Quantitative Approaches to Portfolio Optimisation

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This report presents a study and analysis of quantitative portfolio construction strategies, carried out during my summer internship under the guidance of aforementioned mentor.

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Abstract

This project presents a comprehensive framework for portfolio construction and evaluation by integrating various quantitative techniques. We explore multiple strategies that combine different expected return estimators—namely mean historical returns, exponential moving average (EMA), and CAPM-based returns—with a range of covariance matrix estimators, including sample covariance, exponentially weighted sample covariance and covariance shrinkage matrices. These estimations are then used in classical optimization models such as Maximum Sharpe Ratio and Global Minimum Variance. Our strategies are empirically backtested on historical stock data, and performance is compared against an equally-weighted benchmark portfolio in terms of risk adjusted returns. The study highlights how the choice of return and risk estimators can significantly influence portfolio performance.

Keywords: Portfolio Construction, Quantitative Finance, Mathematical programming

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I look forward to applying the knowledge and skills gained during this internship in my future career.

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1 Introduction

The financial industry has undergone a significant transformation in recent years, driven by technological advancements, the increasing availability of large-scale financial data, and the growing computational power. These developments have led to the growing adoption of quantitative methods and machine learning (ML) techniques across various domains of finance—including algorithmic trading, risk modeling, and portfolio optimization.

The motivation for adopting quantitative techniques stems from the inherent limitations of traditional investing, which is often influenced by behavioral biases that hinder objective decision-making. Loss aversion and endowment effect for example, can cause investors to make irrational investment choices and showcase inconsistent performance. Moreover, discretionary approaches are inherently limited in scalability, relying heavily on subjective judgment and lacking the capacity to process large volumes of data in real time.

In contrast, quantitative investing leverages data-driven, rule-based models that mitigate psychological biases and enable real-time analysis of vast financial datasets. These methods are particularly well-suited for uncovering statistical patterns and predictive signals that may remain hidden in traditional frameworks, thereby supporting more consistent and scalable decision-making.

Objective

This paper focuses on applying quantitative techniques to portfolio construction and optimization. Specifically, it investigates the impact of varying components of the portfolio modeling pipeline—including:

- Expected return estimators: mean historical return, exponentially weighted moving average (EMA), and Capital Asset Pricing Model (CAPM) returns,
- Covariance matrix estimators: sample covariance, exponentially weighted covariance, and Ledoit-Wolf shrinkage estimator, and
- Optimization objectives: maximum Sharpe ratio and global minimum variance.

The study also incorporates dynamic rebalancing strategies using rolling window on weekly data, and benchmarks the resulting portfolio performances against a dynamic equally-weighted portfolio, nifty 500 index and a mutual fund. By systematically comparing different configurations, this paper aims to highlight the benefits and trade-offs involved in building robust, data-driven investment strategies.

2 Data Description

2.1 Data Source and Rationale

The historical stock data used in this study was sourced from the **Yahoo Finance API**, a free and widely-used platform that provides reliable historical OHLCV (Open, High, Low, Close, Volume) data. This source was chosen due to its ease of use, accessibility, and suitability for academic research and backtesting strategies.

To ensure consistency and avoid discrepancies that may arise from using multiple data providers, all data in this study was sourced from the same API. While alternative sources such as `nse-stocks`, `bseindia`, and `nsetools` exist, they are unofficial scrapers prone to breakage in case of structural changes to the source websites. Therefore, Yahoo Finance offered a more stable and consistent solution for data acquisition.

2.2 Time Period

The chosen time period for this analysis spans from **January 2003 to January 2025**. This extended window captures multiple market cycles — including the Global Financial Crisis (2008), the COVID-19 crash and recovery (2020), the Russia–Ukraine war (2022), and several general elections — making it well-suited for evaluating long-term investment strategies across diverse market regimes. The breadth of this timeframe allows for a comprehensive assessment of price behavior, the identification of seasonal patterns, and improves the robustness and generalizability of backtesting results.

2.3 Stock Universe and Selection

The study focuses on the **NIFTY 500** index constituents—a group of 500 of the largest and most actively traded companies on the National Stock Exchange (NSE) of India. These stocks provide a high-quality, liquid, and representative sample of the Indian equity market.

Although using the entire NSE-listed universe might offer broader exposure, doing so would introduce several complications:

- **More noise:** Many smaller companies may have irregular trading patterns or unpredictable behavior, which can obscure meaningful analysis.
- **Data quality issues:** Not all stocks have reliable or complete data, especially smaller or recently listed ones.
- **Survivorship bias and filtering difficulties:** Many small-cap stocks are delisted over time due to mergers, acquisitions, or poor performance, making it harder to ensure consistent historical data.

The universe of NIFTY 500 stocks offers a practical balance between diversification and tractability. Wikipedia's NIFTY 500 page was used to obtain the most up-to-date list of current constituents.

Furthermore, care was taken to include only those stocks for which continuous historical data for atleast 15+ years was available over the selected time frame.

2.4 Final Dataset

After applying all filtering and consistency checks, the final dataset consists of a curated subset of NIFTY 500 stocks with atleast 15 years of uninterrupted historical data from 2003 to 2025.

3 Methodology Overview

We began by downloading the adjusted closing prices for the NIFTY 500 index constituents from January 2003 to January 2025 using the Yahoo Finance API via the yfinance Python library. We specifically used Adjusted Close prices because they account for corporate actions such as dividends, stock splits, and rights offerings. This provides a more accurate measure of a stock's total return and ensures consistency when calculating returns or comparing asset performance over time.

While collecting historical stock data, a few tickers failed to return valid information. These issues were primarily due to:

- Missing timezone metadata, typically associated with delisted or unsupported securities.
- Absence of historical price data for the selected period, often due to recent listings or illiquid stocks.
- Stocks delisted or suspended, resulting in incomplete time series.

The exclusions represented less than 1 percent of the initial stock universe and did not materially impact the quality or scope of our analysis.

To ensure data consistency and robustness, we filtered the dataset to include only stocks with at least 15 years of uninterrupted historical data within the specified time frame. For each stock, we calculated the duration of available data by identifying the earliest and latest valid (non-missing) price observations. Stocks with insufficient historical depth were removed from the analysis.

In doing so, we were left with approximately 300 stocks from the NIFTY 500 with sufficient long-term data, forming the core universe for portfolio construction.

3.1 Preprocessing

To further prepare the data for portfolio construction and backtesting, we performed a series of preprocessing steps including handling missing values, resampling to a weekly frequency, and computing returns.

Dealing with missing values

Stock price data for NIFTY 500 constituents often contain missing values due to several reasons. First, stocks enter and exit the index over time; newly listed companies have no historical data prior to their IPO, while delisted firms stop reporting prices. Second, corporate actions such as stock splits, mergers, or spin-offs can lead to temporary gaps during data adjustments by providers. Third, many mid-cap and small-cap stocks in the index suffer from illiquidity, resulting in days with no trades and hence no recorded prices. Additionally, trading suspensions due to regulatory actions, insolvency, or pending announcements can cause prolonged missing periods. Lastly, exchange holidays and weekends create alignment issues when synchronizing data across multiple assets.

Thus, to address missing values, we applied a forward-fill (ffill) operation, which propagates the last observed valid price forward. This method preserves continuity in the price series and avoids introducing artificial volatility that would result from filling missing values with zeros or dropping them outright. It is a widely accepted practice in financial time series, especially when the last observed price remains valid until a new price is recorded.

Resampling weekly

After forward filling, we resampled the data to a weekly frequency, selecting prices as of each Wednesday's close (W-WED). Wednesday is often preferred in academic research and backtesting because:

- It reduces bias from start-of-week and end-of-week volatility.
- It reflects a "typical" trading day during the week.

Computing returns

Finally, we computed weekly simple returns using the percentage change method.

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} = \frac{P_{i,t}}{P_{i,t-1}} - 1$$

where:

- $R_{i,t}$ is the weekly simple return for asset i at time t,
- $P_{i,t}$ is the closing price of asset i at the end of week t,
- $P_{i,t-1}$ is the closing price of asset i at the end of the previous week.

In this study, we opted to use simple returns instead of logarithmic returns. For assets that exhibit relatively small price changes over short timeframes—such as daily or weekly intervals—the difference between simple and log returns is minimal. In such cases, simple returns can be preferred due to their simplicity and ease of interpretation.

Why work with returns instead of prices?

We work with returns instead of prices because stock prices are typically non-stationary, meaning their mean and variance change over time. In contrast, returns are closer to being stationary, exhibiting more stable statistical properties, which makes them more suitable for modelling, estimation, and portfolio optimization.

Choosing the Return Frequency: Weekly vs. Daily or Monthly

We compute returns at a weekly frequency rather than daily or monthly.

- Daily returns are highly volatile and contain substantial market noise, obscuring meaningful signals.
- Monthly returns reduce noise and highlight long-term trends but require longer testing periods, leading to data loss from missing monthly observations.

Weekly returns provide an optimal balance, reducing noise while maintaining sufficient data coverage for reliable forecasting in our optimization framework.

These steps resulted in a clean, consistent weekly return matrix suitable for further modeling, including expected return estimation, risk modeling, and portfolio optimization.

Training set

We define our training set for estimating expected returns and covariances for assets on a rolling window basis such that it includes 10-year history (520 weeks) of weekly returns. For each rebalancing period we keep only those assets that have full 10 years of history as on the rebalancing day. This filtering helps avoid unstable or biased estimates that could distort portfolio weights during optimization.

The rolling window approach begins in 2003, and as a result, our backtesting period spans 2014 to 2025.

3.2 Expected Returns Estimator

Accurate estimation of expected returns is a critical component of portfolio optimization, as it directly influences asset allocation decisions and portfolio performance. However, return forecasts are notoriously difficult to predict due to the inherent volatility and noise in financial markets. Even small estimation errors can lead to significant shifts in optimized weights, resulting in concentrated and unstable portfolios. To address these challenges, various approaches have been developed for estimating expected returns, ranging from historical mean-based methods to more sophisticated models like exponentially weighted moving averages and CAPM-based estimators.

3.2.1 Mean Historical Return

To compute the **mean historical return** from weekly return data, we use the **geometric mean**, which more accurately reflects compounded growth over time than the arithmetic mean.

Given a time series of simple weekly returns $r_1, r_2, \ldots, r_T$, the weekly mean return is calculated as:

$$\mu = \left(\prod_{t=1}^{T} (1 + r_t) \right)^{\frac{1}{T}} - 1$$

where:

- r_t is the simple return at time t,
- T is the number of non-missing return periods,
- μ is the weekly geometric mean return.

This method is easy to compute and understand, which is why it is often used as a starting point in portfolio analysis. It accurately reflects how returns compound over time, giving a more realistic picture of growth than the arithmetic mean. It adjusts for the number of valid return periods, making it more reliable when some return data points are missing.

However, it has some drawbacks. It assumes that past performance will continue in the future, which may not always be true in changing markets. Also unlike models like CAPM, it doesn't account for risk, volatility, or other market-related explanatory factors, making it less suitable for risk-adjusted return estimation.

3.2.2 Exponentially Weighted Mean Historical Return

To estimate expected returns in a way that is more responsive to recent market behavior, we employ the **Exponentially Weighted Moving Average (EWMA)** method. Unlike the mean historical return, which treats all past returns equally, EWMA assigns greater weight to more recent returns and exponentially less weight to older data.

This approach captures short-term trends more effectively and adapts to structural changes in the market.

Given a time series of returns $r_1, r_2, \ldots, r_t$, the EWMA is computed recursively as:

$$\text{EMA}_t = \alpha \cdot r_t + (1 - \alpha) \cdot \text{EMA}_{t-1}$$

where:

- r_t is the return at time t,
- EMA_t is the exponentially weighted average at time t,
- $\alpha = \frac{2}{\text{span}+1}$ is the smoothing factor controlling the decay rate.

A smaller `span` leads to faster decay and more emphasis on recent returns, while a larger `span` results in a smoother average with longer memory.

We give more weight to recent data because financial markets are dynamic, and asset returns often exhibit time-varying behavior due to changing economic conditions, company performance, and investor sentiment. Older data may no longer reflect the current risk-return characteristics of an asset. By emphasizing recent returns, the estimator becomes more responsive to current market trends, improving its relevance for forecasting future returns. This is particularly important during periods of structural change or volatility, where relying on very old data can lead to outdated and misleading estimates. This approach smooths volatility and captures time-varying trends better than a simple historical average.

