

## 1.

$$A \rightarrow \textcircled{3} \text{Saa} | \textcircled{4} b$$



$$= \{\epsilon, aba, abb\} \oplus_3 \{b, aa, aba, abb\} \oplus_3 \{\epsilon, aba, abb\}$$

6

$$\text{FIRST}_2(\epsilon) \oplus_2 \{\epsilon\} = \{\epsilon\} \quad \text{and} \quad \cap = \emptyset \quad \therefore \text{Cons.}$$

$$T_1 = T_A, \{\varepsilon\}$$

$$T_2 = T_5, \{aa\}$$

aa	$S \rightarrow \epsilon$	—
ab	$S \rightarrow abA$	For A: $\{aa\}, T_3$

$$T_3 = T_A, \{aa\}$$

$$FIRST_2(S_{aa}) \cup \{aa\} = \{aa, ab\} \quad (b) \quad \text{"} = \{ba\} \quad \cap = \emptyset \text{ is cons.}$$

aa	} A → S <sub>aa</sub>	FOR S:
ab		{aa}: T <sub>2</sub>
ba		A → b

∴ G<sub>1</sub> is LL(2).

LL(2) parser:

	aa	ab	ba	bb	a	b	ε
T <sub>0</sub>		abT <sub>1</sub> ②					ε①
T <sub>1</sub>	T <sub>2</sub> aa③	T <sub>2</sub> aa③				b④	
T <sub>2</sub>	ε①	abT <sub>3</sub> ②					
T <sub>3</sub>	T <sub>2</sub> aa③	T <sub>2</sub> aa③	b④				
a	p	p			p		
b			p	p		p	
#							accept

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	stack	input	output
	T <sub>0</sub> #	ababbaa	ε
└	abT <sub>1</sub> #	"	2
└ <sup>2</sup>	T <sub>1</sub> #	abbaa	"
└	T <sub>2</sub> aa #	"	2, 3
└	abT <sub>3</sub> aa #	"	2, 3, 2
└ <sup>2</sup>	T <sub>3</sub> aa #	baa	"
└	baa #	"	2, 3, 2, 4
└ <sup>3</sup>	#	ε	"
└	accept and output 2324		

$$2. \quad S \rightarrow ABCabcd$$

$$A \rightarrow a \mid \epsilon$$

$$B \rightarrow b \mid \epsilon$$

$$C \rightarrow c \mid \epsilon$$

$$T_0 = T_S, \{\epsilon\}$$

$$\text{FIRST}_2(ABCabcd) \oplus_2 \{\epsilon\} = \{ab, ac, aa, bc, ba, ca\}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\{a, \epsilon\} \quad \{b, \epsilon\} \quad \{c, \epsilon\}$$

ab

ac

aa

bc

ba

ca

$$S \xrightarrow{1} ABCabcd$$

FOR A:

$$\text{FIRST}_2(BCabcd) \oplus_2 \{\epsilon\} = \{bc, ba, ca, ab\} : T_1$$

FOR B:

$$\text{FIRST}_2(Cabcd) \oplus_2 \{\epsilon\} = \{ca, ab\} : T_2$$

FOR C:

$$\{ab\} : T_3$$

$$T_1 = T_A, \{bc, ba, ca, ab\}$$

$$\text{FIRST}_2(a) \oplus_2 \{bc, ba, ca, ab\} = \{ab, ac, aa\}$$

( $\epsilon$ )

"

$$= \{bc, ba, ca, ab\}$$

$\cap = \{ab\} \neq \emptyset$

$\therefore$  conflict

$\therefore G_2$  is not  $LL(2)$



3.  $S \rightarrow AB$

$A \rightarrow 0A1 \mid \epsilon$

$B \rightarrow 1B1$

Use theorem to prove  $G_3$  is  $L((z))$ .

For  $S$ : OK  $\because$  only 1  $S$ -production

For  $A$ :

$$\text{FIRST}_2(0A1) \oplus_2 \{1, 11\} \cap \text{FIRST}_2(\epsilon) \oplus_2 \{1, 11\} = \{00, 01\} \cap \{1, 11\} = \emptyset$$

$$\text{" } \{11\} \cap \text{" } \{11\} = \{00, 01\} \cap \{11\} = \emptyset$$

$\therefore$  cons.

For  $B$ :

$$\text{FIRST}_2(1B1) \oplus_2 \{\epsilon\} \cap \text{FIRST}_2(1) \oplus_2 \{\epsilon\} = \{11\} \cap \{1\} = \emptyset$$

$\therefore$  cons.

$\therefore G_3$  is  $L((z))$ .

$$\begin{aligned}
 4. \quad & S \rightarrow AB \mid BC \\
 & A \rightarrow a \mid CBS \mid bSS \\
 & B \rightarrow b \mid bAS \mid bB \\
 & C \rightarrow a \mid aSBC
 \end{aligned}$$

$$(a) \text{ FIRST}_2(S) = \{ab, aa, ba, bb\}$$

$$\text{FIRST}_2(A) = \{a, aa, ab, ba, bb\}$$

$$\text{FIRST}_2(B) = \{b, ba, bb\}$$

$$\text{FIRST}_2(C) = \{a, aa, ab\}$$

(b) For S:

$$\begin{aligned}
 \text{FIRST}_2(AB) \oplus_2 \{\epsilon\} \cap \text{FIRST}_2(BC) \oplus_2 \{\epsilon\} &= \{ab, aa, ba, bb\} \cap \\
 &\quad \{ba, bb\} \\
 &= \{ba, bb\} \neq \emptyset
 \end{aligned}$$

$\therefore G_4$  is not LL(2).

3. Use the theorem below to prove that the following grammar  $G_1$  is LL(2). Show steps.  
Theorem: A CFG  $G$  is LL( $k$ ) iff the following holds: if  $A \rightarrow \beta$  and  $A \rightarrow \gamma$  are distinct productions, then  $\text{FIRST}_k(\beta\alpha) \cap \text{FIRST}_k(\gamma\alpha) = \emptyset$  for all  $w\alpha z$  such that  $S \rightarrow w\alpha z$  in any number of steps.

$$\begin{aligned}
 G_1: \quad & S \rightarrow AB \\
 & A \rightarrow 0A1 \mid \epsilon \\
 & B \rightarrow 1B \mid 1
 \end{aligned}$$

10. 4. For the following grammar  $G_4$ ,

(a) Find  $\text{FIRST}_2(x)$  for all non-terminal  $x$  in  $G_4$ .

(b) Use the above theorem to prove that  $G_4$  is not LL(2).

$$\begin{aligned}
 G_4: \quad & S \rightarrow AB \mid BC \\
 & A \rightarrow a \mid CBS \mid bSS \\
 & B \rightarrow b \mid bAS \mid bB \\
 & C \rightarrow a \mid aSBC
 \end{aligned}$$