**Fundamentals of Geometry**

* Points are the basic units in geometry, an element in space, not even 2 dimensional
* Lines are an infinite collection of collinear (on same line) points, 2D
* Line segments are a portion of a line
* Rays are “lines” originating in 1 direction from a start point, 2D
* Planes are flat 2D collection of infinitely many coplanar (on same plane) points and lines
* Everything has infinite amount of points
* 2 points define a line and are collinear
* 3 noncollinear points define a plane and are coplanar
* Any 2 non parallel planes define a line
* Any 2 non parallel lines define a point
* Any 2 parallel lines define a plane
* Skew lines are non parallel lines on different planes
* Shapes are 2D within an area on a plane
* Objects are 3D, on many planes
* Every point, lines has an infinite amount of lines and planes intersecting it
* Every point has an infinite amount of lines and planes containing it
* Every line has an infinite amount of planes containing it
* Etc… use ur imagination
* Angles are intersection of 2 lines/segments/rays
* Chords have endpoints on circle
* Tangent: touching in 1 point
* Relationship between linear and squared measurements are squared
* Relationship between linear and cubic measurements are cubed
* All radius and diameters have same length, infinite amount present in a circle
* Supplementary angles sum to 180, complementary 90
* Rays are named with endpoint as first letter

**Logic**

* Compounds
  + Conjunction
    - Read as “and”
    - Symbolic connector: ^
  + Disjunction
    - Read as “or”
    - Symbolic connector: v
  + Negation
    - Read as “not”
    - Symbolic connector: ~
  + Conditional
    - Read as “if…, then…”
    - Symbolic connector: ->
    - A converse flips the if and the then
  + Biconditional
    - Read as “…if and only if…”
    - Symbolic connector: <->
* Truth tables
  + Given 2 variables, does the truth value of the 2 variables affect the truth value of the claim
  + *Exercise*: assign statements p = \_\_\_\_ q = \_\_\_\_\_
  + Using symbolic connectors, write a bunch of statements
  + Translate the statements to actual words
  + Create a chart with different combinations of p being true/false and q being true/false, determine if the statement is still true

**Constructions**

* Equilateral
  + Define 2 points
  + Use distance between those points as radius of circle to draw a circle around each point
  + Connect original 2 points and one of the intersections of circle
  + Extending the line containing containing the original endpoints and connecting one of the original endpoints to a point of circle intersection creates a 120 deg angle (supplementary (2 angles summing to 180) to 60)
  + Creates equilateral triangle (all sides in 1:1:1 ration, angles 60-60-60)
* Perpendicular bisector
  + Perform equilateral construction, using endpoints of the line to be perpendicularly bisected
  + Connect the 2 points the circles intersect
  + Perpendicularly bisects a line
  + Connecting the 2 points of intersection of circles with one of the endpoints of the perpendicularly bisected segment forms an isosceles triangle (2 congruent sides and base angles)
  + Connecting all the points creates parallelogram (opposite sides parallel, opposite angles and sides are congruent)
* Angle bisector
  + Center on vertex of angle to be bisected
  + Make arc on each ray/line/segment of angle to be bisected
  + Draw circle using radius of the distance between the intersections around each point of intersection
  + Connect vertex to new intersection of circles
* Copying angle
  + Point at old vertex
  + Make arc on line/ray/segment of old angle
  + Draw a new line/ray/segment
  + Use radius (distance from vertex to arc) to draw circle around endpoint of the new line/ray/segment
  + Use radius (distance between the intersections of the arcs and segments/arcs/lines) to draw circle around intersection of new line and new circle
  + Connect center of new circle to intersection of the previously drawn circles
* Center of a circle
  + Draw a random chord
  + Perpendicularly bisect chord (result: diameter)
  + Perpendicularly bisect diameter
  + Intersection is center of circle
* Partitioning a line
  + Draw a line out from an endpoint of segment to be partitioned
  + Create marks on the new line, the amount in which you want other line to be partitioned, same distance apart
  + Connect last mark to other endpoint of segment to be bisected
  + Copy angle created by original line and the line drawn last step to every mark on the original line