However, it has limitations. The choice of decay factor is subjective, the method is still backward-looking and does not include forward-looking information, and it can overreact to short-term market noise. Also, because it down-weights older data, it might miss meaningful long-term patterns or historical behaviors that still influence returns.

3.2.3 CAPM Return

The Capital Asset Pricing Model (CAPM) provides a widely used framework for estimating expected returns based on an asset's systematic risk exposure to the market. Under the CAPM, the expected return of an asset is determined by its sensitivity to market movements, captured through the asset's β, relative to the risk-free rate and the expected market return.

$$\mathbb{E}[R_i] = R_f + \beta_i \cdot (\mathbb{E}[R_m] - R_f)$$

where:

- $\mathbb{E}[R_i]$ is the expected return of asset i,

- R_f is the annualised risk-free rate,
- β_i is the asset's beta, representing its sensitivity to market returns,
- $\mathbb{E}[R_m]$ is the expected return of the market.

The market return above the risk-free rate is called the market risk premium — it's the extra return you get for taking market risk instead of staying safe.

Beta tells you how much market risk the stock takes compared to the market.

So, for example, if the market gives an extra 6% and your stock takes 1.2 times the market risk, you expect about $1.2 \times 6\% = 7.2\%$ extra return on top of the risk-free rate.

Estimation Procedure:

1. Betas are calculated using the sample covariance matrix:

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}$$

2. The expected market return $\mathbb{E}[R_m]$ is calculated using the geometric mean:

$$\mathbb{E}[R_m] = \left(\prod_{t=1}^{T}(1 + r_{m,t})\right)^{\frac{1}{T}} - 1$$

   where T is the number of return observations.

3. The risk-free rate R_f was set to 0.07 annually, in line with the long-term average yield on Indian 10-year government securities.

CAPM is a popular method to estimate expected returns because it explicitly incorporates systematic risk (via beta), allowing return predictions to adjust based on an asset's sensitivity to overall market movements. It's based on a well-established theoretical framework, making it interpretable and widely accepted in finance for estimating risk-adjusted expected returns.

However, its drawback is that it assumes markets are perfect and uses only one factor (market risk), which is not realistic and often fail in real-world settings, especially during market turmoil. Also, the inputs like beta and market return are hard to estimate accurately, so the predictions can be unreliable, especially when markets are volatile.

3.3 Covariance matrix estimation

The covariance matrix plays a critical role in portfolio optimization as it captures the relationships and co-movements between asset returns. Accurate estimation of this matrix is essential because it directly affects risk assessment and optimal weight allocation. However, estimating covariance is challenging in practice due to noise, limited historical data, and changing market conditions. In this section, we explore different methods for computing the covariance matrix, including sample covariance, exponentially weighted covariance, and shrinkage-based approaches (Ledoit-Wolf).

3.3.1 Sample Covariance

The **sample covariance matrix** is a statistical tool used to measure how pairs of asset returns move together. It plays a central role in portfolio theory, particularly in the Markowitz Mean-Variance Optimization framework.

Given two random variables (e.g., asset returns) $x = \{x_1, x_2, \ldots, x_n\}$ and $y = \{y_1, y_2, \ldots, y_n\}$ with n observations, the sample variance and sample covariance are defined as:

$$\begin{aligned}
\text{Sample Variance:} \quad \mathrm{Var}(x) &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \\
\text{Sample Covariance:} \quad \mathrm{Cov}(x, y) &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})
\end{aligned}$$

Here, $\bar{x}$ and $\bar{y}$ are the sample means of x and y, respectively.

In the multivariate case, these pairwise covariances are assembled into a square symmetric matrix called the **sample covariance matrix**. Each diagonal entry represents the variance of an asset, while each off-diagonal entry represents the covariance between a pair of assets.

This method provides a straightforward method to calculate asset variances and covariances between them directly from historical returns. However, it implicitly assumes that asset return relationships remain constant over time, which may not hold in dynamic markets.

3.3.2 Exponentially weighted Covariance

The **Exponentially Weighted Covariance Matrix (EWCM)** is an alternative to the sample covariance matrix that assigns more weight to recent observations. This approach reflects the intuition that recent asset returns are often more relevant for predicting near-term co-movement and volatility.

Given two return time series $X = \{X_1, X_2, ..., X_T\}$ and $Y = \{Y_1, Y_2, ..., Y_T\}$, the exponentially weighted covariance between X and Y is computed through the following steps:

1. **Demeaning:** Compute the mean of each series and construct demeaned returns:

$$X'_t = X_t - \bar{X}, \quad Y'_t = Y_t - \bar{Y}$$

2. **Covariation Series:** Form the time series of pointwise products (covariation):

$$C_t = (X_t - \bar{X})(Y_t - \bar{Y})$$

   For each time step, this product tells us whether: 1. both assets move in the same direction (both above/below their means - positive value) 2. both assets move in the opposite direction (one above one below - negative value) 3. no relation (zero value)

3. **Exponentially Weighted Moving Average (EWMA):** Apply an exponentially weighted moving average to the covariation series:

$$\mathrm{EWMA}_t = \alpha C_t + (1 - \alpha)\mathrm{EWMA}_{t-1}$$

where $\alpha = \frac{2}{\mathrm{span}+1}$ is the smoothing factor determined by the chosen span.

4. **Final Covariance Estimate:** The exponentially weighted covariance is taken as the last value of the EWMA series:

$$\mathrm{Cov}_{\mathrm{EW}}(X, Y) = \mathrm{EWMA}_T$$

To compute the full covariance matrix, this procedure is applied to all asset pairs (i, j) in the return matrix. The resulting $N \times N$ matrix is symmetric, with exponentially weighted variances on the diagonal and covariances on the off-diagonal.

This method is more responsive to recent changes. It gives higher weight to recent data, making it better at capturing shifts in market volatility or correlations. The exponential weighting dampens the effect of random short-term fluctuations compared to using raw covariation.

However, since the choice of span/decay is subjective, it can lead to very different results, and there's no universally optimal value. Additionally, by down weighting older data, it might overlook persistent historical relationships, especially in stable markets.

3.3.3 Shrinkage estimation - Ledoit Wolf Covariance

In high-dimensional settings—where the number of assets N is large relative to the number of observations T—the sample covariance matrix can become highly unstable. While it is an unbiased estimator, the sample covariance matrix often suffers from high estimation variance, ill-conditioning, and sensitivity to noise. These issues can significantly distort portfolio optimization, resulting in extreme or unstable portfolio weights.

To mitigate this, **shrinkage estimators** offer a practical solution by balancing the trade-off between bias and variance.

Shrinkage Estimators: Concept

The idea behind shrinkage is to improve the estimation of the covariance matrix by "shrinking" the noisy sample covariance matrix S toward a more structured and stable target matrix F. The resulting estimator takes the form:

$$\hat{\Sigma}_{\mathrm{shrunk}} = (1 - \delta)S + \delta F$$

where:

- S is the sample covariance matrix,
- F is a predefined target matrix (structured, low-variance),
- $\delta \in [0, 1]$ is the shrinkage intensity, controlling the balance between S and F.

The goal is to choose F and δ such that the mean squared error (MSE) between the estimate $\hat{\Sigma}_{\text{shrunk}}$ and the true (but unknown) covariance matrix Σ is minimized.

Ledoit-Wolf Shrinkage Estimator

Ledoit and Wolf (2004) proposed a shrinkage method that provides:

1. A choice of structured targets F suitable for financial applications.
2. A data-driven, closed-form solution to compute the optimal shrinkage intensity δ^* that minimizes the MSE.

The Ledoit-Wolf shrinkage estimator is defined as:

$$\hat{\Sigma}_{\text{LW}} = (1 - \delta^*)S + \delta^* F$$

The shrinkage intensity δ^* is computed analytically by minimizing the Mean Squared Error (MSE) between the true covariance matrix Σ and the shrunk estimator $\hat{\Sigma}_{\text{shrunk}}$.

The different types of shrinkage targets are:

- **Constant Variance Target**: Assumes all assets have the same variance; the target is a scaled identity matrix.
- **Constant Correlation Target**: Assumes all pairwise correlations are the same, with individual variances preserved.
- **Single-Factor Model Target**: Assumes asset returns are driven by one common factor (e.g., market return), mimicking a CAPM-style structure.

By shrinking towards a structured target with optimal intensity it lowers the variance of estimates, making them more robust in small-sample (more assets than data points) or high-dimensional settings. A slight bias is introduced, but this trade-off produces more stable and reliable covariance matrices, leading to better-behaved portfolio weights.

However, the performance of this method depends on selecting an appropriate shrinkage intensity and target matrix, which may vary across datasets. Thus, this method faces the risk of introducing bias into the estimation as it pulls estimates toward the target, which can distort true relationships if the target is poorly chosen.

3.4 Portfolio Optimisation Framework

Portfolio construction involves balancing the trade-off between expected return and risk. A foundational approach to this problem is the Markowitz mean-variance optimization framework. The Markowitz Mean-Variance Optimization Model is a mathematical framework first introduced by the economist Harry Markowitz in 1952. This model frames portfolio selection as a convex quadratic programming problem.

The convexity of this problem arises from its quadratic objective function—portfolio variance—and linear constraints, such as budget constraints (the weights summing to one). By solving this optimization, investors can trace out the efficient frontier—a set of optimal portfolios offering the highest expected return for each level of risk.

Let $\mathbf{w} \in \mathbb{R}^n$ be the vector of portfolio weights, $\boldsymbol{\mu} \in \mathbb{R}^n$ the vector of expected returns, and $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$ the covariance matrix of asset returns. The general Markowitz problem is:

$$\max_{\mathbf{w}} \ \boldsymbol{\mu}^\top \mathbf{w} - \lambda \mathbf{w}^\top \boldsymbol{\Sigma} \mathbf{w} \quad \text{subject to} \quad \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0$$

Here, $\lambda \geq 0$ is a risk-aversion parameter. Adjusting λ allows investors to generate different portfolios along the efficient frontier depending on their risk tolerance.

3.4.1 Minimum Variance Portfolio

Let $h \in \{1, \ldots, H\}$ denote the index for each rebalancing (e.g., weekly) period. Let $w_{h-1}^* \in \mathbb{R}^N$ be the vector of actual portfolio weights at the end of the previous period $h-1$, and let $w_h \in \mathbb{R}^N$ denote the new portfolio weights to be chosen at period h.

The total relative transaction cost incurred when moving from w_{h-1}^* to w_h is modeled as:

$$\tau_h = \sum_{i=1}^{N_h} c_{h,i} \left| w_{h,i} - w_{h-1,i}^* \right|, \tag{1}$$

where $c_{h,i}$ denotes the per-unit transaction cost for asset i at time h, and N_h is the number of assets in the investment universe at that time.

The portfolio optimization problem that balances variance and transaction costs is then given by:

$$\min_{w \in \mathbb{R}^N} \quad w^\top \hat{\Sigma}_h w + \lambda \sum_{i=1}^{N_h} c_{h,i} \left| w_{h,i} - w_{h-1,i}^* \right| \tag{2}$$

, subject to the following constraints:

$$\boldsymbol{\mu}_h^\top \mathbf{w} \geq b_h, \tag{3}$$

$$\mathbf{1}^\top \mathbf{w} = 1, \tag{4}$$

$$\|\mathbf{w}\|_1 = \sum_{i=1}^{N} |w_i| \leq \kappa. \tag{5}$$

Here, $\hat{\Sigma}_h$ represents the estimated covariance matrix at time h. The parameter $\lambda > 0$ controls the trade-off between minimizing risk and minimizing transaction costs. Its effect can be interpreted as follows:

- **Small** λ: The optimizer places greater emphasis on minimizing portfolio variance. As a result, it is more willing to incur transaction costs to achieve lower risk.
- **Large** λ: The optimizer penalizes turnover more heavily, placing more emphasis on reducing transaction costs. This may lead to accepting higher risk in exchange for lower portfolio rebalancing activity.

1) The constraint $\boldsymbol{\mu}_h^\top \mathbf{w} \geq b_h$ imposes a lower bound on the expected return of the portfolio at selection time h. The scalar b_h denotes the minimum acceptable level of portfolio return. This constraint ensures that the optimizer selects portfolios that are not only low-risk or low-cost, but also expected to meet a target return threshold.

