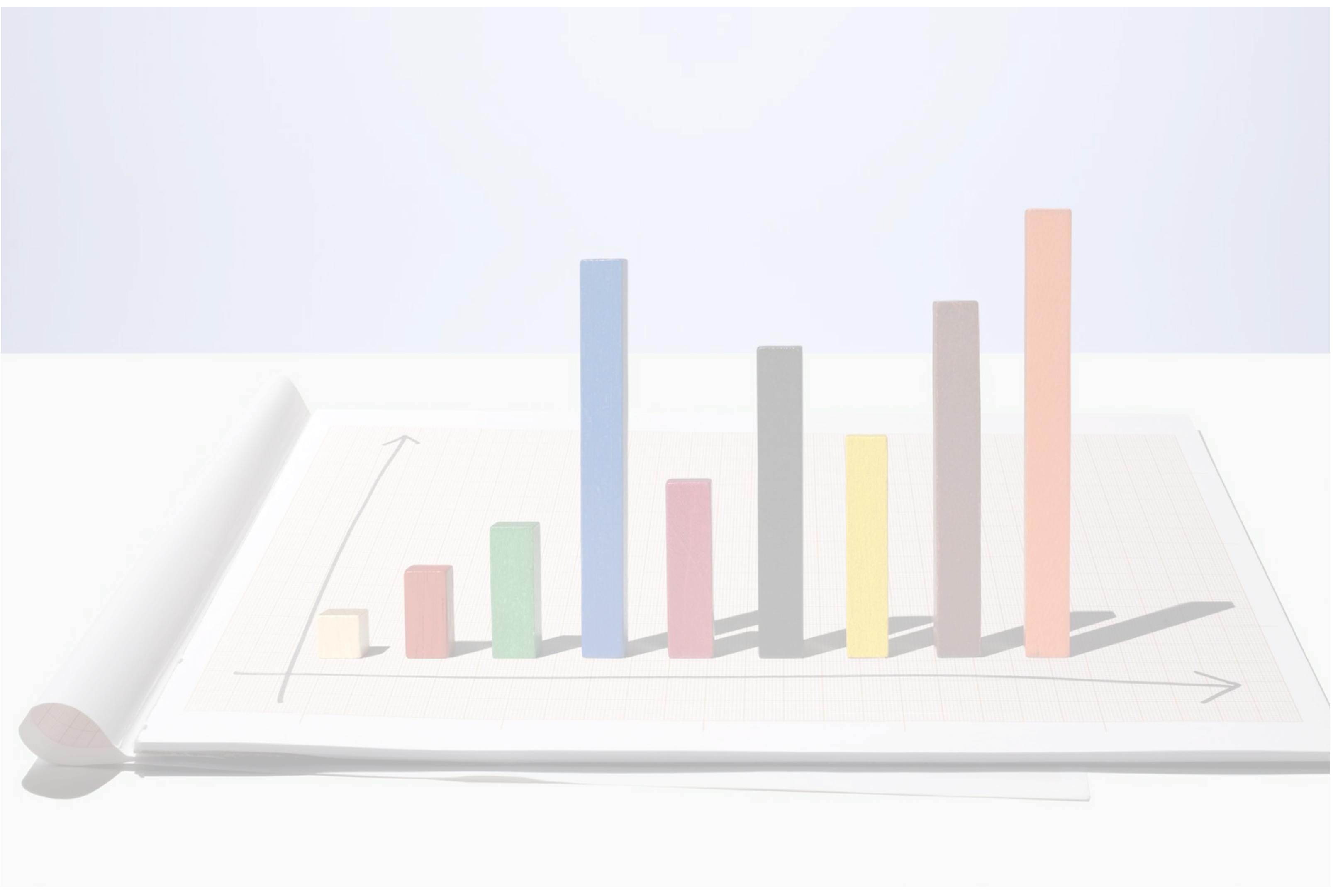
Latin Square Design

Mini-Experimental Design + Results



[Question to tackle: A study was conducted to test the effect of temperature (210F, 215F, 220F, 225F) and four different types of fabric (Fabric1, Fabric2, Fabric3, Fabric4) on shrinkage in a dyeing process. For each combination of temperature and fabric, 2 observations were made. The output for this experiment is the percent shrinkage of the given fabric/ temperature combination. The dataset for this question is fabricexperiment.csv)

*Note: The CSV file is available on the github repository where this PDF file for this project was found.

Fabric	Temperature			
	210F	215F	220F	225F
Fabric1	1.8, 2.1	2.0, 2.1	4.6, 5.0	7.5, 7.9
Fabric2	2.2, 2.4	4.4, 4.8	5.4, 5.6	9.8, 9.2
Fabric3	2.8, 3.2	4.4, 4.8	8.7, 8.4	13.2, 13.0
Fabric4	3.2, 3.6	3.3, 3.5	5.7, 5.8	10.9, 11.1

For this specific experimental design, a factorial design will be used as we are interested in understanding the interactions between each fabric and temperature. Prior to the calculations, the assessment for our parameters are needed. In this case, our parameters involves:

$$a = 4, b = 4, n = 2$$

where **a** refers to the number of fabric groups, **b** refers to the number of temperature groups and **n** refers to the number of observations in each combination of fabric and temperature.

