DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
Properties of Eigen values and Eigen vectors, Cayley Hamilton theorem.

Course Code: MAT2002 **Experiment: 2-A**
Course Name: Application of Differential and Difference Equations **Duration: 90 Minutes**

Eigenvalues and Eigenvectors

We study the problem

$$AX = \lambda X$$

where A is given $n \times n$ square matrix, X is an unknown $n \times 1$column vector, and λ is an scalar.
given an $n \times n$ matrix A, find the value of λ such that $[A - \lambda I]X = 0$ admits non-trival solution, and find those non-trival solution.
This is called the **Eigenvalue Problem**

Solving **characterstic equation** $|A - \lambda I| = 0$, we get n values of λ. These values are known as **eigenvalues**. The vectors corresponding to each of these n values of λ are known as **eigenvectors**.

Properties of Eigenvalues

1) Any square matrix A and its transpose A^T have the same eigen values.
2) The eigenvalues of triangular matrix are just the diagonal elements of the matrix.
3) The eigenvalues of an idempotent matrix are either 0 or 1.
4) The sum of the eigen values of a matrix is the sum of the elements of the principal diagonal.
5) The product of the eigenvalues of a matrix A is equal to its determinant.
6) If λ is an eigenvalues of a matrix A, then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1}.
7) If λ is an eigenvalue of an orthogonal matrix, then $\frac{1}{\lambda}$ is also its eigenvalue.
8) If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigenvalues of a matrix, then A^m has the eigenvalues $\lambda_1^m, \lambda_2^m, \cdots, \lambda_n^m$ (m being a positive integer).

Cayley-Hamilton Theorem

Every square matrix satisfies its own characteristic equation.

1. Find the eigenvalues and eigenvectors of the following matrices:

   (a) $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

   (b) $\begin{bmatrix} 7 & -2 & 2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}$

MATLAB CODE

```
clc
clear

A=input('Enter the Matrix: ');

%Characteristic Equation
cf=poly(A);
disp('Characteristic Equations')
disp(cf)

%Eigenvalues
EV=eig(A);
disp('Eigenvalues')
disp(EV)

%Eigenvectors
[P D]=eig(A);
disp('Eigenvectors')
disp(P)
```

INPUT

```
Enter the Matrix: [3 4;4 -3]
```

OUTPUT

Eigenvalues
-5
5

Eigenvectors
0.4472   -0.8944
-0.8944   -0.4472

INPUT

Enter the Matrix: [7 -2 2;-2 1 4;-2 4 1]

OUTPUT

Characteristic Equations
1.0000   -9.0000   -1.0000   105.0000

Eigenvalues
7.0000
5.0000
-3.0000

Eigenvectors
0.5774   0.0000   -0.2709
-0.5774   -0.7071   -0.7450
-0.5774   -0.7071   0.6096

2. Prove the following statement by MATLAB
   The product of the eigenvalues of a matrix A is equal to its determinant.

   MATLAB CODE

```
clc
clear
```

```
A=input('Enter the Matrix: ');

%Determinant
detA=det(A);
disp('Determinant of A:')
disp(detA)

%Eigenvalues
EV=eig(A);
disp('Eigenvalues:')
disp(EV)

%Product of eigenvalues
prev=prod(EV);
disp('Product of Eigenvalues:')
disp(prev)
```

INPUT

Enter the Matrix: [7 -2 2;-2 1 4;-2 4 1]

OUTPUT

Determinant of A:
-105

Eigenvalues:
7.0000
5.0000
-3.0000

Product of Eigenvalues:
-105.0000

3. Verify Cayley-Hamilton theorem for the matrtix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

MATLAB CODE

```
clc
clear

A=input('Enter the Matrix: ');

%Verification of Cayley-Hamilton theorem
cf=poly(A);
n=length(cf);
CHT=cf(1)*A^(n-1);
for i=2:n
CHT=CHT+cf(i)*A^(n-i);
end
disp('R.H.S of C-H Theorem: ')
disp(round(CHT))

%To find the inverse
INV=cf(1)*A^(n-2);
for i=2:n-1
INV=INV+cf(i)*A^(n-i-1);
end
INV=INV/(-cf(n));
disp('Inverse of A: ')
disp(INV)
```

INPUT

```
Enter the Matrix: [1 4;2 3]
```

OUTPUT

```
R.H.S of C-H Theorem:
0    0
0    0
```

Inverse of A:
-0.6000 0.8000
0.4000 -0.2000

Exercise

4. Prove the following statements:
   (a) The sum of the eigen values of a matrix is the sum of the elements of the principal diagonal.
   (b) If λ is an eigenvalues of a matrix A, then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1}.
   (c) If $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigenvalues of a matrix, then A^m has the eigenvalues $\lambda_1^m, \lambda_2^m, \cdots, \lambda_n^m$ (m being a positive integer).
5. Using Cayley-Hamilton theorem,
   (a) find the inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.
   (b) find A^8, if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.