**Terms Used for Proofing**

Notice:

This article is crafted to enhance simplicity and facilitate comprehension in the realm of mathematical proofing. It may incorporate data from various sources, and due credit will be given to these references as indicated in the concluding section.

In the realm of mathematics, the construction of proofs stands as a cornerstone for affirming and enhancing our comprehension of various concepts through rigorous logical processes. For instance, rudimentary arithmetic statements such as 1 + 1 = 2 or 2 - 1 = 1 are subjects that can be validated through foundational principles. Mathematical proofs serve as the instruments to establish the veracity of theorems. Although these validations tend to be more intricate than elementary arithmetic, they adhere to a comparable structural framework: commencing with accepted knowledge or prior proven results, one systematically unfolds a new demonstration that either extends or builds upon the existing foundation. This article aims to elucidate several general methodologies for proving theories or theorems.

**Proof by Mathematical Induction:**

Indeed, the method of mathematical induction is a classic example of deductive reasoning rather than inductive reasoning. The process begins with the establishment of a base case, which serves as a foundational truth within the scope of the proof. Typically, this base case involves demonstrating the validity of the proposition for the smallest value in the sequence of natural numbers, often n = 1. Once the base case is confirmed, the next step involves the formulation of an inductive hypothesis. This hypothesis posits that if the statement is true for a particular natural number k, then it must also be true for the subsequent number k + 1. By demonstrating this, one constructs a logical chain that links the truth of the base case to the truth of the proposition for all natural numbers greater than or equal to the base case. Through this iterative process, the validity of the proposition for the entire set of natural numbers is established, thereby providing a rigorous proof for the given mathematical statement.

**For Example:**

For all natural numbers n ≥ 1, the statement n≥1 is true.

Start with the base case, n=1.

1≥1

This is true.

Assume the statement is true for some arbitrary natural number k≥1. That is, assume:

k≥1

Now, we need to prove that the statement is true for k+1. We need to show:

k+1≥1

Since we assumed k≥1, adding 1 to both sides gives:

k+1≥1+1

k+1≥2

However, k+1*k*+1 is obviously greater than or equal to 1 because k≥1 and adding 1 keeps it greater than or equal to 1. Hence, the statement k+1≥1 is true.

**Proof by Contraposition:**

The contrapositive of a statement is logically equivalent to the original statement. This means that if the original statement is true, the contrapositive is also true. Similarly, if the original statement is false, the contrapositive is false as well.

For example, consider the statement: “If *x*=*y*, *x*=*y*; then if *x*=1, *x*=1; *y*=1, *y*=1.”

The contrapositive of this statement is: “If *y*≠1, *y*=1; then *x*≠1, *x*=1.”

Both the original statement and its contrapositive are either true or false together.

**Proof by Contradiction:**

Proof of contradiction essentially assumes the opposite of what is being proven. Through processes of logic and other theories, the assumption is then deemed false, which means that the opposite of what was being assumed is true. Typically, there are two potential outcomes, with one generally being false and the other true. For instance, if we postulate that the number 44 is odd, we quickly encounter a contradiction upon observing that 44/2 equals 22. Since division by 2 yielding an integer indicates evenness, this demonstrates that 44 is not odd. Consequently, given that our assumption was incorrect, it follows that 44 is indeed even.

Consider a more complicated example:

To prove the existence of infinite primes, we begin by assuming that there is a finite number of them. Let’s set p\_1 as the first prime number (i.e., 2) and p\_n to be the largest prime. We begin by listing all the primes out  p\_1, p\_2, p\_3,…, p\_n (underscores are used here for establishing subscripts). Let N = p\_1 \* p\_2 \* p\_3 \* … \* p\_n + 1. If N is divided by any prime in our list of primes ({p\_1, p\_2, p\_3, …,p\_n}), there will be a remainder of 1. The Fundamental Theorem of Arithmetic states that any positive number has a prime factorization, and since N isn’t divisible by any of the primes, we can assume that N is either a prime number itself or it is divisible by another prime that isn’t listed in our said list of all the primes. Since this contradicts our previous assumption of having all the primes listed, there are an infinite number of primes. (Since you can add more primes to the list, there will be an infinite amount of Ns, which will lead to an infinite amount of primes).

**Problems for Practice:**

**Prove:**

1. Brahmagupta’s Formula is a formula for determining the area of a cyclic quadrilateral given only the four side lengths, given as follows:

[ABCD] =

where *a, b, c, d* are four side lengths and s =

2. For any triangle with side lengths a, b, c, the area A can be found using the following formula:

A =

where the semi-perimeter s =

3. The *Pythagorean Theorem* states that for a right triangle with legs of length *a* and *b* and hypotenuse of length *c* we have the relationship .

Prove the *Pythagorean Theorem*.

