**Types of Special Numbers**

**特殊数字类型**

**Introduction**

From a subject taught in school to a realm filled with infinite wonders, mathematics is a universe of fascination and surprise, revealing its connections to the world around us. Special numbers are *specific categories of numbers that exhibit* *unique properties*, but they can also take us to a world of deeper mathematical insights. From palindromes to perfect numbers, Fibonacci numbers to happy numbers, we will explore the world of special numbers in this article and learn more about the unique characteristics that make them an intriguing part of mathematics.

**简介**

从学校教授的科目到充满无限奇迹的领域，数学是一个充满魅力和惊喜的宇宙，揭示了它与我们周围世界的联系。特殊数字是展现独特属性的特定数字类别，它们还可以引导我们进入更深层次的数学洞察。本文将探讨从回文数到完全数，斐波那契数到快乐数等特殊数字的世界，并了解它们作为数学中引人入胜部分的独特特征。

**Types of Special Numbers**

**特殊数字类型**

**1. Palindromic numbers**

In mathematics, palindromes, or palindromic numbers, are numbers that read the same backward and forward, for example, 1221 or 7007.

A surprising observation about palindromes is that 10% of two-digit or three-digits numbers are palindromes, but only 1% of four-digit numbers or five-digit numbers are palindromes. Continuing on, you will realize the amount of palindromes keeps decreasing by a factor of 10 for each digit category. Another interesting fact is that all palindromes with an even number of digits are divisible by 11.

To form a palindrome from a given number, we can use the 196-Algorithm. Take a positive integers with two or more digits, reverse its digits, and add it to the

original one. Repeat this until you get a palindrome. As an example, using 5280 we get

5280, 6105, 11121, 23232. Some numbers--Lychrel numbers--cannot transform into a palindrome with this method, such as 196.

**回文数**

在数学中，回文数是指正读和反读都相同的数字，例如1221或7007。有趣的是，两位数或三位数中有10%是回文数，但在四位数或五位数中，这一比例仅为1%。继续深入，您会发现每增加一位数字，回文数的数量将减少十倍。另一个有趣的事实是，所有偶数位的回文数都能被11整除。 要从给定数字形成回文数，我们可以使用196算法：取一个两位或更多位的正整数，将其数字反转并加到原数上，重复此过程直到得到一个回文数。例如，使用5280，我们得到5280, 6105, 11121, 23232。有些数字，如196，被称为Lychrel数，不能通过这种方法变成回文数。

**2. Perfect Numbers**

Next, we have perfect numbers. First discovered by the Pythagoreans, a perfect number  
is a positive integer that is equal to the sum of its proper divisors(the factors of a number excluding the number itself)

**Example**: 6 = 1x6, 2x3 which means it’s factors are 1, 2, 3, 6 Proper divisors: 1, 2, 3 1+2+3=6, Thus 6 is a perfect number.

( 1 is not a perfect number, as it doesn’t have any proper divisors to add up with.)

Later on, philosopher Nicomachus of Gerasa categorized numbers as deficient if the sum is less than the number, perfect if equal, or superabundant if greater.

So how would we be able to find perfect numbers? Before introducing the method, we would need to talk about a few concepts.

-**Primes**: Numbers that can only be divided by 1 or themselves, such as 2, 3, 5, 7...

-**Mersenne Primes**: Prime numbers that are one less than a multiple of 2 (2n). The first numbers in this sequence include 3, 7, 31, 127, 8191.

2000 years ago, Euclid wrote with a formula that represents perfect numbers: **N = 2p-1(2p-1)**

Here, p is a prime for which 2p-1 is a Mersenne prime.

With this, we can find the first few perfect numbers: 6, 28, 496…Yet perfect numbers do not often appear in mathematics, so we have only found around 51 of these perfect numbers, the rest waiting to be uncovered someday.