By explicitly including this condition, the investor communicates a preference for a minimum level of expected performance. This is especially useful when optimizing under additional constraints such as transaction costs or leverage limits, where risk or cost minimization alone could lead to solutions with negligible or even negative expected returns.

2) The constraint $\mathbf{1}^\top \mathbf{w} = 1$ ensures that all the available capital is fully invested — that is, the sum of the portfolio weights equals one:

$$\sum_{i=1}^{N} w_i = 1.$$

This is often referred to as the *budget constraint* in portfolio optimization.

Without the constraint $\mathbf{1}^\top \mathbf{w} = 1$, the optimizer could choose to invest nothing at all (i.e., $\mathbf{w} = \mathbf{0}$), trivially minimizing risk or cost but resulting in an unusable portfolio.

3) The constraint $\|w\|_1 \leq \kappa$ controls the gross exposure of the portfolio:

- $\kappa = 1$: Long-only portfolio (no short positions).
- $\kappa = 1.6$: 130-30 long-short portfolio.
- $\kappa = 2$: 150-50 long-short portfolio.
- $\kappa = \infty$: No constraint on leverage.

By enforcing $\mathbf{1}^\top \mathbf{w} = 1$ along with $\|\mathbf{w}\|_1 \leq \kappa$, we construct a portfolio that is both fully invested and constrained in terms of leverage. This leads to a well-posed and practically implementable solution, where the investor commits their capital while maintaining control over the level of risk introduced by short positions.

3.4.2 Maximum Sharpe Ratio (Tangency Portfolio)

The Sharpe ratio measures the return per unit risk.

$$\begin{aligned}
\max_{\mathbf{w} \in \mathbb{R}^N} \quad & \frac{\hat{\mu}_h^\top \mathbf{w}}{\sqrt{\mathbf{w}^\top \hat{\Sigma}_h \mathbf{w}}} - \lambda \sum_{i=1}^{N_h} c_{h,i} \left| w_{h,i} - w^*_{h-1,i} \right| \\
\text{s.t.} \quad & \mathbf{1}^\top \mathbf{w} = 1, \\
& \|\mathbf{w}\|_1 = \sum_{i=1}^{N} |w_i| \leq \kappa.
\end{aligned}$$

In this project, the risk-free asset is excluded because the portfolio is constructed entirely from risky assets, with no allocation to cash or money market instruments. The investment universe does not include a cash position, and the optimizer is designed to fully invest the available capital across the risky assets. Additionally, we assume the risk-free rate is small and relatively constant, meaning its inclusion would not significantly impact the portfolio weights under mean-variance optimization. Since the objective is to evaluate and compare the performance of portfolios composed solely of risky assets, using the raw Sharpe ratio—without adjusting for the risk-free rate—is both appropriate and practically relevant.

Note: Sharpe ratio is not a convex optimisation problem so we cannot directly use cvxpy we need to reformulate if we wanna use it. To convert it into a convex problem, we apply the method of **homogenization**.

Step 1: Change of variables

Let us define a new scaled variable:

$$\mathbf{y} = \alpha \mathbf{w}, \quad \text{where } \alpha > 0$$

Substituting this into the original objective:

$$\frac{\hat{\mu}_h^\top (\mathbf{y}/\alpha)}{\sqrt{(\mathbf{y}/\alpha)^\top \hat{\Sigma}_h (\mathbf{y}/\alpha)}} = \frac{\hat{\mu}_h^\top \mathbf{y}}{\alpha \sqrt{\mathbf{y}^\top \hat{\Sigma}_h \mathbf{y}/\alpha^2}} = \frac{\hat{\mu}_h^\top \mathbf{y}}{\sqrt{\mathbf{y}^\top \hat{\Sigma}_h \mathbf{y}}}$$

So the α cancels out in the Sharpe ratio, and we now work directly with $\mathbf{y}$.

Step 2: Fixing the numerator

We fix the numerator (expected return) to 1:

$$\hat{\mu}_h^\top \mathbf{y} = 1$$

This transforms the objective to:

$$\min_{\mathbf{y}, \alpha > 0} \quad \mathbf{y}^\top \hat{\Sigma}_h \mathbf{y}$$

Step 3: Transforming the constraints

The original constraints were:

1. **Budget constraint:**
$$\mathbf{1}^\top \mathbf{w} = 1 \quad \Rightarrow \quad \mathbf{1}^\top \mathbf{y} = \alpha$$

2. ℓ_1 **norm constraint:**
$$\|\mathbf{w}\|_1 \leq \kappa \quad \Rightarrow \quad \|\mathbf{y}\|_1 \leq \alpha\kappa$$

Step 4: Transforming transaction cost

The original transaction cost term:

$$\lambda \sum_{i=1}^{N_h} c_{h,i} \left| w_{h,i} - w_{h-1,i}^* \right|$$

becomes,

$$\lambda \sum_{i=1}^{N_h} c_{h,i} \left| y_{h,i} - \alpha w_{h-1,i}^* \right|$$

Final Reformulated Convex Optimization Problem

$$\begin{aligned}
\min_{\mathbf{y}\in\mathbb{R}^N,\,\alpha>0} \quad & \mathbf{y}^\top \hat{\Sigma}_h \mathbf{y} + \lambda \sum_{i=1}^{N_h} c_{h,i} \left| y_{h,i} - \alpha w^*_{h-1,i} \right| \\
\text{s.t.} \quad & \hat{\mu}_h^\top \mathbf{y} >= 1 \\
& \|\mathbf{y}\|_1 \leq \alpha\kappa \\
& \mathbf{1}^\top \mathbf{y} = \alpha
\end{aligned}$$

Recovering the Portfolio

Once the optimal $\mathbf{y}^*$ and α^* are found, we recover the optimal portfolio weights by:

$$\mathbf{w}^*_h = \frac{\mathbf{y}^*}{\alpha^*}$$

This yields the tangency portfolio, which lies on the efficient frontier and is tangent to the capital market line in a return-risk plot.

Note: I experimented with two convex reformulations of the Sharpe ratio optimization problem: one that fixes the portfolio risk and maximizes expected return, and another that fixes the expected return and minimizes risk. While both methods are theoretically valid and produced similar volatility profiles, the fixed-return approach consistently delivered higher risk-adjusted returns (Sharpe ratio) and better overall performance in backtesting. Additionally, it aligns more closely with traditional mean-variance theory. This outcome indicates that the target return parameter used in the second optimization guided the model to a portfolio closer to the true tangency portfolio—the unique point on the efficient frontier that globally maximizes risk-adjusted returns. Based on these results, I chose to adopt the fixed-return formulation for the final portfolio construction.

Choice of objective function

For our study, we chose two of the most foundational and widely adopted objective functions in modern portfolio theory:

- Global Minimum Variance Portfolio (MVP)
- Maximum Sharpe Ratio Portfolio (Tangency Portfolio)

These objective functions represent two distinct investment philosophies—risk minimization and risk-adjusted return maximization. Their complementary nature allows us to comprehensively analyze portfolio performance under differing investor preferences.

- The Global Minimum Variance Portfolio aims to construct a portfolio with the lowest possible volatility, regardless of return. It does not rely on expected returns, which are notoriously difficult to estimate accurately.
- Empirical studies have shown that minimum variance portfolios tend to outperform in real-world settings, especially out-of-sample. (Try to find some research paper to link) MVP represents a baseline for any efficient frontier—any other optimal portfolio must carry more risk.

- Thus, minimum variance optimization serves both as a robust default strategy and a benchmark for comparison.
- However, one of the main drawbacks of investing in a Minimum-Variance Portfolio is that it may underperform other portfolios during market upswings. Additionally, the portfolio may not necessarily be diversified, which could lead to concentration risk if a few assets in the portfolio are negatively impacted.

That's why MVP is often used alongside or compared with other strategies that optimize both return and risk, like the maximum Sharpe ratio approach.

- The Sharpe ratio is one of the most commonly used measures of risk-adjusted return. It looks for the mix where return over the risk-free rate is high compared to volatility. This helps investors see which asset mix is the most efficient for its level of risk.
- The Maximum Sharpe Ratio Portfolio/ tangency portfolio that maximizes this ratio.
- Despite its sensitivity to input estimation—especially expected returns—we include this objective to explore the potential upside of optimal risk-adjusted allocation.

By optimizing for both the minimum variance and maximum Sharpe ratio, we get a comprehensive understanding of portfolio behavior under different optimization philosophies:

- MVP targets risk minimization, making it suitable for conservative or passive investment strategies.
- The Sharpe-optimizing portfolio targets return maximization relative to risk, aligning with more aggressive, performance-seeking strategies.

CVXPY was the most appropriate choice for my portfolio optimization project because it is specifically built for solving convex optimization problems like mean-variance and Sharpe ratio maximization. It provides a high-level mathematical interface, allows for clean constraint modeling, supports multiple solvers, and guarantees globally optimal solutions for convex formulations—all of which are crucial for realistic and robust portfolio construction.

3.5 Portfolio Evaluation Metrics

To evaluate the effectiveness and robustness of the constructed portfolios, we employ a comprehensive suite of performance and risk-adjusted return metrics. These include Compound Annual Growth Rate (CAGR), Annualized Volatility, Maximum Drawdown, Sharpe Ratio, Sortino Ratio, and Treynor Ratio. Each of these metrics captures a distinct dimension of portfolio performance, allowing for a multidimensional assessment that balances return generation, risk exposure, and benchmark-relative behavior.

3.5.1 Compounded Annual Growth Rate

The Compound Annual Growth Rate (CAGR) is a fundamental metric used to evaluate the average annual growth of an investment or portfolio over a specified time period, assuming the investment grows at a steady rate on a compounding basis. It represents the constant rate of return required for a portfolio to grow from its initial value to its final value, thereby smoothing out the effects of market volatility and short-term fluctuations.

In simpler terms, CAGR answers the question: "At what fixed annual rate must my investment grow to move from the beginning value to the ending value over the given period?"

Mathematically, the CAGR over n years is calculated as:

$$\text{CAGR} = \left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)^{\frac{1}{n}} - 1 \tag{6}$$

where:

- V_{final} is the portfolio value at the end of the period,
- V_{initial} is the portfolio value at the beginning of the period,
- n is the number of years.

In this study, we compute CAGR using weekly portfolio returns. Let $r_1, r_2, \ldots, r_T$ denote the weekly returns over T weeks. The cumulative return over the entire period is:

$$\text{Total Growth Factor} = \prod_{t=1}^{T}(1 + r_t) \tag{7}$$

To annualize this return, we compute the number of years as $n = \frac{T}{52}$, and the CAGR is given by:

$$\text{CAGR} = \left(\prod_{t=1}^{T}(1 + r_t)\right)^{\frac{1}{n}} - 1 \tag{8}$$

One of the primary advantages of using CAGR is that it removes the irregularities and noise introduced by year-to-year fluctuations in returns. By presenting a single, smoothed-out annual growth rate, it allows for easier comparisons across different portfolios, strategies, or time periods.

However, a notable limitation of CAGR is that it does not account for the volatility or risk of the portfolio. Two portfolios may have identical CAGR values but very different return paths—one might have had stable growth, while the other experienced sharp drawdowns and recoveries. This is why CAGR is typically used in conjunction with other risk-adjusted performance metrics, such as the Sharpe ratio, Sortino ratio, and maximum drawdown, to obtain a complete picture of portfolio behavior.

3.5.2 Annualised volatility

Volatility is a fundamental measure of the risk associated with a portfolio. It quantifies how much the portfolio's returns fluctuate over time, capturing the degree of unpredictability or instability in its performance. In simple terms, higher volatility indicates that returns vary widely from their average, often leading to more uncertainty and risk for investors. On the other hand, lower volatility suggests more consistent and stable returns.

Technically, volatility is measured using the standard deviation of returns. Standard deviation answers the question: "On average, how far are the returns from their mean?" It provides a statistical gauge of the dispersion in returns over a given period.

In this study, we compute annualized volatility based on weekly returns. To do this, we first calculate the standard deviation of weekly returns and then scale it by the square root of 52 (the number of trading weeks in a year). This standardization allows us to express volatility on an annual basis, making it easier to compare across portfolios and time horizons.

Volatility plays a critical role in portfolio evaluation because it provides insight into the total risk taken to achieve a given return. While return metrics like CAGR tell us how much the portfolio grew, they do not account for how consistent or smooth that growth was. Volatility fills this gap by measuring the intensity of fluctuations around the average return.