**Theorems and Postulates**

* Postulates: statement accepted without proof
* Theorem: accepted statement, that is provable
* Segment addition postulate
  + The length of 2 segments added together equal the length of the segment composed of the 2 segments that were added (ex. AB + BC = AC)
* Angle addition postulate
  + The measure of 2 angles added together equal the measure of the angle composed of the 2 angles that were added
* Addition/subtraction/multiplication/division property of equality
  + Result of things being added/subtracted/multiplied/divided is equal to the things that were added/subtracted/multiplied/divided on the other side
* Substitution
  + Things of equal value can be used interchangeably
* Perpendicular lines create right angles, which are 90 deg, which all right angles are congruent
* Reflexive property
  + Something is equal to itself
* Transitive
  + If a=b and b=c, then a=c
* If 2 parallel lines are cut by a transversal, alternate interior/exterior and corresponding angles are congruent (and its converse)
* If 2 parallel lines are cut by a transversal, adjacent and same side interior angles are supplementary (and its converse)
* Vertical angles are congruent

**Polygons**

* Closed shape with 3+ sides
* Interior angle sum to 180(number of sides - 2)
* An exterior angle is 360/n
* Named with letters in clockwise or counter orientation
* Triangles
  + 3 sides and angles
  + Angles sum to 180
  + 1 rotational symmetry (360 - except for equilateral)
  + Largest angle is opposite longest side, smallest angle opposite shortest side
  + Sum of length of any 2 sides must exceed the length of the 3rd side
    - Triangle inequalities
      * a+b>c
      * a+c>b
      * b+c>a
  + Right
    - 90 angle
    - a2+b2=c2
  + Acute
    - Angles <90
    - a2+b2>c2
  + Obtuse
    - Angles >90
    - a2+b2<c2
  + Isosceles
    - Legs of triangle are congruent
    - Base angles are congruent
    - 1 reflection (perpendicular bisector)
      * Ways in which shape can be rotated or reflected to map on itself
  + Equilateral
    - All sides and angles are congruent
      * angles: 60-60-60
    - 3 reflection symmetry (each perpendicular bisector) and 3 rotational symmetry (intervals of 60 up to 180)
  + Scalene
    - No sides or angles are congruent
  + Third angle theorem
    - If there are 2 sets of congruent angles shared between 2 triangles, then the last angle of the triangles are congruent
  + Isosceles bisector
    - A line perpendicularly bisecting the base of an isosceles triangle also bisects its vertex angle (and its converse)
  + Remote interior angle
    - In a triangle, the supplement of 1 angle is equivalent to the sum of the other 2 angles
  + Corresponding parts of congruent/similar polygons are congruent/similar
  + SSS similarity/congruence postulate
    - If 3 sides of a triangle are similar/congruent to the corresponding ones on another triangle, then those triangles are similar/congruent
  + ASA similarity/congruence postulate
    - If 2 consecutive angles and the contained side are congruent/similar to the corresponding ones on another triangle, then those triangles are similar/congruent
  + SAS similarity/congruence postulate
    - If 2 consecutive sides and the contained angle are congruent/similar to the corresponding ones on another triangle, then those triangles are similar/congruent
  + AAS similarity/congruence theorem
    - If 2 angles and a side are consecutive and are congruent/similar to the corresponding ones on another triangle, then those triangles are similar/congruent
  + Hypotenuse-leg similarity/congruence theorem
    - If the hypotenuse and a leg of a right triangle is congruent/similar to the corresponding ones on another triangle, then those triangles are similar/congruent
  + AA(A) similarity postulate
    - If 2/3 angles are congruent to the corresponding ones on another triangle, then those triangles are similar/congruent
  + If a line in a triangle cuts 2 sides proportionally and is parallel to the 3rd side, then the line created similar triangles
