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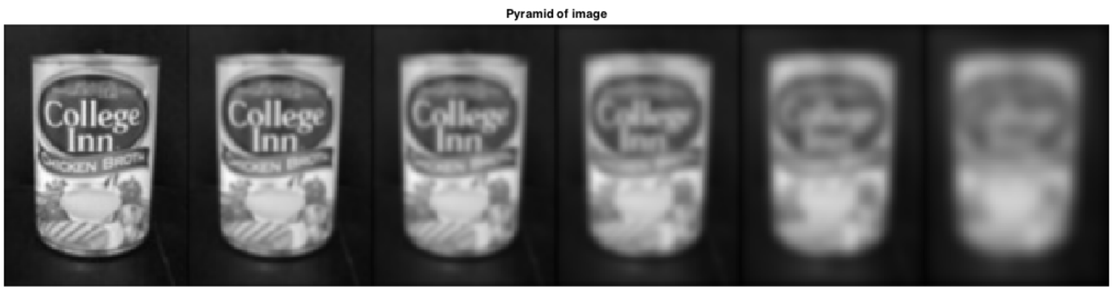
Computer Vision 16-720
Assignment 2
October 9, 2016

Assignment #2

1.1 Gaussian Pyramid of *model_chickenbroth.jpg*



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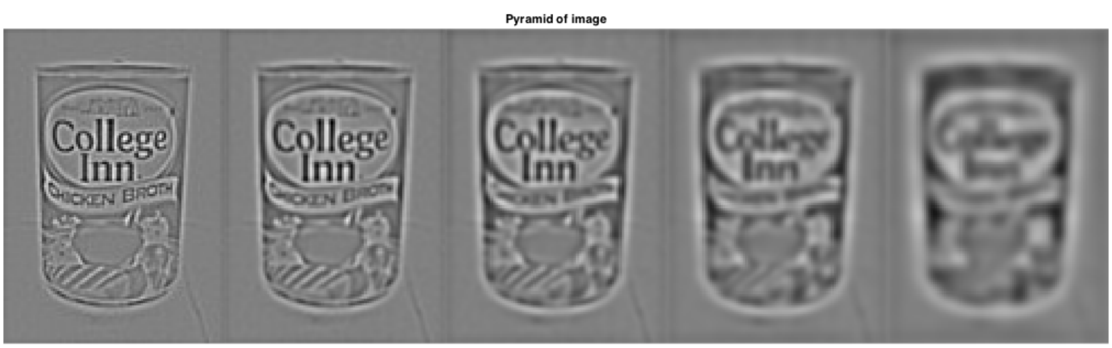
1.2 Diference in Guassian Pyramid of *model_chickenbroth.jpg*



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1.5 Image of detected keypoints with edge suppresions on *model_chickenbroth.jpg*