It is also a key input to many risk-adjusted return metrics, such as the Sharpe Ratio and Sortino Ratio, which assess whether higher returns are justified by the level of risk taken.

3.5.3 Max Drawdown

Maximum Drawdown (MDD) is a key risk metric used to quantify the largest observed loss from a portfolio's peak value to its lowest point (trough) over a specific time period. It provides a clear measure of downside risk by capturing the worst-case decline an investor would have experienced before the portfolio recovered.

To calculate drawdown, we first track the highest value the portfolio has reached up to each point in time—this is referred to as the running peak. At each point, we compare the current portfolio value to this running peak. The drawdown at that point is the percentage decline from the peak. The maximum drawdown is simply the largest such decline observed in the entire time series.

Mathematically, the maximum drawdown is given by:

$$\text{Max Drawdown} = \frac{\text{Trough Value} - \text{Peak Value}}{\text{Peak Value}} \tag{9}$$

One of the most valuable aspects of MDD is its ability to convey both the magnitude and the duration of the worst historical decline. This helps investors assess not only how much the portfolio value dropped, but also how long it remained under water before recovering to a new high.

In this study, to enable a fair comparison across portfolios with varying levels of volatility, we normalize the Maximum Drawdown by dividing it by the portfolio's annualized volatility. A normalized MDD metric provides a more apples-to-apples comparison of downside risk, independent of overall risk levels.

MDD has certain limitations. It focuses solely on the single worst decline, without accounting for the frequency of such drawdowns, the speed of recovery, or the overall variability in returns. A portfolio may exhibit a low MDD but still experience frequent moderate losses, which may not be captured by this metric alone.

Therefore, while Maximum Drawdown is a critical component of portfolio risk assessment, it should be used in conjunction with other measures—such as volatility, Sharpe ratio, and recovery time—to obtain a more comprehensive view of a portfolio's performance and stability under stress.

3.5.4 Sharpe Ratio

The Sharpe Ratio is one of the most widely used measures of risk-adjusted return. It quantifies the excess return earned by a portfolio over the risk-free rate, per unit of total risk, as measured by the standard deviation of portfolio returns.

It answers the question: *"How much excess return am I getting for each unit of risk I am taking?"* This makes it particularly useful for comparing investments with different levels of risk on an equal footing, enabling more informed decision-making in portfolio construction and capital allocation.

Mathematically, the Sharpe Ratio is defined as:

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p} \tag{10}$$

where:

- R_p is the average return of the portfolio,
- R_f is the risk-free rate of return,
- σ_p is the standard deviation (volatility) of portfolio returns.

In this analysis, we assume the risk-free rate $R_f = 0$. Under this assumption, the Sharpe Ratio reflects the raw return earned per unit of risk.

Interpretive guidelines for the Sharpe Ratio are as follows:

- Sharpe Ratio < 1.0 $\rightarrow$ Sub-par risk-adjusted performance
- $1.0 \leq$ Sharpe Ratio < 2.0 $\rightarrow$ Acceptable performance
- $2.0 \leq$ Sharpe Ratio < 3.0 $\rightarrow$ Strong performance
- Sharpe Ratio ≥ 3.0 $\rightarrow$ Excellent risk-adjusted returns

A higher Sharpe Ratio indicates that a portfolio is delivering better returns for the same or lower levels of risk, which is highly desirable.

However, the metric has its limitations. Notably, it treats both upside and downside volatility equally, which may understate the appeal of investments with asymmetric return profiles (e.g., strategies that yield frequent small losses and occasional large gains).

3.5.5 Sortino Ratio

The Sortino Ratio is a refinement of the Sharpe Ratio that measures the risk-adjusted return of a portfolio, but with a crucial difference — it only considers downside risk rather than total volatility. In doing so, it avoids penalizing an investment for positive fluctuations, which are typically desirable.

In our analysis, we assume the risk-free rate to be zero, which means we are measuring excess returns relative to zero rather than to a government bond or similar low-risk asset. Under this assumption, the downside deviation — the key risk input in the Sortino Ratio — focuses exclusively on negative returns, since those are the only observations that fall below the risk-free rate of 0. In other words, only losses contribute to the risk measure in the Sortino Ratio, making it more aligned with an investor's actual concerns: the risk of losing money.

$$\text{Sortino Ratio} = \frac{R_p - R_f}{\sigma_D}$$

where:

- R_p = Portfolio return (mean or annualized)
- R_f = Risk-free rate (assumed to be 0 in our analysis)
- σ_D = Downside deviation (standard deviation of negative returns)

While both the Sharpe and Sortino Ratios are used to evaluate the risk-adjusted performance of an investment, they differ in how they define and handle risk:

- Sharpe Ratio considers total volatility, using the standard deviation of all returns — both positive and negative. This means that large gains can increase the standard deviation and reduce the Sharpe Ratio, even though such gains are beneficial to the investor.
- Sortino Ratio, on the other hand, focuses only on downside deviation, measuring the variability of negative returns. As a result, it gives a more realistic picture of risk from the perspective of an investor who is primarily concerned with losses rather than all types of fluctuations.

The Sortino Ratio is particularly useful for evaluating strategies that have asymmetric return distributions, such as those that generate steady returns with occasional large gains. In such cases, the Sharpe Ratio might unfairly penalize the strategy due to high volatility from those gains. The Sortino Ratio avoids this issue and provides a more accurate measure of performance in terms of risk of loss.

By isolating downside risk, the Sortino Ratio gives investors a more targeted understanding of how well a portfolio compensates them for the risk they truly care about — the risk of negative returns.

3.5.6 Treynor Ratio

The Treynor Ratio is a performance metric that measures the excess return of a portfolio per unit of systematic risk—that is, the portion of risk that is linked to overall market movements and cannot be eliminated through diversification.

$$\text{Treynor Ratio} = \frac{R_p - R_f}{\beta_p}$$

where:

- R_p = Portfolio return
- R_f = Risk-free rate (assumed to be 0 in our analysis)
- β_p = Portfolio beta (systematic risk)

Unlike the Sharpe Ratio, which considers total risk (including both systematic and unsystematic risk), the Treynor Ratio isolates only systematic risk by using the portfolio's beta as the risk measure. Beta represents the sensitivity of the portfolio's returns to movements in the overall market, and therefore captures how much market risk the portfolio is exposed to.

This makes the Treynor Ratio particularly useful for evaluating well-diversified portfolios, where unsystematic risk has largely been diversified away. In such cases, only systematic risk remains relevant, and the Treynor Ratio provides a more accurate assessment of how effectively the portfolio has compensated investors for the market risk they've taken.

In our study, we use an equally weighted benchmark portfolio as a proxy for market returns when computing beta. This was an appropriate benchmark as it better represents the opportunity set available to the investor.

Ratio	**Return per Unit of**	**Formula**
Sharpe Ratio	Total Risk	Sharpe Ratio $= \frac{R_p - R_f}{\sigma_p}$
Sortino Ratio	Downside Risk	Sortino Ratio $= \frac{R_p - R_f}{\sigma_D}$
Treynor Ratio	Systematic Risk	Treynor Ratio $= \frac{R_p - R_f}{\beta_p}$

Table 1: Risk-Adjusted Return Metrics

The selected combination of metrics provides a well-rounded framework for evaluating portfolio performance. CAGR summarizes overall return potential over time, while volatility and max drawdown capture different aspects of risk—variability and peak-to-trough losses, respectively. The Sharpe, Sortino, and Treynor ratios provide varying perspectives on risk-adjusted returns, differentiating between total, downside, and systematic risk. This blend of metrics ensures that we do not focus solely on returns or risk in isolation, but instead evaluate performance holistically.

4 Backtesting Design

The backtesting framework was designed to evaluate the performance of portfolio optimization strategies over a historical period. We employed a **rolling window approach** to dynamically update model inputs and portfolio weights, simulating real-world conditions where market data evolves over time.

The backtest spans from **2013 to 2025**, with a **training window of 10 years (520 weeks)** for return estimation and **1 year (52 weeks)** for risk estimation, preceding each rebalancing point.

We use a long window (10 years) for estimating expected returns because a long history smooths out temporary fluctuations giving a more stable mean. Using 10 years captures multiple market regimes, helping avoid overfitting to recent noise.

For covariance matrix estimation we use a shorter window (1 year) because volatility and correlations are time-varying and adapt much faster than expected returns. Using a shorter rolling window ensures the covariance matrix reflects the current market risk structure.

In industry practice, return estimation uses longer (5-10 years) training windows to capture structural return characteristics. Covariance estimation use shorter windows (6–24 months) to remain sensitive to recent shifts in risk and correlation.

At every weekly rebalancing date, the following steps are performed:

1. **Data Selection**
   - From the available assets, we select those with complete return histories over the previous 520 weeks to ensure consistency in covariance and expected return estimation.
2. **Expected Return and Covariance Estimation**
   - Expected returns (μ) are estimated using different methods, such as:
     - Mean Historical Return
     - Exponentially Weighted Moving Average (EMA)
     - CAPM-based estimates
   - The covariance matrix (Σ) is computed using one of three models:
     - Sample Covariance
     - Exponential Covariance
     - Shrinkage Estimator (Ledoit-Wolf)
3. **Optimization and Constraints**
   - At each rebalancing, an optimization model determines portfolio weights:
     - Global Minimum Variance
     - Max Sharpe Ratio
4. **Portfolio Update and Performance Tracking**
   - The previous portfolio is *drifted forward* based on realized returns before rebalancing, mimicking how weights naturally evolve.
   - Transaction costs are applied based on the absolute change in weights, scaled by per-asset trading cost rates.
   - Portfolio value is updated multiplicatively using net portfolio returns after costs.
5. **Performance Metrics**
   - At the end of the backtesting horizon, we compute risk-return statistics such as:
     - CAGR

  – Annualized Volatility
  – Sharpe, Sortino and Treynor Ratios
  – Max Drawdown

This design ensures that our backtest reflects practical considerations such as **time-varying inputs, rolling estimation windows, and trading frictions**, providing a realistic assessment of portfolio performance.

5 Results and Performance Analysis

5.1 Expected Return Estimators

All portfolios were constructed using the same settings:

- **Covariance Estimator:** Shrinkage
- **Optimisation Objective:** Maximize Sharpe Ratio

Metric	**Mean Historical Return**	**Exponentially Weighted Mean Historical Return**	**CAPM Return**
Compounded Annual Growth Rate (CAGR)	20.17%	15.30%	23.94%
Annualised Volatility	17.65%	16.16%	18.37%
Max Drawdown	2.53	2.38	2.48
Sharpe Ratio	1.13	0.97	1.26
Sortino Ratio	0.99	0.82	1.09
Treynor Ratio	0.22	0.20	0.24