  + In a right triangle, if a altitude is drawn perpendicular to the hypotenuse, congruent triangles are formed
  + Triangle sum theorem: triangle angles sum to 180
  + Points of concurrency
    - Circumcenter
      * Point where perpendicular bisectors of all sides meet
      * If a circle is drawn around circumcenter with radius of distance between center and an angle of triangle, the circle will be circumscribed (circle around triangle, angles tangent to circle) the triangle -ctr equidistant to angles
      * In acute triangle, it’s inside triangle
      * In right triangle, it’s on the midpoint of the hypotenuse
      * In obtuse triangle, it’s outside the triangle
    - Incenter
      * Point where angle bisectors of all angles meet
      * If a circle is drawn around incenter with radius of distance from incenter to where a perpendicular line passing through the incenter intersects the other side, then the circle will be inscribed (circle in triangle, tangent in 3 points) in the triangle
      * Always inside triangle
    - Centroid
      * Point where medians (angle connect to midpoint of opposite side) meet
      * Divides median to 1:2 ratio -2 side closer to angle
      * Inside when triangle is acute
      * On the right angle when triangle is right
      * Outside when triangle is obtuse
    - Orthocenter
      * Point where altitudes (height - perpendicular from a side up until highest extremity - draw line through that point parallel to lowest extremity - original line that has perpendicular line drawn from) meet
      * Inside triangle when it’s acute and right
      * For obtuse triangles, it’s inside when it’s isosceles, and outside when it’s scalene
      * Coincides with orthocenter in equilateral triangles
  + Midsegment: line connecting midpoints of 2 sides
    - Parallel to 3rd side
    - ½ length of 3rd side
  + Right triangle trigonometry
    - Sine
      * Complementary to cosine
      * Opposite side length divided by hypotenuse length
    - Cosine
      * Adjacent side length divided by hypotenuse length
    - Tangent
      * Sine divided by cosine
  + Geometric mean
    - In cases of right triangles with altitude drawn to the hypotenuse
      * Altitude = sqrt(left section of hyp \* right section of hyp)
      * Leg = sqrt(adjacent section of hyp \* hyp)
  + Special right triangles
    - 45-45-90
      * Legs are congruent
      * Hypotenuse is sqrt(2) \* leg
    - 30-60-90
      * Hypotenuse is double shortest side (opposite of 30)
      * 2nd longest side (opposite 60) is sqrt(3) \* shortest side
* Quadrilaterals
  + Parallelograms
    - All opposite sides parallel and congruent
    - Opposite angles congruent
    - Each diagonal forms 2 congruent triangles
    - Diagonals perpendicularly bisect each other
    - 2 rotational symmetries (intervals of 180 up to 360)
  + Rectangles
    - Opposite sides are congruent and parallel
    - All angles are right
    - Diagonals form 2 congruent triangles
    - Diagonals are congruent and bisects
    - 2 reflectional symmetry (perpendicular bisectors) and 2 rotational symmetries (same as parallelogram’s)
  + Rhombus
    - Opposite sides are parallel and congruent
    - Opposite angles congruent
    - Diagonals form 2 congruent triangles
    - Diagonals perpendicularly bisect each other
    - All sides are congruent
    - 2 reflectional symmetry (diagonals) and 2 rotational symmetries (intervals of 180 up to 360)
  + Kite
    - Opposite angles congruent
    - One diagonal bisects the other
    - 1 rotational symmetry (360)
  + Trapezoid
    - At least 1 set of parallel opposite sides
    - 1 rotational symmetry (360)
    - Isosceles trapezoid
      * At least 1 set of opposite congruent leg sides
      * 1 additional reflective symmetry (perpendicular bisector of base)
    - Midsegment: connects midpoints of of legs
      * Mean of the bases
  + Square
    - Has characteristics of all quadrilaterals
    - 4 reflectional symmetries (diagonals and perpendicular bisectors) and 4 rotational symmetries (intervals of 90 up to 360)