接下来，我们来谈谈完全数。完全数最初由毕达哥拉斯学派发现，它是一个正整数，等于其所有适当除数之和（即除了该数本身外的因子之和）。 例如：6 = 1x6, 2x3，这意味着它的因子有1, 2, 3, 6，而它的适当除数为1, 2, 3，1+2+3=6，因此6是一个完全数。 （1不是完全数，因为它没有任何适当除数可以相加。）

后来，哲学家尼科马库斯将数字分类为不足数（因数和小于该数）、完全数（相等）或过剩数（因数和大于该数）。 那么我们如何找到完全数呢？在介绍这个方法之前，我们需要讨论几个概念：

* 素数：只能被1或其本身整除的数字，例如2, 3, 5, 7等。
* 梅森素数：素数中比2的倍数少1的那些，例如2^n - 1形式的素数。这一序列的前几个数字包括3, 7, 31, 127, 8191。 2000年前，欧几里得提出了一个表示完全数的公式：N = 2^(p-1)(2^p - 1) 这里的p是一个素数，

2^p - 1是一个梅森素数。

利用这个公式，我们可以找到一些完全数，如6, 28, 496…然而，完全数在数学中并不常见，因此我们迄今为止只发现了大约51个这样的完全数，其他的仍有待发掘。

**3. Fibonacci Numbers**

Discovered by the famous Italian mathematician Leonardo Fibonacci, the fascinating Fibonacci numbers open us to a whole new realm in the world of mathematics. The Fibonacci numbers form the Fibonacci sequence, which is a series of numbers where every number is the sum of the two numbers before it, starting with 0 and 1. The first few numbers in the sequence include:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34…

In a book, Leonardo wrote: Suppose that some rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?

The correct answer is 144, and the method used to obtain the answer is none other than the Fibonacci sequence. The total number of rabbits written in a sequence

would be 1, 1, 2, 3, 5…You probably have noticed, but the total number of rabbits every month follows the sequence of the Fibonacci sequence.

There is also a relationship between the Fibonacci Numbers and the golden

ratio. The ratio of any two consecutive numbers within the Fibonacci sequence would be close to the golden ratio, which is approximately 1.618. (e.g. 3/2= 1.5, 5/3≈ 1.67...)

The Fibonacci sequence also appears in nature. For example, many flowers have a number of petals that is a Fibonacci number. Seeds, fruits, and vegetables frequently show spiral patterns related to Fibonacci numbers as well.

Fibonacci numbers also have practical applications. They are used in search algorithms in coding and agile development. Specifically, Fibonacci numbers help convert positive integers into binary code (a system using only 0 and 1), making it more convenient for programmers to organize and search for data.

Fibonacci numbers are also commonly used in architecture. Designers use it to make sublime designs that humans like. It is also common in human anatomy, found in the ratio of facial features, and is sometimes used to as beauty standards.

1. **斐波那契数** 由著名意大利数学家列奥纳多·斐波那契发现的斐波那契数开启了我们对数学世界的全新认识。斐波那契数形成斐波那契序列，这是一个系列，其中每个数字都是前两个数字的和，从0和1开始。序列中的前几个数字包括： 0, 1, 1, 2, 3, 5, 8, 13, 21, 34…

列奥纳多在一本书中写道：假设一些兔子永不死亡，且每对母兔从第二个月开始每月都能产下一对新兔子（一雄一雌），一年内会有多少对兔子？ 正确答案是144，使用的方法正是斐波那契序列。序列中兔子的总数为1, 1, 2, 3, 5…您可能已经注意到，每月的兔子总数都遵循斐波那契数列。 斐波那契数和黄金比之间也存在关系。斐波那契序列中任意两个连续数字的比率接近黄金比，大约为1.618。（例如3/2= 1.5, 5/3≈ 1.67...） 斐波那契数列也出现在自然界中。例如，许多花的花瓣数是斐波那契数。种子、果实和蔬菜经常显示出与斐波那契数相关的螺旋图案。 斐波那契数在实际应用中也很常见。它们被用于编码和敏捷开发中的搜索算法。具体来说，斐波那契数有助于将正整数转换为二进制代码（一个仅使用0和1的系统），使程序员能更方便地组织和搜索数据。 斐波那契数还常用于建筑设计。设计师使用它们来创造人们喜爱的崇高设计。它在人类解剖学中也很常见，出现在面部特征的比例中，有时还用作美学标准。

**4. Happy Numbers**

Happy numbers are an interesting category that belongs to number theory.

Imagine you add up the square of each digit in a number, ifyou do this repeatedly and end up with the number 1, the original number is what we call a happy number. As an example, let’s take the number 13:

13’s two digits: 1 and 3.