Table 2: Performance comparison across different expected return models

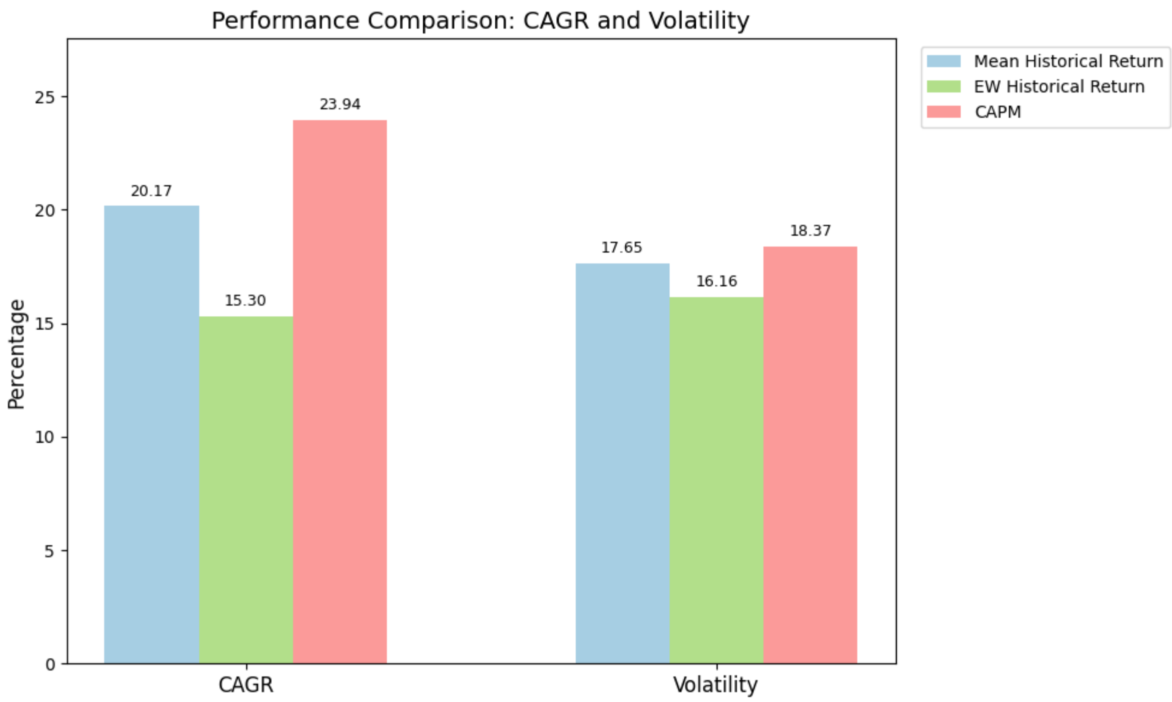

Figure 1: Portfolio Performance comparison across different return estimators

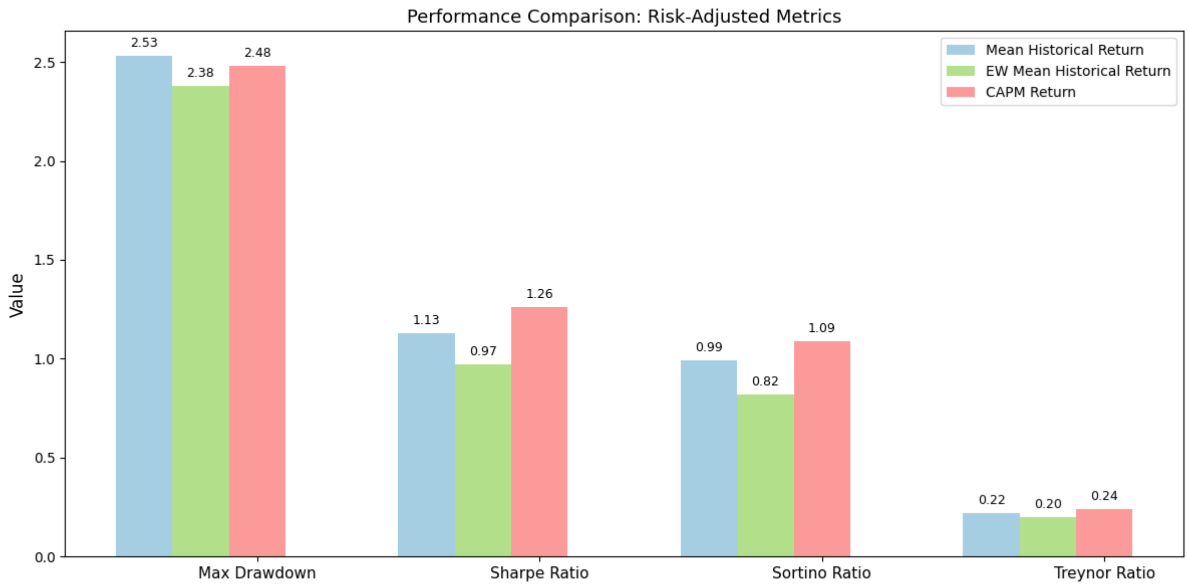

Figure 2: Portfolio Performance comparison across different return estimators

The choice of return estimation method significantly affected portfolio performance. The CAPM-based approach delivered the highest growth and best risk-adjusted returns because it tilted towards high-beta stocks during a period when taking extra market risk was rewarded. Its reliance on long-term market relationships kept portfolio weights stable, controlling volatility and limiting drawdowns. The Mean Historical Return method produced balanced results, offering moderate growth and risk but lacking adaptability to changing market conditions, which led to missed opportunities. The Exponentially Weighted Mean (EWMA) method was the most cautious, quickly adjusting weights based on recent performance. This kept volatility and drawdowns low, but in a market where recent trends often reversed, it resulted in lower returns and weaker risk-adjusted performance.

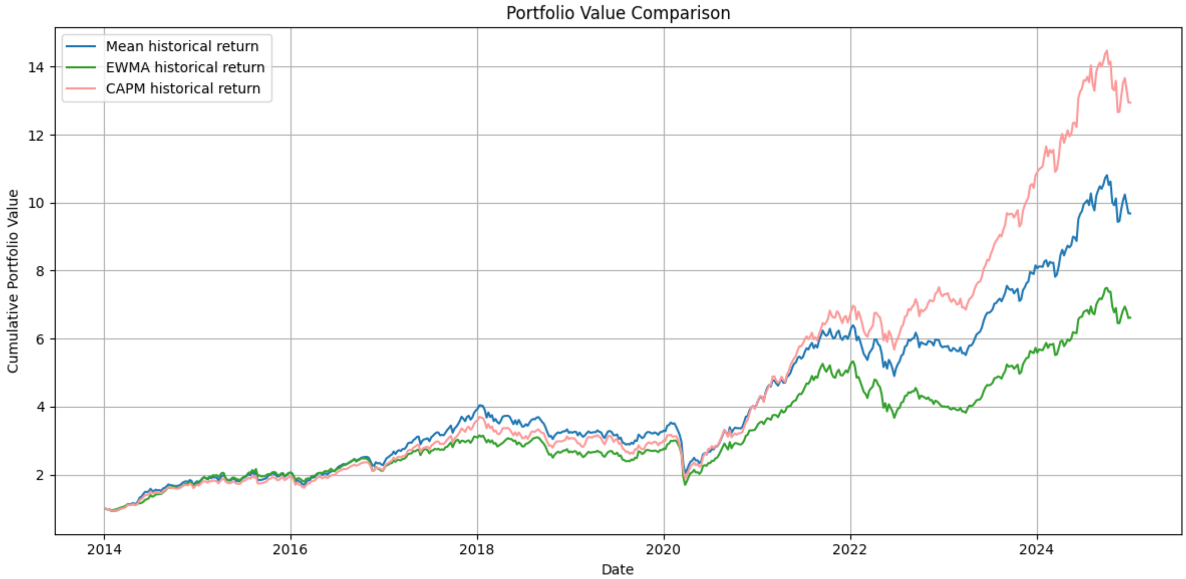

Figure 3: Performance comparison of portfolios over time

Figure 3 visually confirms that the CAPM portfolio steadily pulls ahead, particularly after 2020, reflecting its stable, market-driven weighting approach that captured strong rallies while controlling volatility.

The Mean Historical Return method shows a portfolio path that initially tracks CAPM closely but diverges in later years. This aligns with its slower reaction to changing conditions—stable but missing part of the post-2020 upside.

The EWMA approach manifests as a lagging growth curve. While its adaptability helped reduce drawdowns during downturns, the frequent allocation changes meant it often missed rebounds, limiting long-term growth.

Together, the data and chart show that a stable, market-aware method like CAPM outperformed more reactive or purely historical approaches over the sample period.

5.2 Covariance Matrix computations

All portfolios were constructed using the same settings:

- **Return Estimator:** CAPM
- **Optimisation Objective:** Global Minimum Variance (GMV)

Metric	Sample Covariance	Exponential Covariance	Covariance Shrinkage (Ledoit Wolf)
Compounded Annual Growth Rate (CAGR)	11.64%	11.98%	15.84%
Annualised Volatility	12.75%	12.57%	11.50%
Max Drawdown	2.36	2.43	2.16
Sharpe Ratio	0.92	0.96	1.33
Sortino Ratio	0.83	0.88	1.22
Treynor Ratio	0.29	0.30	0.34

Table 3: Performance comparison across different covariance matrix computation models

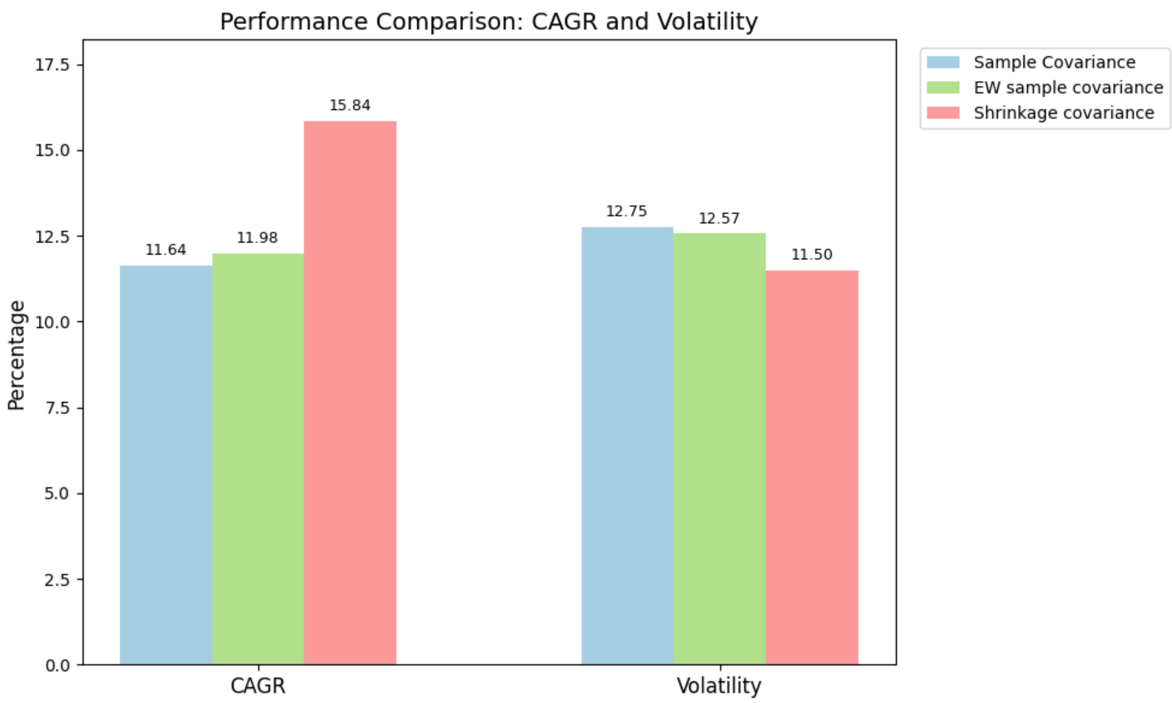

Figure 4: Portfolio Performance comparison across different covariance matrix

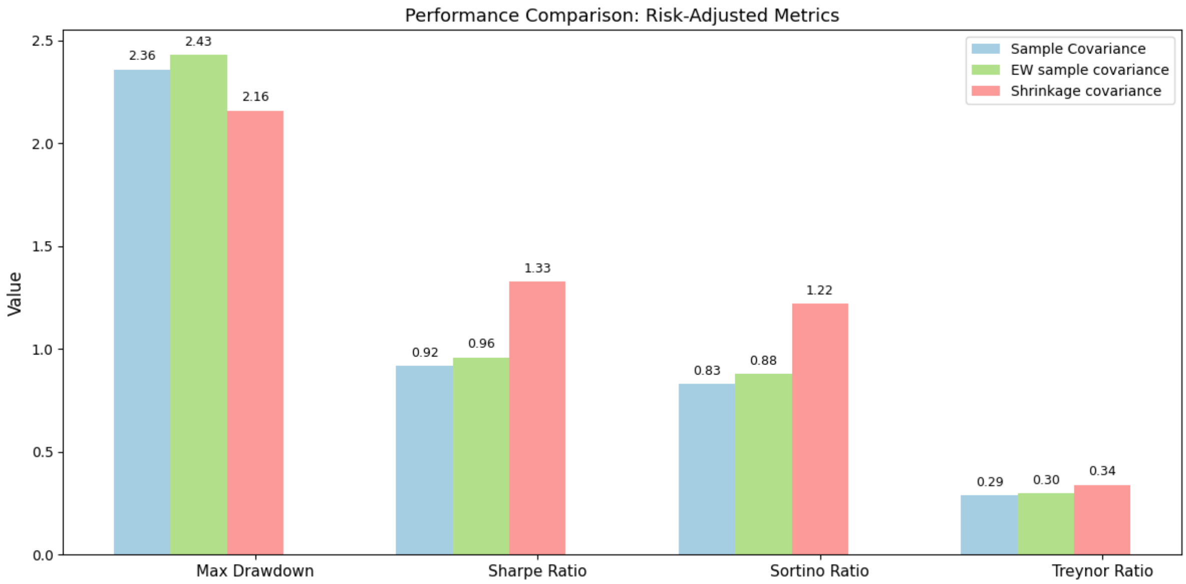

Figure 5: Portfolio Performance comparison across different covariance matrix

The choice of covariance estimation method had a clear impact on portfolio performance. The Covariance Shrinkage (Ledoit-Wolf) approach delivered the highest growth and strongest risk-adjusted returns by producing more stable and reliable covariance estimates. By blending historical data with a structured target, it reduced estimation errors, leading to lower volatility, better diversification, and smaller drawdowns. The Exponential Covariance method provided moderate results, slightly improving returns over the sample covariance by giving more weight to recent market behaviour. However, this responsiveness also increased vulnerability to sudden reversals, contributing to the highest drawdown among the three. The Sample Covariance method was the weakest performer, as its sensitivity to noise in historical data led to less stable portfolio weights and reduced efficiency, resulting in lower returns and weaker risk-adjusted metrics.