The variances for each group were computed by making use of R Studio as shown in **Figure 1.** The variance for Variable A was 1.744 and Variable B was 11.83068. Using the formulas of the sum of squares, we obtained 41.891152 and 283.895152 for Variable A and Variable B respectively.

$$SS_A = bn \sum_{i=1}^{a} (\overline{y_{i.}} - \overline{y})^2$$

$$SS_B = an \sum_{i=1}^{b} (\overline{y_{.j}} - \overline{y})^2$$

```
> data
   Fabric Temperature Shrinkage
1  Fabric1        210F       1.8
2  Fabric1        210F       2.1
3  Fabric1        215F       2.0
4  Fabric1        215F       2.1
5  Fabric1        220F       4.6
6  Fabric1        220F       5.0
7  Fabric1        225F       7.5
8  Fabric1        225F       7.9
9  Fabric2        210F       2.2
10 Fabric2        210F       2.4
11 Fabric2        215F       4.2
12 Fabric2        215F       4.0
13 Fabric2        220F       5.4
14 Fabric2        220F       5.6
15 Fabric2        225F       9.8
16 Fabric2        225F       9.2
17 Fabric3        210F       2.8
18 Fabric3        210F       3.2
19 Fabric3        215F       4.4
20 Fabric3        215F       4.8
21 Fabric3        220F       8.7
22 Fabric3        220F       8.4
23 Fabric3        225F      13.2
24 Fabric3        225F      13.0
25 Fabric4        210F       3.2
26 Fabric4        210F       3.6
27 Fabric4        215F       3.3
28 Fabric4        215F       3.5
29 Fabric4        220F       5.7
30 Fabric4        220F       5.8
31 Fabric4        225F      10.9
32 Fabric4        225F      11.1
> Ameans <- aggregate(Shrinkage~Fabric,FUN=mean,data=data)
> s2_Ameans <- var(Ameans[,2])
> Bmeans <- aggregate(Shrinkage~Temperature,FUN=mean,data=data)
> s2_Bmeans <- var(Bmeans[,2])
> s2_Ameans
[1] 1.744844
> s2_Bmeans
[1] 11.83068
```

Figure 1: The dataset was added into R Studio and then the aggregate and var() functions were used to obtain the variances for both of the Variable A [Fabrics] and Variable B [Temperature].

Following this, we produce the ANOVA table specifically for the Factorial Design on R Studio and compare the values we obtained through manual calculations to ensure that our table is accurate. This acts as a process of verification. The ANOVA table obtained can be noticed in **Figure 2.**

Using the ANOVA table, the p-value for the interactions between the Fabric and the Temperature will be $7.09e^{-09}$ (0.008749) which is less than the alpha value (0.05). The alpha value wasn't provided in the experiment so we have assumed the value to be 0.05. In this case, we can conclude that the values are statistically different from each other and we have rejected the null hypothesis as well.

```
> summary(aov(Shrinkage~Fabric*Temperature,data=data))
                   Df Sum Sq Mean Sq F value   Pr(>F)    
Fabric              3  41.88   13.96  279.18 5.05e-14 ***
Temperature         3 283.94   94.65 1892.91  < 2e-16 ***
Fabric:Temperature  9  15.86    1.76   35.24 7.09e-09 ***
Residuals          16   0.80    0.05                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 2: The ANOVA table for the Factorial design obtained with Variable A [Fabrics] and Variable B [Temperature]. The table consists of the degree of freedom, sum of squares, mean of squares and the F-value accordingly.

Now, our experiment has decided to treat the temperature as a single continuous variable. The plot of the data indicated that the shrinkage increases quadratically with temperature with a potential linear interaction between fabric and temperature. The visualization of the regression model applied can be seen as follows.