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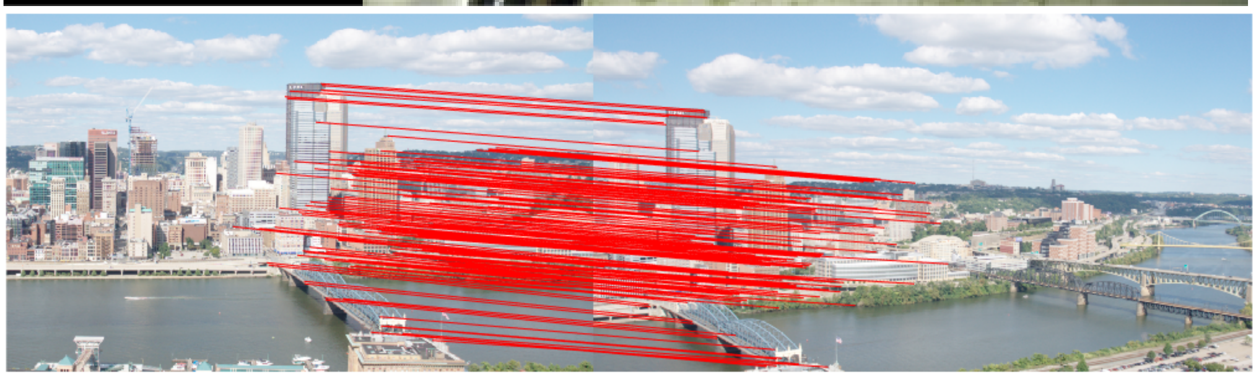
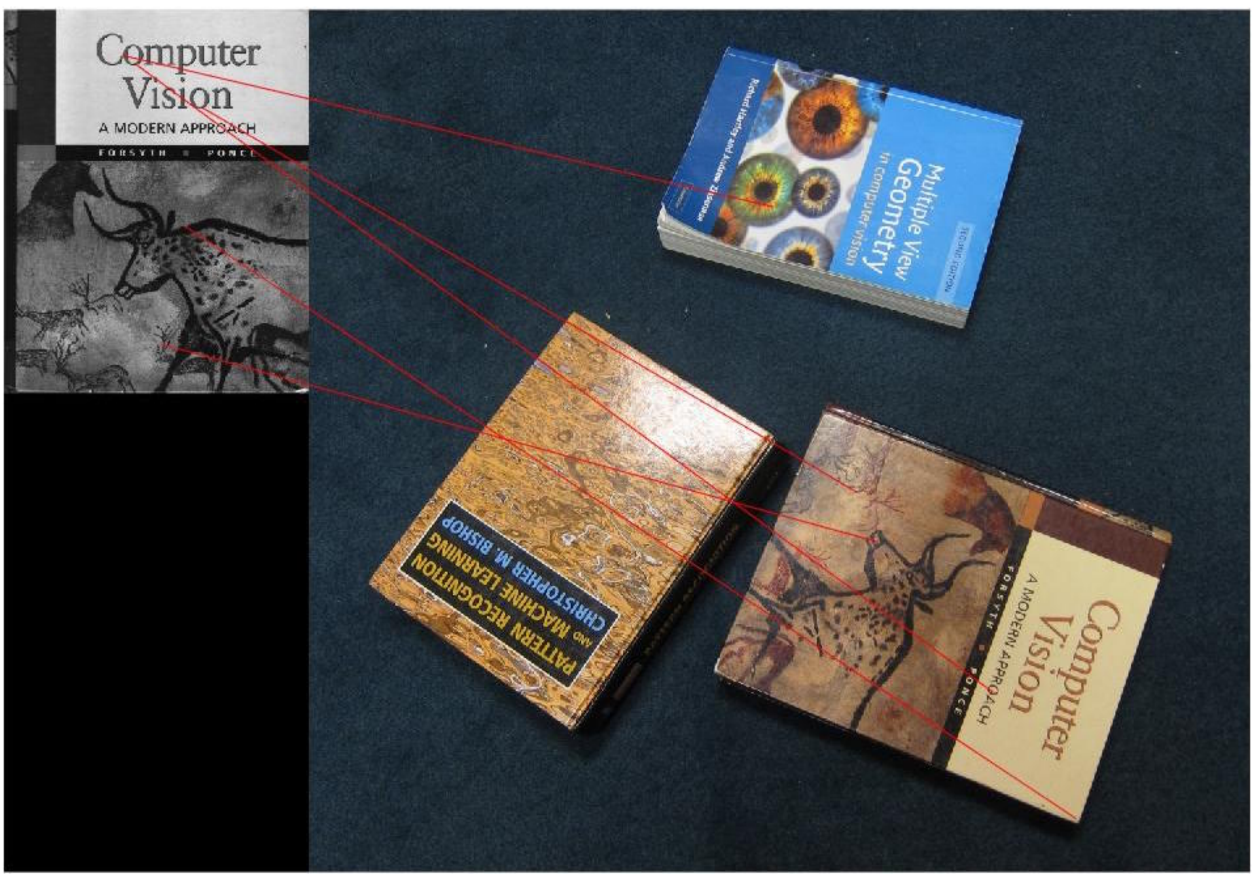
2.4 Results of two images matching on *chickenbroth.jpg*, *inline.jpg*, and *textbook.jpg*.

