1 Visualization of Power of a Hypothesis Test

Let us consider the simplest of case of calculating the power of a hypothesis test. This will assume the distributions are normal with a known population standard deviation and that we are doing a one-sided hypothesis test. For all the plots, we will consider the null hypothesis as $H_0 : \mu = \mu_0 = 0$ and the alternative hypothesis $H_a : \mu < 0$.

In the first plot (figure 1) we show the case where we determine the power of detecting an effect size $(\mu_a - \mu_0)/\sigma = 1/2$, and where $\alpha = 0.05$ and $n = 20$. The black curve is the distribution of $\bar{X}$ under the null hypothesis $\mu = \mu_0$ and the blue curve is the distribution of $\bar{X}$ under the hypothesis $\mu = \mu_a$. The vertical line marks the rejection region; it is chosen so that the area under the black curve to the left of the line is α, the probability of a type 1 error. The shaded area under the blue curve to the left of the line is the power: the probability that we will reject the null hypothesis given that the true mean is μ_a. For this example, the power is about 72%.

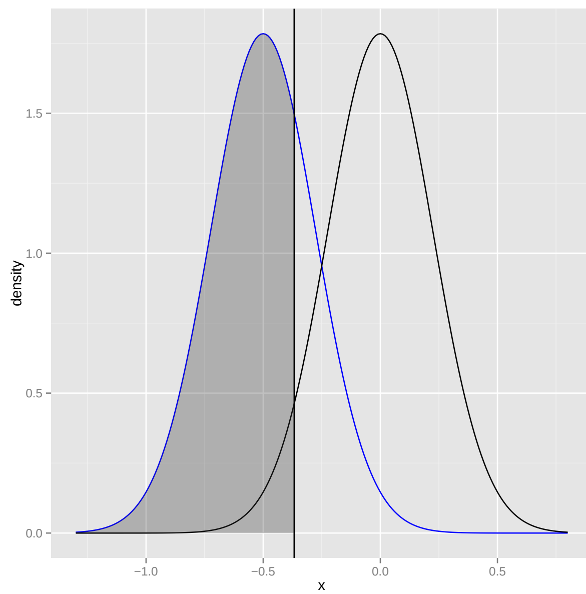

Figure 1: Effect size = 1/2; $n = 20$; $\alpha = .05$

2 Vary the Type 1 Error Rate

In figure 2 we increase α to 0.10. This moves the cut-off line for the rejection region to the right, and thus increases the power of the hypothesis test. It's worth noting at this point that the unshaded area under the blue curve is β, the type 2 error rate; the power of the test is $1 - \beta$. Thus we see that if we are willing to accept a higher type 1 error rate (α), we will have a higher power and a smaller type 2 error rate (β). For this example, the power is now about 83%.

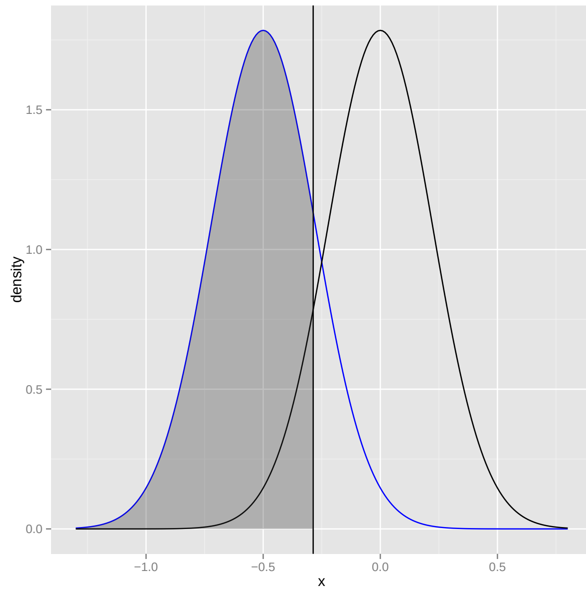

Figure 2: Effect size = $1/2$; $n = 20$; $\alpha = .10$

3 Vary the Effect Size

In figure 3 we return α to its previous value of 0.05, but now reduce the effect size that we want to detect to $1/4$. This moves the blue curve to the right, and thus less of the curve is to the left of the rejection cut-off line, so reducing the power. Note that if we were to reduce the effect size to 0 by moving the blue curve until it overlapped with the black curve, the power would be equal to α. Thus the limit of the power as the effect size goes to 0 is α. For this example, the power is now only about 30%.

