**How Expected Value is Used in Game Theory**

**期望值在博弈论中的应用**

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**Introduction**

It is likely that you have previously engaged in a game that provided you with entertainment. I am not referring to electronic games, but rather to those found in casinos, involving dice. These games share a common feature: they all hinge on the selection of a random outcome within a collective context. Luck plays an indispensable role. You may have witnessed on television individuals experiencing an extraordinary stroke of fortune, winning millions of dollars. Similarly, a visit to a casino will reveal patrons successfully amassing monetary gains. The sums awarded to these fortuitous individuals are considerable. Nevertheless, the question arises: how do casinos generate profits? Moreover, what motivates people to partake in these games when they are aware that the casino is poised to earn substantial revenues?

**引言**

你大概率玩过让你十分感兴趣的游戏。这里指的并不是电子游戏，而是像赌场中常见的骰子游戏。这类游戏有一个共同点：它们都依赖于在一个平等背景设置下选择一个随机的结果，运气在其中则起着至关重要的作用。你或许曾在电视上看到某些人因运气极好而赢得了数百万美元。同样的，在赌场里也能看到一些顾客成功地赢得了大量金钱，这些幸运儿所获得的奖金额相当可观。然而，问题来了：赌场是如何盈利的？而人们明明知道赌场可以赚取可观的利润，为什么还会去参与这些游戏呢？

**Definitions**

To answer these questions, we need to learn what is expected value. Expected value is the most likely value you would get in the next repeated trail. In gaming, it is the expected character or rarity you would get in the next roll.

**定义**

为了回答这些问题，我们需要了解什么是期望值。期望值是指在接下来的多次试验中，最有可能出现的结果。在游戏中，它指的是下一次掷骰子时你最有可能获得的结果。

**Example**

If you have a fair dice and you calculate the outcomes you would find.

|  |  |
| --- | --- |
| Number of Roll | Number |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |

The total of all outcomes is 21 which divide the number of rolls is 3.5. So, the expected value is going to be 3.5 You can notice that this number at the same time is the mean of the group.

The equation is While E(x) is the expected value and is the event.

Another example is a spinner. Prizes are: 0, 0, 50, 50, 50, 100, 100, 100, 100, 150, 150, 200, 5000. Entrance fee is 500. Some people might think it’s a chance to win 5000 dollars. Calculate the expected value

|  |  |
| --- | --- |
| Number of Spin | Amount |
| 1 | 0 |
| 2 | 0 |
| 3 | 50 |
| 4 | 50 |
| 5 | 50 |
| 6 | 100 |
| 7 | 100 |
| 8 | 100 |
| 9 | 100 |
| 10 | 100 |
| 11 | 150 |
| 12 | 200 |
| 13 | 5000 |

You will find the total is 6000 and the fee for 13 times is going to be 6500. 6000 divided by the number of spins is around 462, so for each roll, the person who owns the spinner earns 48. Therefore, some people who think that it’s a chance to win money will lose money. Yes, there is a chance to win the 5000, but there is a bigger chance is that you will lose money.

**示例**

假设你有一个公平的骰子，来计算可能的结果。

|  |  |
| --- | --- |
| 投掷次数 | Number |
| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |
| 5 | 5 |
| 6 | 6 |

**掷骰次数与结果：**

所有结果的总和为21，除以掷骰次数得到3.5。因此，期望值是3.5。你可以注意到，这个数字恰好是所有结果的平均值。  
期望值的公式是：

其中，E(x)是期望值，​是事件的具体结果。

另一个例子是转盘游戏。奖品包括：0、0、50、50、50、100、100、100、100、150、150、200、5000。入场费是500。有人可能会认为这是一个赢得5000美元的机会。我们来计算期望值。  
**转盘次数与金额：**

|  |  |
| --- | --- |
| 转盘次数 | 奖金 |
| 1 | 0 |
| 2 | 0 |
| 3 | 50 |
| 4 | 50 |
| 5 | 50 |
| 6 | 100 |
| 7 | 100 |
| 8 | 100 |
| 9 | 100 |
| 10 | 100 |
| 11 | 150 |
| 12 | 200 |
| 13 | 5000 |

我们发现所有金额的总和是6000，13次转盘的总费用是6500。6000除以转盘次数得到约462，因此每转一次，转盘的持有者会赚取48。因此，尽管有机会赢得5000，但你更有可能会亏钱。

**Extensions**

Casinos adeptly exploit the concept of expected value by commonly pricing their offerings above this calculated expectation. Consequently, they secure a profit over an extended period. Individuals who fail to compute potential results and outcomes often entertain the misconception that they, too, can amass substantial winnings akin to those experiencing extraordinary luck. One can ascertain the profitability of games by evaluating the expected value and juxtaposing it with the price; if the value exceeds the price, a profit is likely to be realized. Casinos use games like slot machines, where the expected value for each spin is less than the amount bet because the house has a built-in advantage. For instance, if a slot machine has an RTP (Return to Player) of 95%, and you bet $1 per spin, the expected value is $0.95. Over time, the casino will keep the extra $0.05 from each bet, ensuring a profit. Gamblers who don't understand expected value may think they can win big, but the odds are stacked against them. If a game had an expected of $1.10 per $1 bet, players would, on average, make a profit, but such games are not typically found in casinos.