Reference Websites / Articles:

<https://artofproblemsolving.com/>

<https://www.wikipedia.org/>

**证明的体系**

注意：

本文旨在简明阐述数学证明领域并促进理解。它可能包含来自不同来源的数据，并将在结论部分对这些参考文献给予应有的重视。

在数学领域，证明的构建是通过严格的逻辑过程肯定和增强我们对各种概念理解的基石。例如，1+1=2或2-1=1等基本算术语句是可以通过基本原理进行验证的主题。数学证明是建立定理准确性的工具。尽管这些验证往往比基本算术更复杂，但它们遵循一个类似的结构框架：从公认的知识或先前经过验证的结果开始，系统地展开一个新的演示，要么扩展现有的基础，要么建立在现有的基础上。本文旨在阐明证明理论或定理的几种一般方法。

**数学归纳法证明**

事实上，数学归纳法是演绎推理而非归纳推理的经典例子。这一过程始于建立一个基础案例，作为证据范围内的基本公理。通常，这个基本情况涉及证明自然数序列中最小值的命题的有效性，通常n=1。一旦基本情况得到确认，下一步就是制定归纳假设。假设如果该陈述对特定的自然数k为真，那么对后续的数k+1也必须为真。通过证明这一点，我们构建了一个逻辑链，将所有大于或等于基本情况的自然数的基本情况的真理与命题的真理联系起来。通过这个迭代过程，建立了命题对整个自然数集的有效性，从而为给定的数学陈述提供了严格的证明。

例如：

对于所有n≥1的自然数，n≥1是真的。

从基本情况开始，n=1。

1≥1

这是真的。

假设对于某个任意自然数k≥1，该陈述为真。也就是说，假设：

k≥1

现在，我们需要证明该陈述对k+1是正确的。我们需要展示：

k+1≥1

由于我们假设k≥1，两边加1得到：

k+1≥1+1

k+1≥2

所以，k+1明显大于或等于1，因为k≥1，加1使其大于1。因此，k+1≥1的陈述是正确的。

**反证证明**

陈述的换质换位句在逻辑上与原始陈述等价。这意味着，如果原始陈述为真，则换质换位句也为真。同样，如果原始语句为假，则换质换位句也为假。

例如，考虑以下语句：“如果x=y，x=y；那么如果x=1，x=1；y=1，y=1。”

这句话的换质换位句是：“如果y≠1，y=1；那么x≠1、x=1。”

原始陈述及其换质换位句合在一起要么是真的，要么是假的。

**矛盾证明**

矛盾的证明本质上假定与被证明的相反。通过逻辑和其他理论的过程，假设被认为是错误的，这意味着与假设相反的是真的。通常，有两种潜在的结果，一种通常为假，另一种为真。例如，如果我们假设数字44是奇数，那么在观察到44/2等于22时，我们很快就会遇到矛盾。由于除以2得到一个整数表示均匀性，这表明44不是奇数。因此，鉴于我们的假设是不正确的，因此44确实是偶数。

考虑一个更复杂的例子：

为了证明无限素数的存在，我们首先假设它们的数量是有限的。让我们将p\_1设置为第一个素数（即2），p\_n设置为最大素数。我们首先列出所有素数p\_1、p\_2、p\_3、…、p\_n（这里使用下划线来建立下标）。设N=(p\_1\*p\_2\*p\_3\*…\*p\_n)+1。如果N除以素数列表中的任何素数{p\_1，p\_2，p\_3，…，p\_N}，则余数为1。算术基本定理指出，任何正数都有一个素数因式分解，由于N不能被任何素数整除，我们可以假设N本身是一个素数，或者它可以被我们所说的所有素数列表中没有列出的另一个素数整除。由于这与我们之前列出所有素数的假设相矛盾，因此素数的数量是无限的。（由于你可以在列表中添加更多的素数，因此将有无限数量的n，这将导致无限数量的素数）。

**实践问题**

证明：

1.Brahmagupta公式是一个用于确定仅给定四条边长的循环四边形面积的公式，如下所示：

[ABCD] =

其中a、b、c、d是四个边长，s =

2.对于边长为a、b、c的任何三角形，可以使用以下公式找到面积a：

A =

其中半周长s =

3.勾股定理指出，对于一个边长为a和b、斜边长度为c的直角三角形，我们有关系式 。

参考网站：

https://artofproblemsolving.com/

<https://www.wikipedia.org/>

Question 1:

If we draw *AC*, we find that [ABCD]= + = . Since B+D=180˚, . Hence, [ABCD] = . Multiplying by 2 and squaring, we get:

4 [ABCD]2 =

Substituting results in

4 [ABCD]2 =

By the law of cosines, ,

So, a little rearranging gives:

Question 2:

Using basic Trigonometry, we have

,

Which simplifies to

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The Law of Cosines states that in triangle ABC, , which can be written as . Thus, .

Now, we can simplify:

Question 3:

In these proofs, we will let *ABC* be any right triangle with a right angle at angle ACB.

We use [ABC] to denote the area of triangle ABC.

Let *H* be the perpendicular to side *AB* from *C*.

A triangle with letters and numbers

Description automatically generated

Since *ABC, CBH, ACH* are similar right triangles, and the areas of similar triangles are proportional to the squares of corresponding side lengths,

But since triangle *ABC* is composed of triangles *CBH* and *ACH*, , so .

**题1：**

连接AC，可以得到[ABCD]= + = 。由于B+D=180˚，。所以，[ABCD] = 。乘以2然后平方可以得到：

4 [ABCD]2 =

带入，化为：

4 [ABCD]2 =

根据余弦定理，。所以，经过整理：

**题2：**

根据基本三角函数，我们有，能化简成。余弦定理表明，在三角形ABC中。这个又可以被写成。所以，现在就可以化简为：

题3：

在这个证明中，我们让ABC是任何以角ACB为直角的直角三角形。

我们用[ABC]表示三角形ABC的面积。

设H为C与AB边的垂线。

A triangle with letters and numbers

Description automatically generated

由于ABC、CBH、ACH是相似的直角三角形，并且相似三角形的面积与相应边长的平方成正比，

而由于三角形 *ABC*是由三角形*CBH* 和*ACH*组成的, , 所以可以得到 .