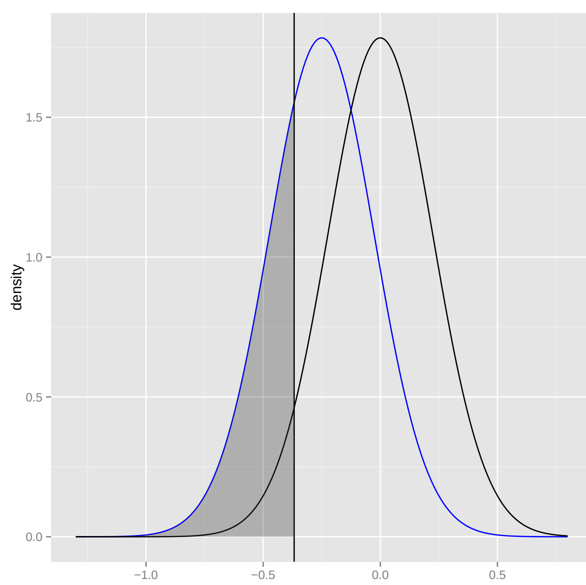

Figure 3: Effect size = $1/4$; $n = 20$; $\alpha = .05$

4 Vary the Sample Size

Finally in figure 4 we increase the sample size n to 100, while leaving the effect size at 1/4. This narrows the distribution curves, since the standard error is reduced. It has two effects on the diagram, both of which increase the power of the test. Firstly, the rejection cut-off line moves to the right, because with a narrower distribution for the null hypothesis more of the area under curve is to the right. Secondly, because the distribution of the alternative hypothesis is narrower, more of its area will be to the left of the line (at least for the usual case where μ_a is left of the cut-off value). For this example, the power is now about 80%, even with the smaller effect size.

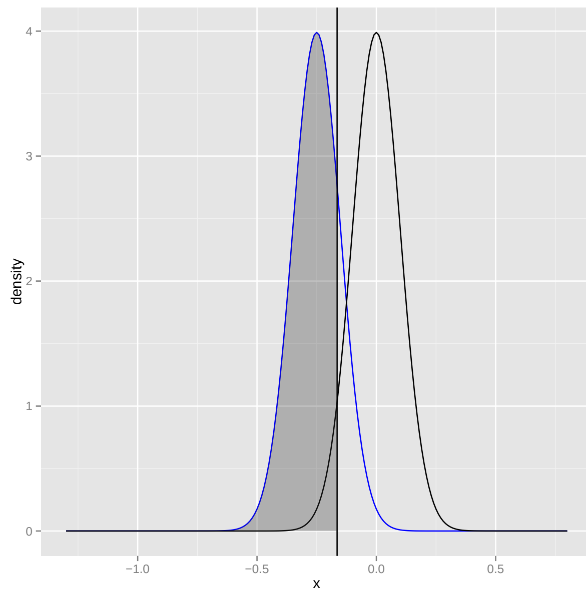

Figure 4: Effect size = 1/4; n = 100; α = .05

5 Code for the Figures

Function definition to produce plots.

```
makeplt <- function(effect, n, alpha) {
    xlower <- -1.3
    xupper <- .8
    x <- seq(xlower, xupper, by=.01)
    sigma <- 1
    se <- sigma / sqrt(n)
    y0 <- dnorm(x, 0, se)
    ya <- dnorm(x, effect, se)
    cutoff <- qnorm(alpha, 0, se)
    lines.df <- data.frame(x=c(x, x), density=c(y0, ya),
                           hyp=c(rep("null", length(x)), rep("alt", length(x))))
    plt <- ggplot(lines.df, aes(x, y=density, col=hyp)) +
        geom_line() +
        scale_colour_manual(values=c("blue", "black")) +
        guides(colour=FALSE)
    plt <- plt + geom_vline(data=data.frame(cutoff), aes(xintercept=cutoff))
    polys.df <- data.frame(x=c(x[x<cutoff], cutoff, xlower), y=c(ya[x<cutoff], 0, 0))
```

```
    plt + geom_polygon(data=polys.df, aes(x, y, col=NULL), alpha=0.3)
}
```

Figure 1.

```
effect <- -1/2
n <- 20
alpha <- 0.05
sigma <- 1
makeplt(effect, n, alpha)
# Power
se <- sigma / sqrt(n)
cutoff <- qnorm(alpha, 0, se)
pnorm(cutoff, effect, se)

## [1] 0.7228
```

Figure 2.

```
effect <- -1/2
n <- 20
alpha <- 0.1
sigma <- 1
makeplt(effect, n, alpha)
# Power
se <- sigma / sqrt(n)
cutoff <- qnorm(alpha, 0, se)
pnorm(cutoff, effect, se)

## [1] 0.8301
```

Figure 3.

```
effect <- -1/4
n <- 20
alpha <- 0.05
sigma <- 1
makeplt(effect, n, alpha)
# Power
se <- sigma / sqrt(n)
cutoff <- qnorm(alpha, 0, se)
pnorm(cutoff, effect, se)

## [1] 0.2992
```

Figure 4.

```
effect <- -1/4
n <- 100
alpha <- 0.05
sigma <- 1
makeplt(effect, n, alpha)
# Power
se <- sigma / sqrt(n)
```

```
cutoff <- qnorm(alpha, 0, se)
pnorm(cutoff, effect, se)

## [1] 0.8038
```