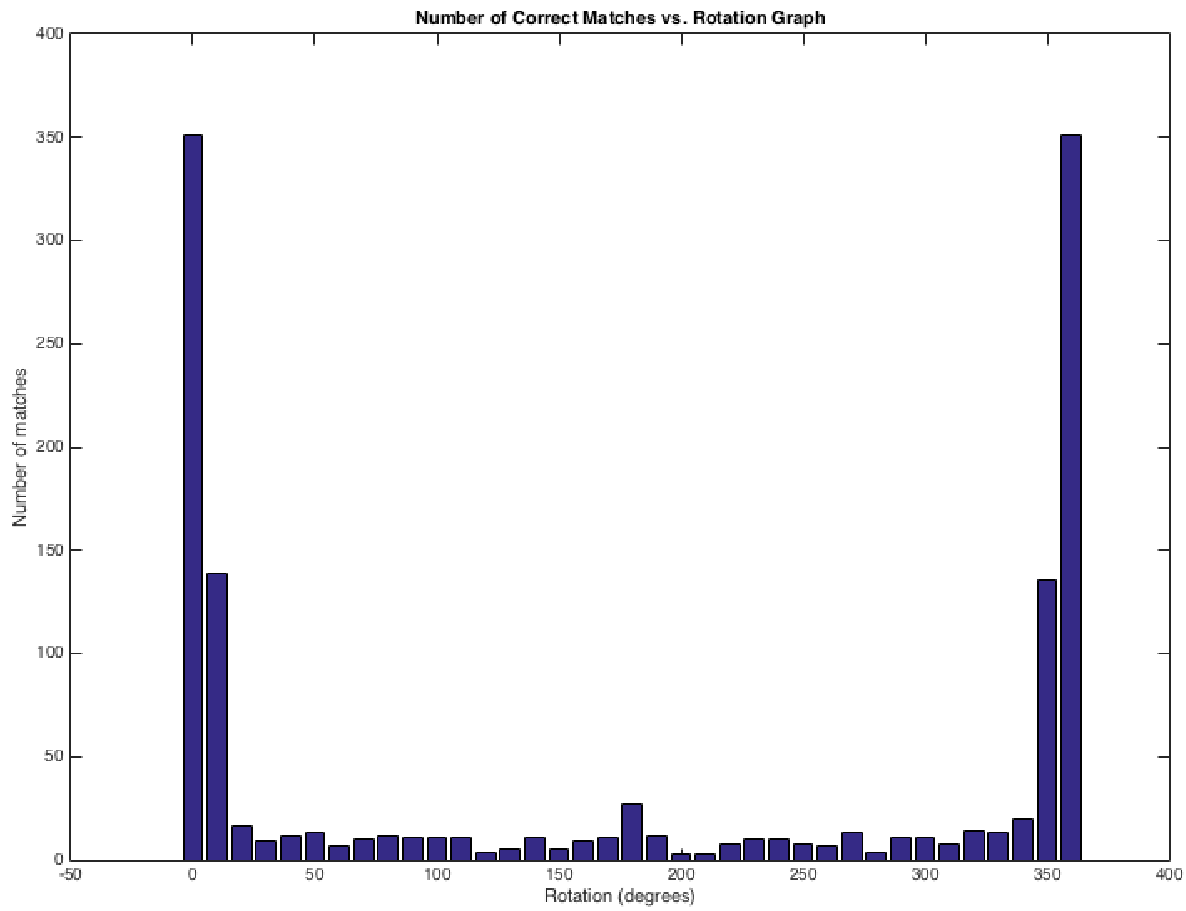
The matching results are below (Running all the matching with ratio = 0.7)





As we can see from above, after rotation, the book is not matched well. This is because BRIEF is not rotation-invariant (extra credit). This is because the tests pairs that we created do not calculate the same histogram after the image is rotated. One possible way to fix this is to rotate the test pairs too so the local environment in a descriptor will be consistent around all the interest points. ∎

2.5 Show the histogram of performance of matching with different rotation of *model_chickenbroth.jpg*.



As discussed above (2.4), the main reason that BRIEF is not rotation invariant is because the testpairs don't allow correct histogram calculation upon rotation. ∎

3.1

(a) Write out the expression of $\mathbf{A}$, in homography $\mathbf{Ah} = 0$.

(b) How many elements are in $\mathbf{h}$?

(c) How many point pairs (correspondences) are required to solve this system? *Hint:* How many degrees of freedom are in $\mathbf{h}$? How much information does each point correspondence give?

