**The Road to Infinity**

Daniel 8-7

*One, two, three…*

Most modern kindergarteners can grasp counting simple natural numbers, and many can go much further.

*Eleven, twelve, thirteen…*

And they would continue, scaling up to thousands, millions, and even billions.

**Beginning: The Invention of Numerals**

How did this all begin?

The people of the Stone Age would have a good answer. Initially, they used sticks to carve stripes on walls, marking quantities—two stripes meant two apples or two deer hunted that day. These early humans recorded simple numbers using pictographs in their daily lives, marking the earliest form of numerals.

However, as their need for numbers grew, they faced a challenge: how to represent larger quantities. This problem remained unsolved until the first sparks of civilization ignited.

The Sumerians, often regarded as the first notable civilization on Earth, addressed this issue by developing the sexagesimal system, or "base 60." This number was chosen for its twelve factors, making calculations easier.

The early Romans adapted this system, using Latin symbols to represent numbers. They introduced the base-10 system, which cycles a digit every tenth place. Numbers that once required extensive pictographs, like 114514, could now be expressed simply as CXIVDXIV. The Arabs improved this further, limiting their symbols to just ten—the numerals we know and love today.

**A Leap Away from Perception: Climbing the Number Mountain**

Now that we’ve explored the beginnings of numerals, we can consider how our perception of large numbers has changed. Once limited in knowledge, we now encounter vast quantities daily. Let’s examine the true size of numbers.

At one meter, everything feels normal; your height is likely between 1.5 and 2 meters, and most objects fit within a meter.

What comes next? Ten? A hundred? No, according to standard notation, one thousand follows. Each increment in the naming of large numbers means adding three more digits. A leap to 1,000 meters can take you across town.

A million meters is substantial, equating to the diameter of medium-sized countries. At a billion meters, or one million kilometers, we traverse distances comparable to stacking 100 Earths. Trillions represent even greater distances—traveling that far in meters would mean four round trips to the sun.

Most elementary schools stop here, and students often wonder what comes after trillions. The American system continues with quadrillion, quintillion, sextillion, and so on, while the British use a different naming convention, counting trillions as billions and quadrillions as thousand billion.

The numbers keep growing, becoming almost unfathomable. To illustrate, the universe is approximately 93 billion light-years in diameter, with a light-year being about 9.46 trillion kilometers. The scale of these numbers is truly monstrous.

Is that the end of big numbers? Not even close.

**Munching Up Big Numbers: The Ultimate Monstrosity**

Next up is the googol, representing 10100—an astonishingly large number. The tech company Google took its name from this term. A modern field of study, Googology, explores the nomenclature of large numbers. Following the googol is the centillion, which has 303 zeros after the initial 1.

But even a googol and a centillion are small compared to the googolplex, which is 10 with a googol of zeros following it. Writing out this number is virtually impossible; the universe might end before you finish!

You may wonder, what’s the point of discussing such meaningless numbers? While many of these numbers are abstract, some serve specific purposes. Take Graham’s number, for instance.

Defining Graham’s number requires a new notation called arrow notation. In its simplest, a↑b = ab and a↑↑b = a↑a↑a…. ↑a b times. To show the sheer size of it even a simple expression such as 3↑↑↑2 equals an enormous number like 7,625,597,484,987. The base of the Graham’s number is 3↑↑↑↑3, already an impossibly large number. The next iteration would involve a number of arrows equal to the previous result. This is incomprehensible, and the ultimate result is far beyond a googolplex.

Despite its size, Graham’s number has mathematical significance, answering a complex hypothesis, making it respected as the largest number ever used in a proof.

So, we’ve traveled from the smallest to the largest numbers. One final question remains: where is infinity?

一、二、三……

大多数现代幼儿园的孩子都能掌握简单自然数的计数，很多孩子还能更进一步。

十一、十二、十三……

他们会继续数下去，数到数千、数百万甚至数十亿。

**数字的发明**

这一切是如何开始的呢？

石器时代的人们会有一个很好的答案。最初，他们用棍子在墙上刻条纹，标记数量——两条条纹意味着两个苹果或当天猎获的两头鹿。这些早期人类在日常生活中使用象形文字记录简单的数字，这标志着数字的最早形式。

然而，随着他们对数字的需求增长，他们面临一个挑战：如何表示更大的数量。这个问题一直没有得到解决，直到第一道文明的火花被点燃。

苏美尔人，通常被认为是地球上第一个著名的文明，通过发展六十进制系统（即“以 60 为基数”）解决了这个问题。选择这个数字是因为它有十二个因数，使得计算更加容易。

早期的罗马人改编了这个系统，使用拉丁符号来表示数字。他们引入了十进制系统，每十个数字循环一次。曾经需要大量象形文字的数字，比如 114514，现在可以简单地表示为 CXIVDXIV。阿拉伯人进一步改进了这个系统，将他们的符号限制在仅仅十个——也就是我们今天所熟知和喜爱的数字。

**超越感知的飞跃：攀登数字之山**

现在我们已经探索了数字的起源，我们可以考虑一下我们对大数字的感知是如何变化的。曾经知识有限，现在我们每天都会遇到大量的数字。让我们来审视一下数字的真正大小。

在一米的时候，一切都感觉很正常；你的身高可能在 1.5 到 2 米之间，大多数物体都能放在一米之内。

接下来是什么呢？十？一百？不，根据标准记数法，接下来是一千。大数字的命名每增加一次，就意味着再增加三个数字。跃升到 1000 米可以带你穿过城镇。

一百万米是相当大的，相当于中等国家的直径。在十亿米，即一百万公里的时候，我们跨越的距离相当于把 100 个地球堆叠起来。万亿代表着更大的距离——以米为单位旅行那么远意味着往返太阳四次。

大多数小学就教到这里，学生们经常想知道万亿之后是什么。美国的计数系统继续是万万亿、千万亿、百亿亿等等，而英国使用不同的命名惯例，把万亿算作十亿，把万万亿算作千亿。

数字不断增长，变得几乎难以想象。例如，宇宙的直径大约是 930 亿光年，一光年大约是 9.46 万亿公里。这些数字的规模确实巨大。

这就是大数字的尽头吗？差得远呢。

**吞噬大数字：终极巨兽**

接下来是古戈尔，代表10的100次方——一个惊人的大数字。科技公司谷歌就是从这个术语中取的名字。一个现代的研究领域“大数数学”探索大数字的命名法。在古戈尔之后是 centillion，它在开头的1后面有303个零。

但是，即使是古戈尔和 centillion 与古戈尔普勒克斯相比也很小，古戈尔普勒克斯是1后面跟着一个古戈尔个零。写出这个数字几乎是不可能的；在你写完之前，宇宙可能就终结了！

你可能会想，讨论这些毫无意义的数字有什么意义呢？虽然这些数字中的许多是抽象的，但有些有特定的用途。以葛立恒数为例。

定义葛立恒数需要一种叫做箭头表示法的新符号。在最简单的情况下，a↑b = ab，a↑↑b = a↑a↑a……↑a b 次。即使是像 3↑↑↑2 这样简单的表达式也等于一个巨大的数字，如 7625597484987。葛立恒数的底数是 3↑↑↑↑3，这已经是一个无法想象的大数字了。下一次迭代将涉及的箭头数量等于前一个结果。这是难以理解的，最终的结果远远超过古戈尔普勒克斯。

尽管葛立恒数很大，但它具有数学意义，回答了一个复杂的假设，使其作为在证明中使用过的最大数字而受到尊重。

所以，我们已经从最小的数字走到了最大的数字。最后一个问题仍然存在：所谓的无限在哪里？