```
> model <- lm(Shrinkage~Fabric + Temperature + Fabric:Temperature + I(Temperature^2) + Fabric:I(Temperature^2),data=data)
> model

Call:
lm(formula = Shrinkage ~ Fabric + Temperature + Fabric:Temperature + 
    I(Temperature^2) + Fabric:I(Temperature^2), data = data)

Coefficients:
                (Intercept)                FabricFabric2                FabricFabric3                FabricFabric4  
                  1240.8250                    -295.4750                      12.1125                    1137.5975  
                Temperature             I(Temperature^2)    FabricFabric2:Temperature    FabricFabric3:Temperature  
                   -11.7800                       0.0280                       2.6700                      -0.3675  
  FabricFabric4:Temperature  FabricFabric2:I(Temperature^2)  FabricFabric3:I(Temperature^2)  FabricFabric4:I(Temperature^2)  
                   -10.5545                      -0.0060                       0.0015                       0.0245  
```

```
> ggplot(data, aes(x = Temperature, y = Shrinkage, color = Fabric)) + geom_point(size = 3, alpha = 0.7) + geom_smooth(method = "lm", f
ormula = y ~ poly(x,2), se = FALSE) + labs(title = "Visualization of Regression Model", x = "Temperature (°F)", y = "Shrinkage (%)") +
theme_linedraw()
```

Figure 3: R Studio code for obtaining the coefficients of our regression model. Next R Studio code refers to the code required to plot the data points to obtain the relationship between the Fabrics and the Temperatures.

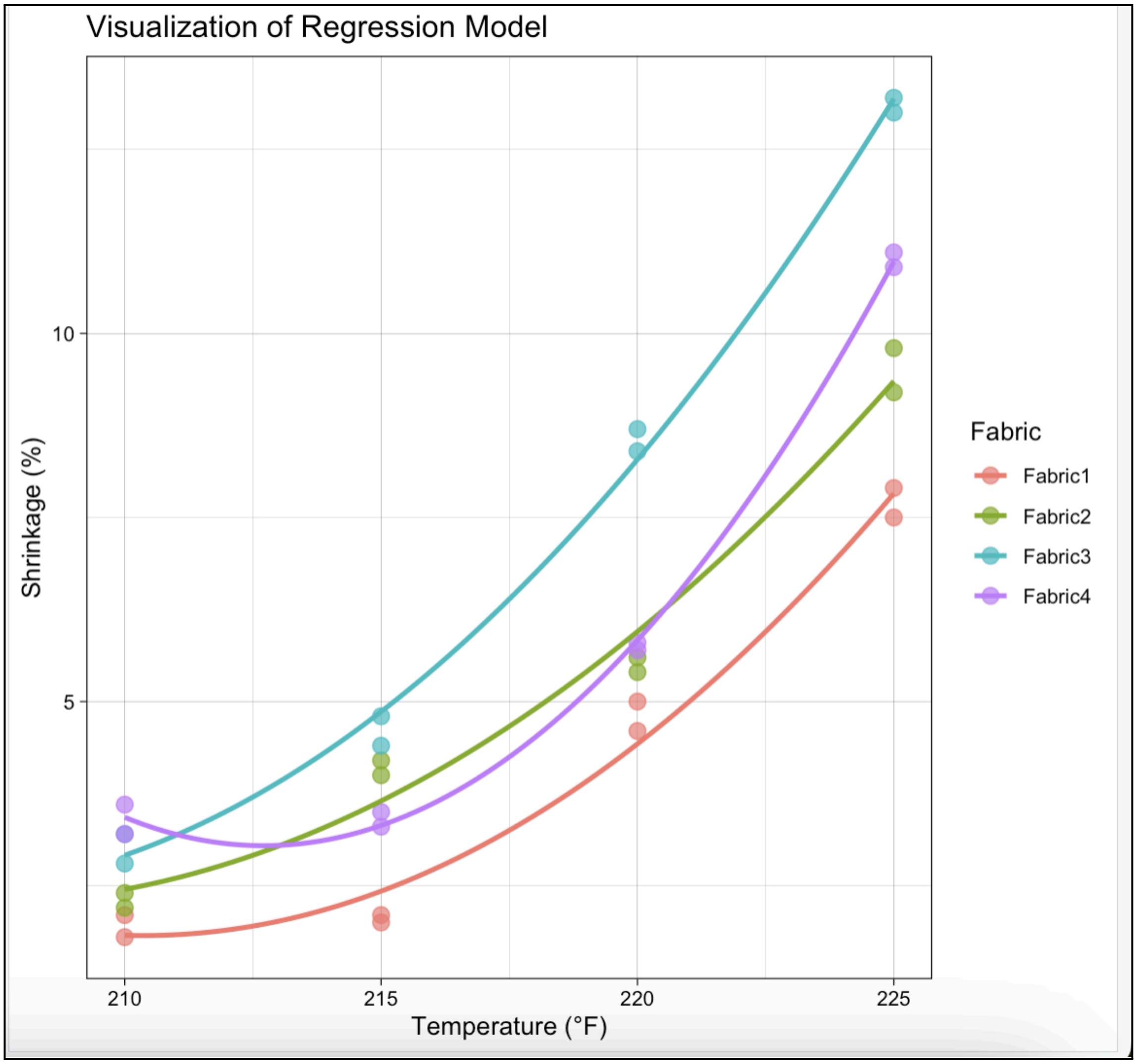


Figure 4: Visualizations of the plots containing the interactions between each Fabric and the respective data points on temperatures.