(d) Show how to estimate the elements in $\mathbf{h}$ to find a solution to minimize this homogeneous linear least squares system. Step us through this procedure. *Hint:* Use the Rayleigh quotient theorem (homogeneous least squares).

(a) Given that $p^i \equiv \mathbf{H}q^i$, where $p^i = [x_i', y_i', 1], q^i = [x_i, y_i, 1]$, we can expand it out and get the equation as:

$$\begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

so divide x_i' and y_i' with the third row, we get:

$$\frac{x_i'}{1} = \frac{h_{11}x_i + h_{12}y_i + h_{13}}{h_{31}x_i + h_{32}y_i + h_{33}}$$
$$\frac{y_i'}{1} = \frac{h_{21}x_i + h_{22}y_i + h_{23}}{h_{31}x_i + h_{32}y_i + h_{33}}$$

Rearrange, we get:

$$x_i'(h_{31}x_i + h_{32}y_i + h_{33}) = h_{11}x_i + h_{12}y_i + h_{13}$$
$$h_{31}x_ix_i' + h_{32}y_ix_i' + h_{33}x_i' - h_{11}x_i + h_{12}y_i + h_{13} = 0$$
$$y_i'(h_{31}x_i + h_{32}y_i + h_{33}) = h_{21}x_i + h_{22}y_i + h_{23}$$
$$h_{31}x_iy_i' + h_{32}y_iy_i' + h_{33}y_i' - h_{21}x_i + h_{22}y_i + h_{23} = 0$$

Now put it in the matrix $\mathbf{Ah} = 0$, we have the desired homogeneous system for all N points:

$$\begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' & -y_1' \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_Nx_N' & -y_Nx_N' & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -x_Ny_N' & -y_Ny_N' & -y_N' \end{pmatrix} \begin{pmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{pmatrix} = 0$$

Now, if we have N point, then the matrix A will have $2N \times 9$ dimension.

(b) From above, we can see that **h** is 9×1, which has 9 elements.

(c) **h** has 9 unknown, however, it is scale-invariant, which means that we can multiply by a constant that eliminates one of the degree of the freedom. So **h** has 8 degress of freedom. Since from part (a) we see that each point provides two equations, we need a total of $\boxed{4}$ non-colinear point pairs to solve the unique system.

(d) We are trying to find **h** such that it minimizes **Ah**. Using the given hint and Rayleigh's quotient theorem for solving homogeneous least squares (let $\mathbf{h} = x$). First we set up our problem as:

$$argmin_h \frac{||Ax||^2}{||x||^2}, \text{where } ||x||^2 = 1$$

Rearrange, we get:

$$\frac{||Ax||^2}{||x||^2} = \frac{x^T A^T A x}{x^T x}$$

which is in Rayleigh's quotient's form. We now express $A^T A$ with its eigenvalues and eigenvectors as:

$$A^T A v_i = \lambda_i v_i$$

, we can also express x as the sum of on the same basis of eigenvectors v_i,

$$x = \sum \alpha_i v_i$$

So we can rewrite above as:

$$\begin{aligned}
\frac{||Ax||^2}{||x||^2} &= \frac{x^T A^T A x}{x^T x} \\
&= \frac{(\sum \alpha_i v_i)^T A^T A (\sum \alpha_i v_i)}{(\sum \alpha_i v_i)^T (\sum \alpha_i v_i)} \\
&= \frac{\sum \alpha_i^2 \lambda_i}{\sum \alpha_i^2}
\end{aligned}$$

Note we can simplified $v_i v_j = 0$ for $i \neq j$ because eigenvectors are orthogonal. To minimize this, we can cearly see that we want to choose the smallest eigenvalues λ_{min} in $A^T A$. This is to choose **h** to be the the corresponding eigenvector in $A^T A$. Thus we have shown that **h** is the eigenvector corresponding to the smallest eigenvalues in A. (In SVD decomposition of $A = U \Sigma V^T$, this is a column of V that corresponds to the smallest eighenvalues in Σ.)

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5.1 Show the warped *incline_R.png*.

below is the wraped *incline_R.png*:

also the stiched image:

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5.2 Show the warped *incline_R.png* with no image clipping.

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6.2 Show the final panorama image with image mask blending:

The default iteration and tolerance for RANSAC are 2000 and 5:

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