**延伸**

赌场巧妙地利用期望值的概念，通常将其游戏的定价定得高于这一计算出的期望值。因此，它们能够在长期内获得利润。那些没有计算潜在结果和后果的玩家，往往会误以为自己也能像那些幸运的人一样赢得巨额奖金。我们可以通过评估期望值并将其与游戏的价格进行对比，来判断一款游戏是否能盈利；如果期望值大于价格，盈利就有可能实现。赌场利用像老虎机这样的游戏，其中每次旋转的期望值低于下注金额，因为赌场本身就有一个内在的优势。例如，如果一台老虎机的玩家回报率（RTP, Return to Player）是95%，而每次下注1美元，那么期望值就是0.95美元。随着时间的推移，赌场将从每次下注中保留额外的0.05美元，从而确保盈利。不了解期望值的赌徒可能认为自己能赢得巨额奖金，但实际上赔率对他们不利。如果一款游戏每下注1美元的期望值是1.10美元，那么玩家平均能获得利润，但这种游戏通常不会出现在赌场中。

**Conclusion**

It’s true that if you are extremely lucky, you might be able to win massive amounts of money. Still, there is an extremely small percentage you will win. Even if you spend lots of money on it, it won’t be a guaranteed win. If you play games in the casino or games like lottery, consider the expected value of the outcomes and then decide whether to spend money or not.

Gambling can be quite the adventure, offering a thrilling rush that many find irresistible. However it’s crucial to approach it with a clear mind and an understanding of the mathematics at play. The expected value is essentially a weighted average of all possible outcomes of an event, factoring in the probability of each outcome. In the case of most casino games or lotteries, the expected value is negative when you factor in the house edge or the costs of playing.

So before you pick your numbers, take a moment to think about the thrill you’re seeking and the experience you’re paying for. The odds may be long, but the joy of playing can be its own kind of win.

**总结**

如果你非常幸运，确实有可能赢得巨额奖金，但这种概率极低。即使你投入大量资金，也不一定能保证赢得大奖。如果你去赌场或者买彩票，最好先考虑一下每个结果的期望值，然后决定是否花钱参与。  
赌博确实充满了冒险，给很多人带来了刺激的快感。但更重要的是，以清晰的头脑和对数学原理的理解来对待它。期望值本质上是所有可能结果的加权平均数，考虑到每个结果发生的概率。对于大多数赌场游戏或彩票来说，期望值是负数，因为你必须考虑到赌场的优势或参与游戏的成本。  
所以，在你选择数字之前，先想一想你追求的刺激和你为此付出的代价。即使赔率很小，游戏的乐趣本身可能就是一种胜利。

**Practice Problems to Test Yourself**

（The answers will be revealed in the next article, so please subscribe to our account!）

1. Albert gives Chris 3 spinners each with a equal probability of rolling any integer from 1-6, inclusive. If the three spinners spin on the same number, Chris gets $10000. If the entrance fee is 40$, does Chris usually win money or lose money?
2. Michelle gives Chris 6 fair dices, if the sum of the number on the 6 fair dices is smaller than7 then Chris gets $1000. What is the minimum integer value for the entrance fee if Michelle wants to make money?
3. Cathy gives Chris 4 spinners, each with a equal probability of getting a integer from 1-5, inclusive. If the product of the 4 numbers on the spinner is greater than 600, Chris gets $10000. What is the lowest integer entrance fee if Cathy wants to make money?

**考验自己的小练习：**

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1. Albert 给 Chris 3 个转盘，每个转盘转到 1-6 中的任何整数的概率相等，包括 1-6。如果三个转盘转到同一个数字，Chris 将获得 10000 美元。如果入场费是 40 美元，Chris 通常是赢钱还是输钱？
2. Michelle 给 Chris 6 个公平的骰子，如果 6 个公平骰子的数字之和小于7，则 Chris 获得 1000 美元。如果Michelle想赚钱，入场费的最小整数值是多少？
3. Cathy 给了 Chris 4 个转盘，每个转盘转到 1-5中的任何整数的概率相等，包括 1-5。如果转盘上 4 个数字的乘积大于 600，Chris 将获得 10000 美元。如果 Cathy 想赚钱，入场费的最小整数值是多少？