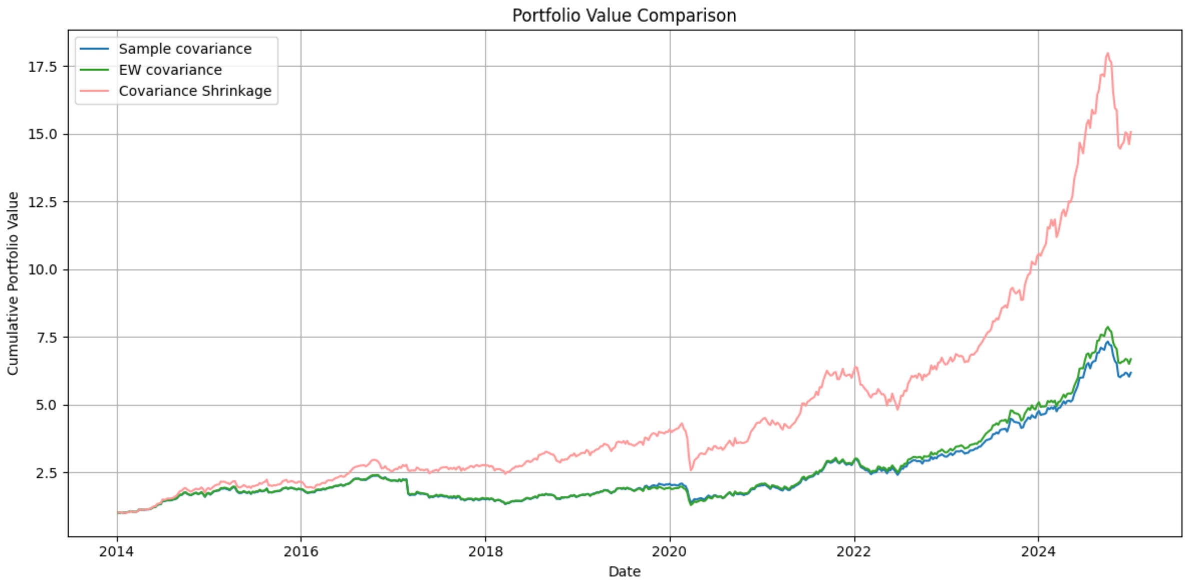

Figure 6: Performance comparison of portfolios over time

Figure 6 confirms visually, that the shrinkage-based portfolio pulled ahead consistently—especially after 2020—thanks to its ability to balance recent market conditions with long-term structure, leading to more stable and efficient risk estimates.

The EWMA covariance method was responsiveness to recent market volatility patterns, which helped it adapt to changing risk environments. In Figure 6, this manifests as the EWMA portfolio tracking closely to the Sample Covariance early on but performing marginally better in the later years.

The Sample Covariance method, while straightforward, lags behind in both metrics and cumulative growth. Its reliance on equal weighting of all historical data made it slower to adjust to new volatility regimes, leaving it less effective in capturing risk-adjusted gains compared to more adaptive methods.

Together, the data and chart show that incorporating shrinkage or adaptive weighting into covariance estimation significantly improves portfolio performance over purely historical approaches.

5.3 Different investment objectives

All portfolios were constructed using the same settings:

- **Return Estimator:** CAPM
- **Covariance Computation:** Shrinkage

Metric	**Max Sharpe Ratio**	**Global Minimum Variance**
Compounded Annual Growth Rate (CAGR)	23.94%	15.84%
Annualised Volatility	18.37%	11.50%
Max Drawdown	2.48	2.16
Sharpe Ratio	1.26	1.33
Sortino Ratio	1.26	1.33
Treynor Ratio	1.09	1.22

Table 4: Performance comparison across different investment objective

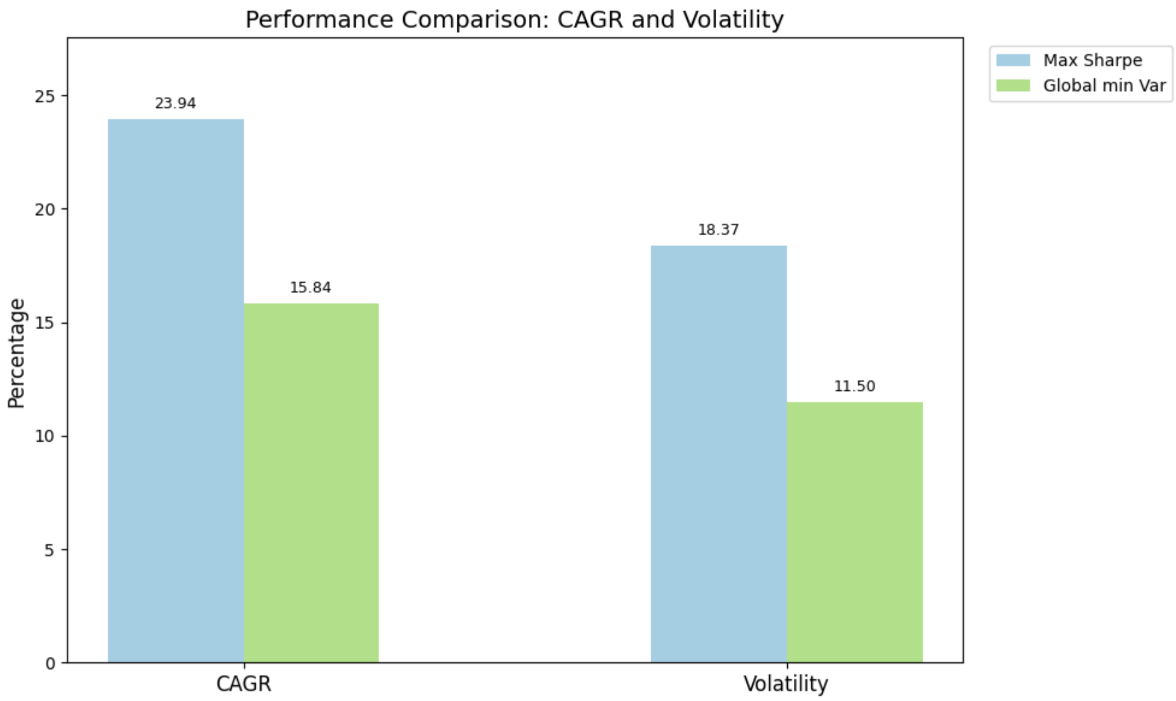

Figure 7: Portfolio Performance comparison across different objective functions

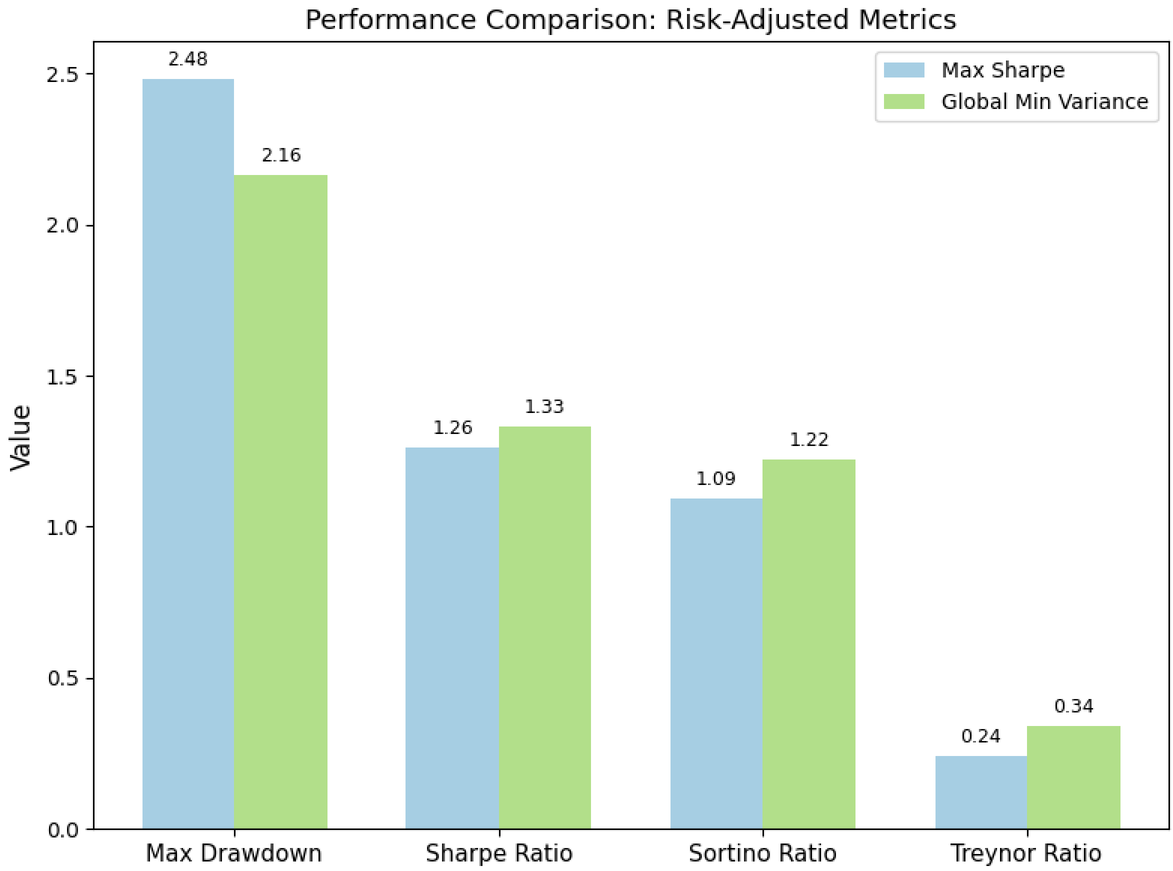

Figure 8: Portfolio Performance comparison across different objective functions

The choice of investment objective had a substantial effect on portfolio outcomes. The Max Sharpe Ratio approach delivered the highest growth by actively seeking the best return per unit of risk. However, this pursuit of higher returns came with elevated volatility and a slightly larger drawdown, reflecting its more aggressive allocation profile.

The Global Minimum Variance strategy prioritised stability, producing the lowest volatility and smallest drawdown among the two. While its CAGR was lower, it achieved marginally better risk-adjusted metrics (Sharpe, Sortino, and Treynor ratios), indicating a more efficient balance between return and risk.

Overall, the Max Sharpe Ratio approach excelled in absolute performance during the period but required tolerating greater fluctuations, whereas the Global Minimum Variance method offered steadier growth with stronger downside protection.

