profit maximization problem

$$\frac{\partial}{\partial m} \Pi_R(\theta(m),\, w(m),\, m) = -2m + a - c = 0, \tag{2.2.4}$$

for which the second order derivative is

$$\frac{\partial^2 \Pi_R}{\partial m^2} = -2, \tag{2.2.5}$$

which is negative satisfying the second order condition for a maximum.

Solving (2.2.4) results in the following margin

$$m = \frac{a - c}{2}, \tag{2.2.6}$$

and the corresponding wholesale price and effort can be obtained from (2.2.3):

$$w = \frac{b(a + 3c) - c}{4b - 1}, \qquad \theta = \frac{a - c}{2(4b - 1)}. \tag{2.2.7}$$

It should be observed from (2.2.6) that for the retailer's margin to be nonnegative, the following condition has to be satisfied:

$$c \leq a, \tag{2.2.8}$$

which is same as that of the MS case (2.1.9). Also, this condition guarantees a nonnegative demand as can be seen in Table 1.

Table 1 summarizes other pertinent quantities such as demand and profit levels.

Table 1		
Equilibrium	MS	RS
w	$\frac{4b(a+c)-c}{8b-1}$	$\frac{b(a+3c)-c}{4b-1}$
q	$\frac{2(a-c)}{8b-1}$	$\frac{a-c}{4b-1}$
p	$\frac{2b(3a+c)-c}{8b-1}$	$\frac{2b(3a+c)-(a+c)}{2(4b-1)}$
θ	$\frac{a+c}{8b-1}$	$\frac{a-c}{2(4b-1)}$
m	$\frac{2b(a-c)}{8b-1}$	$\frac{a-c}{2}$
Π_M	$\frac{(a-c)^2}{8b-1}$	$\frac{(a-c)^2}{4(4b-1)}$
Π_R	$\frac{4b(a-c)^2}{(8b-1)^2}$	$\frac{(a-c)^2}{2(4b-1)}$

2.3 Nash Bargaining

Under the assumption NB, the wholesale price gets determined through bargaining between the manufacturer and retailer. The manufacturer chooses the effort taking the retailer's reaction function into consideration. The retailer chooses the demand to maximise profit given the effort and wholesale price. The retailer's reaction function given the wholesale price and effort is the same as that under MS. The Nash Bargaining Stage is given as:

$$\max_{w} \Pi_M^{1/2} \Pi_R^{1/2}. \tag{2.3.1}$$

Substituting q from (2.1.3) and solving the optimisation problem gives

$$(-2w + a + c + \theta)(a + \theta - w) - 2[(w - c)(a + \theta - w) - 2\theta^2] = 0. \tag{2.3.2}$$

While we get closed form solutions, they are algebraically involved and hence not reported. Substituting q from (2.1.3) and w from (2.3.2) into Π_M, the manufacturer chooses θ.

3 Discussion

This section discusses several implications of the results derived in the previous section. We compare the prices and profits among different power structures. In the following discussion, we use superscripts *MS* and *RS* to denote that the corresponding quantities are for the MS (Manufacturer Stackelberg) and RS (Retailer Stackelberg) cases, respectively. Also, we assume $b > \frac{1}{4}$, to satisfy both the cases.

The wholesale price can be higher or lower in the MS case according to the given b.

$$w^{MS} - w^{RS} = \frac{b(8b-3)(a-c)}{(8b-1)(4b-1)}$$

From the above equation we observe that

$$w^{MS} < w^{RS} \quad \text{when} \quad b < \frac{3}{8}.$$

When $b = \frac{3}{8}$, the wholesale prices in both cases is equal.

The retail price can be lower or higher in the MS case according to the given b.

$$p^{MS} – p^{RS} = \frac{(2b - 1)(a - c)}{2(8b - 1)(4b - 1)}$$

From the above equation we observe that

$$p^{MS} < p^{RS} \quad \text{when} \quad b < \frac{1}{2}$$

The demand level is always lower in the MS case.

$$q^{MS} – q^{RS} = \frac{(-1)(a - c)}{(8b - 1)(4b - 1)}$$

From the above equation we observe that

$$q^{MS} < q^{RS}$$

The manufacturer makes a greater profit in the MS case

$$\Pi_M^{MS} – \Pi_M^{RS} = \frac{(8b - 3)(a – c)^2}{4(8b - 1)(4b - 1)}$$

From the above equation we observe that

$$\Pi_M^{MS} < \Pi_M^{RS} \quad \text{when} \quad b < \frac{3}{8}$$

The retailer makes a greater profit in the RS case

$$\Pi_R^{MS} – \Pi_R^{RS} = \frac{(-1)(a – c)^2}{2(8b - 1)}$$

From the above equation we observe that

$$\Pi_R^{MS} < \Pi_R^{RS}$$

The channel profit is higher in the RS case

$$(\Pi_R^{MS} + \Pi_M^{MS}) – (\Pi_R^{RS} + \Pi_M^{RS}) = \frac{(1 - 16b)(a – c)^2}{4(8b – 1)^2(4b –) \ 1}$$

From the above equation we observe that

$$(\Pi_R^{MS} + \Pi_M^{MS}) < (\Pi_R^{RS} + \Pi_M^{RS})$$

4 Conclusion

This paper analyzes a game theoretic model. We have considered a simple supply chain with a manufacturer and a retailer. We identify the Subgame Perfect Nash equilibrium outcomes. We find that the optimal solutions under both the structures depend on the structure's parameters. We have also compared the optimal solutions of both the cases. We have also formulated the Nash Bargaining solution. Due to analytical complexity we don't have closed form solutions.

References

Choi,S. Chan(1991), "Price Competition in a Channel Structure with a Common Retailer", Marketing Science, 10(4):271-296.