Square the digits and add them together: 12+32= 1+9= 10

Continue: 12+02= 1

Using this method, we usually end up with one of the ten numbers: 0, 1, 4, 16, 20, 37, 42, 58, 89, 145.

Similar to the name *happy numbers*, numbers that do not end up with 1 using this method are considered unhappy numbers.

Once we know if a number is happy or not, the rest of the numbers that we obtained when using the method are also happy numbers. The sequence of numbers for 13 was 13 -> 10 -> 1. Since 13 was a happy number, that would mean all other numbers in the sequence after 13 are happy, making 10 a happy number as well.

**快乐数** 快乐数是数论中的一个有趣类别。 想象一下，如果您不断地将一个数字的每个数字的平方相加，并最终得到数字1，那么原始数字就是我们所说的快乐数。例如，让我们看看数字13： 13的两个数字：1和3。 将这些数字的平方相加：1^2 + 3^2 = 1 + 9 = 10 继续操作：1^2 + 0^2 = 1 使用这种方法，我们通常最终会得到以下十个数字之一：0, 1, 4, 16, 20, 37, 42, 58, 89, 145。 与快乐数的名称相似，使用此方法未能以1结束的数字被认为是不快乐的数字。 一旦我们知道一个数字是否为快乐数，我们使用该方法得到的其他数字也都是快乐数。对于13的数列是13 -> 10 -> 1。由于13是一个快乐数，这意味着序列中13之后的所有其他数字也是快乐的，这使得10也是一个快乐数。

**Questions**

1. Find the sum of all three-digit palindromic numbers that are divisible by 11. Explain the steps you used to find your answer.
2. 求所有能被 11 整除的三位数回文数的总和。

1. What is the sum of all of the reciprocals of a perfect number's factors (including the perfect number itself)?

2.一个完全数的所有因数（包括完全数本身）的倒数之和是多少？

1. Given that F14 = 377 calculate the next Fibonacci number.

3.已知斐波那契数列里的第14个数=377, 求数列中下一个斐波那契数。

1. A five-digit palindrome is a positive integer with respective digits , where  is non-zero. Let  be the sum of all five-digit palindromes. What is the sum of the digits of ? (2014 AMC12)
2. 9 B. 18 C. 27 D. 36 E. 45

1. 一个五位回文数为abcba，其中 a 为非零。令 S 等于所有五位回文数的总和。 S 的各位数字之和是多少？ （2014 年 AMC12）
2. 9 B. 18 C. 27 D. 36 E. 45

**Answers**

1. **Answer: 4400**  
   A three-digit palindrome has the form "aba," such as 121, 232, 343... We only need to find which of these three-digit palindromic numbers are divisible by 11. We know that “a number is divisible by 11 if the difference between the sum of its digits in odd places and the sum of its digits in even places is 0 or a multiple of 11” (divisibility rule of 11), so the only numbers we are left with are:

121, 242, 363, 484, 616, 737, 858, 979

When you add them together, you get **4400**.

1. **Answer: 2**

We can use the perfect number 6 as an example.

Factors of 6: 1, 2, 3, 6

Adding their reciprocals: 1+1/2+1/3+1/6=2. This works for other perfect numbers, such as 28, as well.

Thus, the sum of the reciprocals of a perfect number’s factors equals to **2**.

1. **Answer: 610**

As mentioned in the article, the ratio of any two consecutive numbers within the Fibonacci sequence would be close to the golden ratio, which is approximately 1.618. Meaning if F14 was 377, and F15=x, then x/377≈1.618, x≈610

Since the golden ratio is estimated, the precise answer wouldn’t matter. And because x≈610, the 15th number would be **610**.

1. **Answer: 18**

For each digit a=1, 2, ..., 9, there are 10·10 (ways of choosing b/c) palindromes. So the “a”s contribute(1+2+...+9)(100)(104+1)to the sum. For each digit b=0, 1, 2, ..., 9, there are 9·10 palindromes. So the “b”s add (0+1+2+...+9)(90)(103+10) to the sum. For each c=0, 1, 2, ..., 9 there are 9·10 palindromes, so “c”s make up (0+1+2+...+9)(90)(102)in the sum.

(1+2+...+9)(100)(104+1)+(0+1+2+...+9)(90)(103+10)+(0+1+2+...+9)(90)(102)=49500000

The sum of the digits of 49500000=**18**.