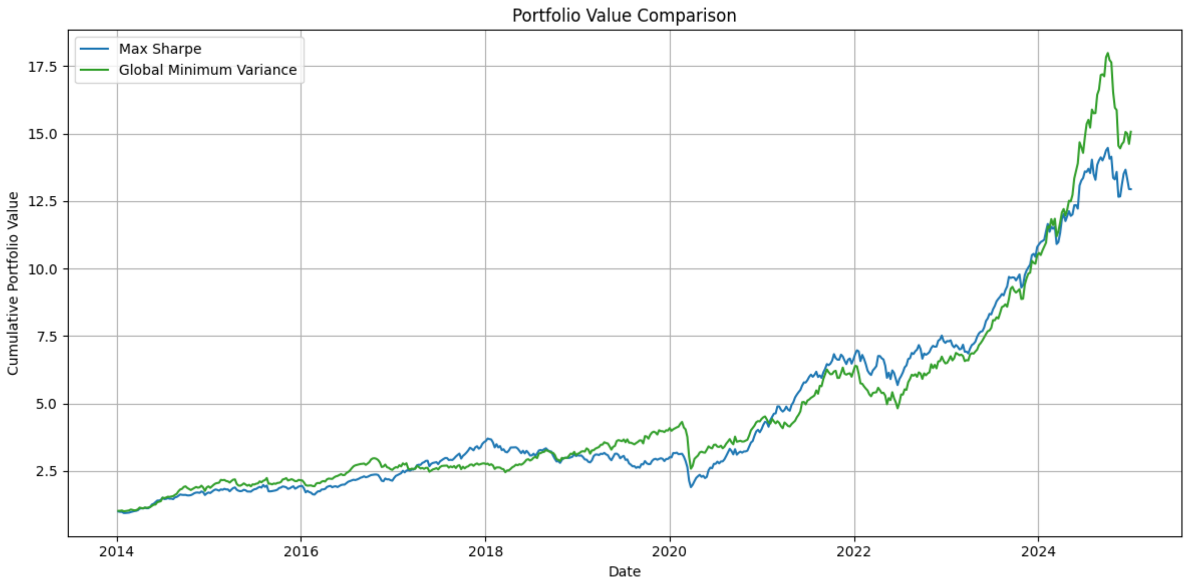

Figure 9: Performance comparison of portfolios over time

The Max Sharpe portfolio generally maintained a performance edge through much of the sample period, especially during strong market rallies, reflecting its focus on maximising returns relative to risk. However, its higher volatility also led to deeper dips during downturns, visible in the sharper drawdowns in 2018 and 2020.

The Global Minimum Variance portfolio, while slower to climb during bull phases, displayed greater resilience in volatile periods, avoiding the deepest drawdowns and recovering faster. Its steadier trajectory from 2023 to 2024 allowed it to overtake the Max Sharpe portfolio in cumulative value near the end of the period.

Overall, the Max Sharpe strategy excelled in high-growth conditions but at the cost of greater fluctuations, while the Global Minimum Variance approach provided smoother, more consistent growth with superior downside protection.

The results are in line with the observation that the Global Minimum Variance (GMV) portfolio often outperforms the Max Sharpe portfolio out-of-sample because it does not heavily rely on the highly unstable and noisy expected return estimates (). While the Max Sharpe strategy aggressively tilts towards assets with the highest estimated returns, GMV focuses primarily on the more stable and reliable covariance structure. Although an expected return still enters the GMV framework as a constraint (ensuring a feasible portfolio with a minimum target return), it plays a much smaller role in driving weights. By minimising risk first and treating return expectations conservatively, GMV builds a more robust portfolio that is less prone to performance decay in real-world conditions.

Note: Expected returns are much harder to estimate than covariances because returns fluctuate a lot. Covariance, by contrast, looks at how two assets move relative to each other rather than their overall direction. Even when individual returns are noisy, the pattern of moving together (or in opposite directions) tends to be more stable over time. So, if Stock A and Stock B both bounce around a lot but usually move in the same direction, the covariance will pick that up reliably, whereas the average return for either stock will be much less certain.

5.4 Comparison to benchmarks

Metric	Max Sharpe Ratio	Global Minimum Variance	Equally weighted portfolio (benchmark	Nifty 500 Index	Motilal Oswal Mutual Fund
Compounded Annual Growth Rate (CAGR)	23.94%	15.84%	35.34%	14.68%	24%
Annualised Volatility	18.37%	11.50%	24.63%	15.70%	47.94%
Max Drawdown	2.48	2.16	2.59	2.12	1.65
Sharpe Ratio	1.26	1.33	1.35	0.95	0.74
Sortino Ratio	1.09	1.22	1.28	0.86	0.75
Treynor Ratio	0.24	0.34	0.33	0.19	0.32

Table 5: Performance comparison across different portfolios

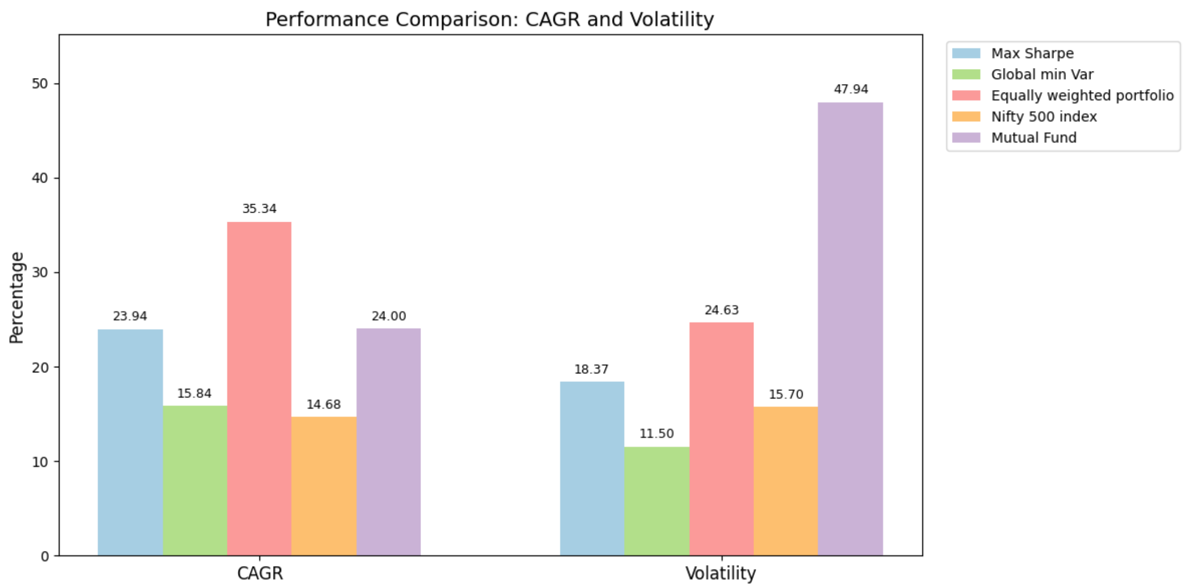

Figure 10: Portfolio Performance comparison to benchmarks

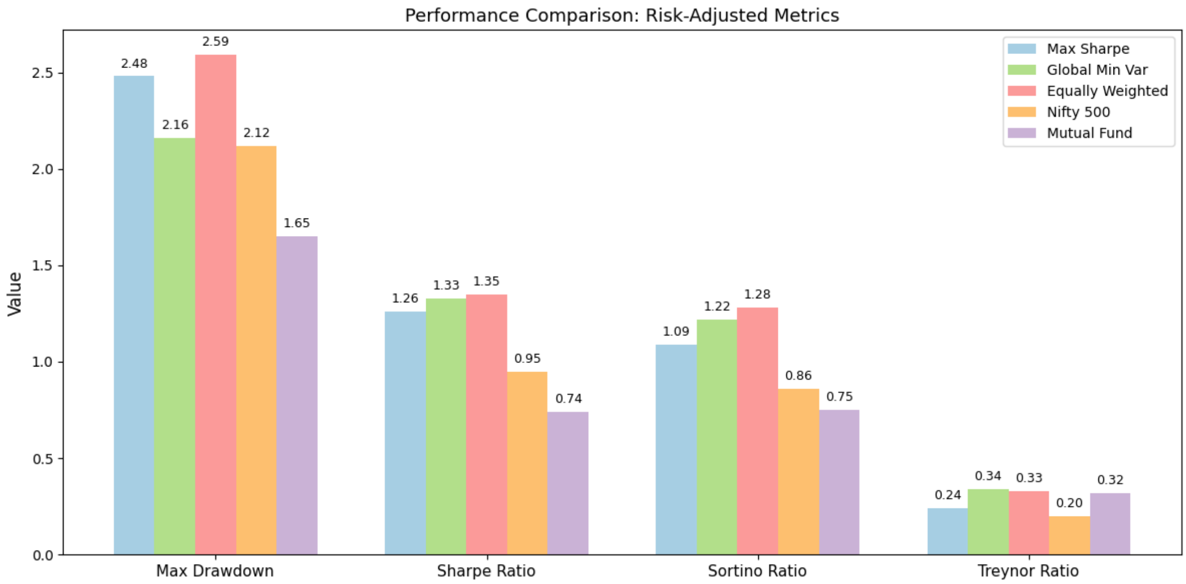

Figure 11: Portfolio Performance comparison to benchmarks

The performance comparison highlights the trade-offs between return potential and risk management across different portfolio strategies. The equally weighted portfolio achieved the highest growth but at the cost of the highest volatility among the tested strategies. The Global Minimum Variance (GMV) portfolio delivered the lowest volatility and the highest risk-adjusted returns, reflecting its strength in stability and downside protection, though at the expense of lower absolute returns. The Max Sharpe portfolio balanced growth and risk more aggressively, generating strong returns with higher volatility than GMV but lower than the benchmark.

In comparison, the Nifty 500 index underperformed both optimised portfolios on a risk-adjusted basis, and while the Motilal Oswal Mutual Fund delivered high absolute returns, it carried extreme volatility that eroded its efficiency, as reflected in its low Sharpe ratio. Overall, optimisation methods, particularly GMV produced more efficient portfolios than passive or high-volatility active strategies.

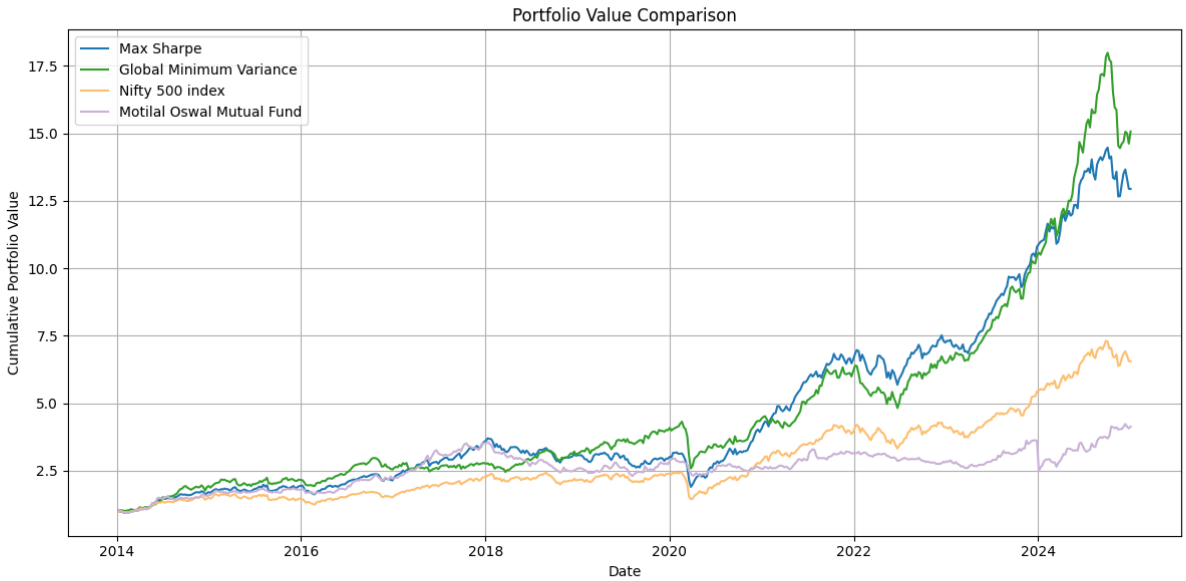

Figure 12: Performance comparison of portfolios over time

The observed performance differences stem from how each strategy treats risk, diversification, and return estimation. The equally weighted portfolio produced the highest returns primarily because it spreads capital evenly across assets without regard for volatility or correlation. This inadvertently results in large exposures to high-volatility stocks, which boosted performance during strong market upswings but also led to higher risk.

The Global Minimum Variance (GMV) portfolio's superior risk-adjusted metrics arise from its exclusive focus on minimising overall portfolio volatility using the covariance structure. By avoiding reliance on unstable expected returns, GMV builds a more robust portfolio that remains resilient in varying market conditions, even if its absolute returns are lower.