**Transformations**

* Read sequence equations from the innermost then outwards
* Commute = doing sequence in different order result in same image
* Rigid motion
  + Preserves congruence between preimage and image
  + Reflections
    - Image and preimage equidistant from line of reflection
    - Line connecting image and preimage is perpendicular to line of reflection
    - Equation: rline of reflection(point reflected)=point reflected prime
    - Construction: line perpendicular to line of reflection through preimage. Then, copy distance from preimage to line of reflection to the other side of the line of reflection
  + Rotation
    - Image and preimage equidistant from center of rotation
    - Line connecting image to center of rotation and line connecting preimage to center of rotation forms and angle with the measure of angle of rotation
    - Equation: R(center of rotation, angle of rotation - CCW)(preimage) = image
    - Construction: connect preimage to center of rotation. Then, construct angle of rotation, with the center of rotation as vertex. Then, copy distance from preimage to center of rotation onto the other segment of the angle
  + Translation
    - Image is parallel to preimage unless vector is parallel to preimage -coincide
    - Coordinate geometry
      * Equation: T(amount to add to x, amount to add to y)(preimage) = image
    - Non coordinate geometry
      * Equation: T(vector: line containing the desired direction and distance)(preimage) = image
* Non rigid
  + Makes image similar to preimage
  + Dilation
    - Scale factor
      * <1: shrink, closer to center of dilation
      * >1: enlarge, farther from center of dilation
      * Dist from image to ctr of dilation / dist from preimage to ctr of dilation = scale factor
    - Dilated lines are parallel
    - Center of dilation, image and preimage are collinear
    - Equation: Dcenter of dilation, scale factor(preimage) = image
    - Coordinate geometry
      * Multiply coordinate by scale factor
    - Non coordinate geometry
      * Partition distance from preimage to center of rotation and copy the scale factor (distance to ctr to preimg = 1)

**Circles**

* All similar
* Relations in circles
  + - Vertex on center
      * Intercepted arc is same measure as angle
    - Inscribed in circle(vertex on circle, each rays pass once)
      * Intercepted arc’s measure is double the angle’s measure
      * Angle formed by chord intersecting tangent line has same relationship
      * Angle is 90 if inscribed in semicircle
    - 2 tangents, or 2 secant (rays pass circle twice) or one of each intersecting
      * ½ (major-minor arc)
    - 2 chords intersecting
      * Mean of arcs contained by an angle formed and its vertical angle
    - When 2 chords intersect, products of the parts of 1 chord is equal to products of the parts of the other chord
    - When 2 secants interact with a circle, the product of the inside part and outside part of one secant is equal to the product of the inside part and outside part of the other secant
    - When a secant and a tangent interact with a circle, the square of the dist from point of tangency to endpoint is equal to the product of the inside part and outside part of the secant
  + Lines tangent to circle are perpendicular to radius
  + The point of intersection of 2 tangents is equidistant to the points of tangency
  + If a quadrilateral is inscribed, opposite angles are supplementary
  + Chords closer to center is longer, closer to diameter (longest chord) length
* Equation
  + (x-a)2+(y-b)2=r2
  + a = x of center, b = y of center, r = radius
* 1 deg = 180/pi radians
* Radian = intercepted arc length/radius

**Area and Volume**

* ½\*apothem\*perimeter for regular polygons
* Prisms
  + Volume: Base area \* height
  + Surface area: lateral faces areas + base areas
  + Lateral faces are rectangles
  + Named by their base shape
  + Bases, cross sections parallel to them, are parallel and similar
  + Right means slant is perpendicular to base
* Cone, pyramid
  + Volume is ⅓ of prism volume formula
  + Surface area of cone: pi \* r \* slant
* Area is squared, volume is cubed, from the proportion, which is at 1st power
* Capelleri
  + If objects have the same height, equal base areas, and cross sections at equal height have equal area, then the objects have equal volume
* Cylinder
  + Volume: pi \* r2 \* h
  + Surface area: 2 \* d \* h + 2 \* pi \* r2
* Sphere
  + Surface area: 4 \* pi \* r2
  + Volume: (4 pi r3)/3