The Max Sharpe portfolio's stronger returns compared to GMV are due to its attempt to exploit estimated return differences between assets, tilting towards historically better performers. However, this approach increases exposure to estimation errors and market reversals, leading to higher volatility and lower risk-adjusted returns compared to GMV.

The Nifty 500 index underperforms the optimised portfolios because it is a cap-weighted strategy dominated by a few large, volatile constituents, while the Motilal Oswal Mutual Fund's high volatility suggests concentrated bets or sector tilts, which amplified both returns and drawdowns.

Conclusion

This project demonstrates that the choice of quantitative techniques has a significant impact on portfolio performance. The study systematically evaluated various strategies by combining different estimators for expected returns and covariance within two primary optimization frameworks.

The key findings indicate that:

- **CAPM as a return estimator** consistently outperformed both mean historical returns and exponentially weighted moving averages. Portfolios constructed using CAPM-based return estimates achieved a superior Sharpe Ratio.
- **Covariance shrinkage** proved to be the most effective method for estimating the covariance matrix. It resulted in a portfolio with the best risk-adjusted returns.
- When comparing investment objectives, the **Global Minimum Variance (GMV) portfolio, delivered superior risk-adjusted performance** with a higher Sharpe Ratio. This aligns with existing empirical evidence suggesting that GMV portfolios are more robust out-of-sample because they do not rely on notoriously noisy expected return estimates.
- The optimized portfolios, demonstrated competitive performance when compared against benchmarks like the Nifty 500 index, an equally weighted portfolio and mutual fund.

In essence, a combination of CAPM for return estimation and a shrinkage methodology for covariance, particularly within a Global Minimum Variance optimization framework, yielded the most robust and risk-efficient portfolio in this study.

Key Takeaways

- **Estimator Selection is Crucial**: The performance of a quantitative portfolio is highly sensitive to the choice of return and risk estimators. Simplistic models like mean historical returns can be easily outperformed by more theoretically grounded approaches like CAPM.
- **Robust Risk Modeling is Key**: The instability of the sample covariance matrix can harm portfolio performance. Shrinkage estimators, like Ledoit-Wolf, offer a more stable and reliable alternative, leading to better-behaved portfolios with lower volatility and improved risk-adjusted returns.
- **Focus on Risk Can Beat Chasing Returns**: The Global Minimum Variance strategy, which ignores noisy expected return forecasts and focuses solely on the more stable covariance structure, often yields better out-of-sample risk-adjusted returns than strategies that explicitly target return maximization (like Maximum Sharpe Ratio).
- **Quantitative Strategies Can Outperform Passive Benchmarks**: The back-tested quantitative portfolios, particularly the Global Minimum Variance strategy, demonstrated superior risk-adjusted performance compared to passive benchmarks like the Nifty 500 index, highlighting the value of systematic portfolio construction.
- **Beware of Model Limitations**: The reported results are subject to limitations such as survivorship bias, simplified transaction cost models, and assumptions about short-selling and a constant risk-free rate. These factors could lead to overly optimistic performance estimates.

Limitations

Survivorship Bias and Its Handling Survivorship bias arises when only assets that have persisted until the end of the study period are included in the analysis, excluding those that were delisted, merged, or went bankrupt. This bias inflates performance estimates since poorly performing assets are omitted. In our study, historical price data was obtained for the current constituents of the NIFTY 500 index via Yahoo Finance, so companies that were removed from the index in the past are not included in our dataset. This introduced survivorship bias, which is a common limitation in studies using publicly available datasets. Almost all sources that provide survivorship-bias-free data are paid because they require maintaining historical index constituents and delisted stocks, which is costly.

Fixed Transation Cost Transaction costs are modeled as a fixed rate per trade (e.g., 1%). In reality, costs vary with liquidity, market impact, and volume.

Short Selling restrictions The backtesting framework assumes unrestricted short-selling without incorporating practical limitations. In reality, not all stocks can be shorted, and short-selling involves additional costs such as borrowing fees, margin requirements, and sometimes regulatory constraints. Ignoring these factors can lead to overly optimistic performance estimates, as the simulated portfolio may achieve allocations and returns that are not feasible in real-world conditions.

Risk free rate We chose a constant risk-free rate of 7% annually based on the long-term average yield of Indian government securities, which are commonly used as a proxy for the risk-free rate in financial models. While this provides a reasonable benchmark for expected returns, it remains unrealistic because interest rates fluctuate over time due to monetary policy changes, inflation, and macroeconomic conditions.

Future Improvements

Based on the findings and methodologies adopted in this project, the following recommendations are proposed to enhance future work and improve the practical robustness of quantitative portfolio construction:

1. We can conduct sensitivity analysis on transaction cost assumptions, shrinkage intensity, and decay factors in EMA estimators to test model stability under varying market conditions.

2. We can include alternative smart beta strategies (e.g., equal risk contribution, volatility-weighted), risk-parity portfolios, or even a 60/40 equity-debt blend to provide broader performance context.

3. We can introduce liquidity screening (e.g., minimum average traded volume) to address liquidity constraints.

4. We can try using machine learning models like random forests, gradient boosting, or LSTM to predict future returns. These models may capture patterns that traditional methods miss.

References

1. Ahmed Fahad. *Understanding Walk-Forward Validation in Time Series Analysis.* Medium, 2023.

2. Oliver Ledoit, & Michael Wolf. *Markowitz Portfolio under Transaction Cost.* Journal of Computational Finance, 2025.

3. Theo Diamandis. *Making the Sharpe Ratio Maximisation Problem Convex*, 2022.

4. PyPortfolioOpt Documentation. Accessed 2025.

Appendix

Data source and tools

- Data Source: Yahoo Finance.
- Tools and Packages: Python (NumPy, Pandas, yfinance, PyPortfolioOpt, CVXPY, Matplotlib, Seaborn).

Github repository

https://github.com/shrutiss14/Summer-internship-project-2025.

Log vs simple returns

Table 6: Comparison of Simple vs Log Returns

Ticker	**Simple Mean**	**Log Mean**	**Simple Std**	**Log Std**	**Cum Simple**	**Cum Log**	**Mean Diff (%)**
RELIANCE.NS	0.004612	0.003718	0.042295	0.042044	70.39	70.39	-0.089

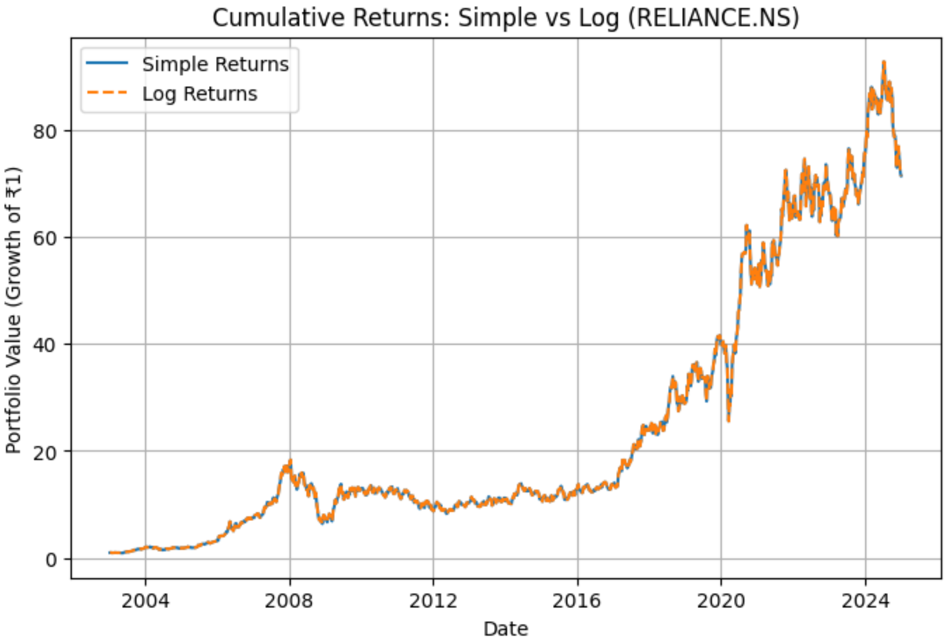

Figure 13: Cumulative returns from simple vs log returns for RELIANCE.NS show negligible difference over the sample period.

The negligible differences confirm that using simple returns instead of log returns does not affect the robustness of the portfolio optimization results.

Stationary vs non-stationary

Stock prices are typically non-stationary, meaning their mean, variance, and other statistical properties change over time. This makes direct modeling of prices problematic, as most statistical and optimization methods assume stationary inputs. In contrast, returns are closer to stationary, making them more suitable for modeling, forecasting, and optimization. Returns also allow for comparisons across assets with different price levels and enable better risk measurement.

To illustrate this, Figure 15 shows the adjusted closing price and weekly returns for RELIANCE.NS from 2014–2025. The price series exhibits a clear trend (non-stationary), while the returns fluctuate around zero with relatively stable variance (closer to stationary).

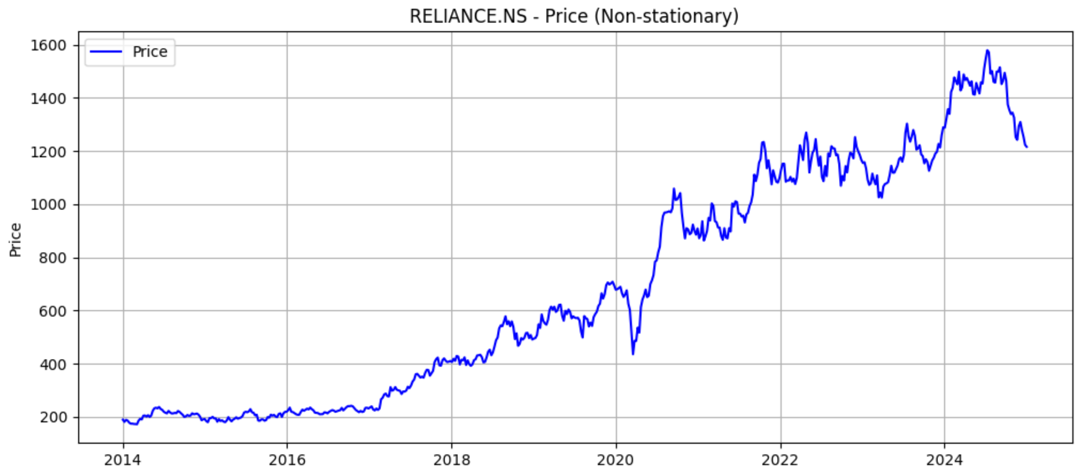

Figure 14: Price (Non-stationary) for RELIANCE.NS

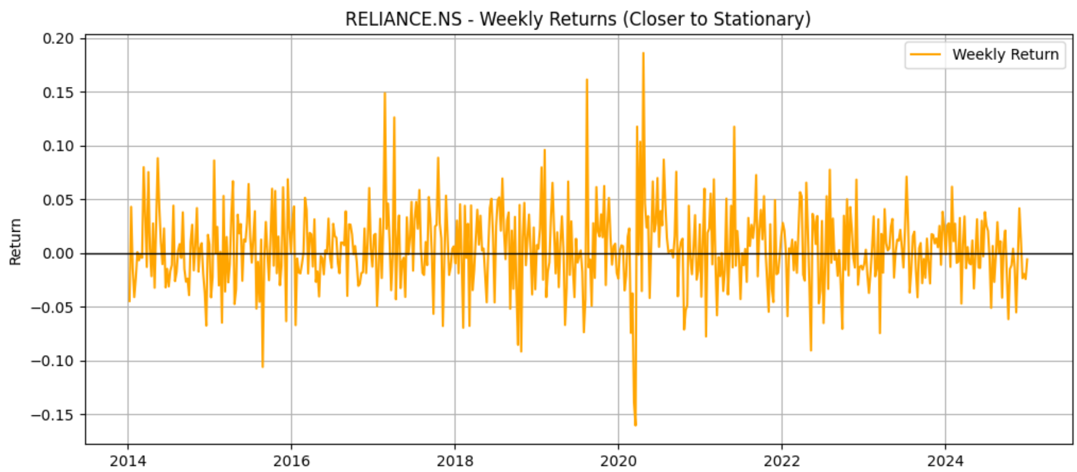

Figure 15: Weekly Returns (Closer to Stationary) for RELIANCE.NS