CS411 HW#5 Solution

1. (a)
$S \xrightarrow{1} XYXY$
$X \xrightarrow{2} 0X \mid \xrightarrow{3} \varepsilon$
$Y \xrightarrow{4} 1Y \mid \xrightarrow{5} \varepsilon$

LR(0) tables and parser:

$A_0 = V_0(\varepsilon)$
- ✓ $I: [S \to .\, XYXY]$ $\quad \varepsilon \overset{?}{\in} EFF_0(XYXY) = \{\varepsilon\}$ yes
- ✓ $[X \to .\, 0X]$ $\quad (0X) = \{0\}$ yes
- ✓ $[X \to .\,]$ $\quad \therefore$ S/R conflict

$A_1 = V_0(X) = goto(A_0, X)$
- ✓ $I: [S \to X.\, YXY]$ $\quad \varepsilon \overset{?}{\in} EFF_0(YXY) = \{\varepsilon\}$ yes
- ✓ $[Y \to .\, 1Y]$ $\quad (1Y) = \{1\}$ yes
- ✓ $[Y \to .\,]$ $\quad \therefore$ S/R conflict

$A_2 = V_0(0) = goto(A_0, 0)$
- ✓ $I: [X \to 0.\, X]$ $\quad \varepsilon \overset{?}{\in} EFF_0(X) = \{\varepsilon\}$ yes
- ✓ $[X \to .\, 0X]$ goto A_2 $\quad (0X) = \{0\}$ yes
- ✓ $[X \to .\,]$ $\quad \therefore$ S/R conflict

$A_3 = V_0(XY) = goto(A_1, Y)$
- ✓ $I: [S \to XY.\, XY]$ $\quad \varepsilon \overset{?}{\in} EFF_0(XY) = \{\varepsilon\}$ yes
- ✓ $[X \to .\, 0X]$ goto A_2 $\quad (0X) = \{0\}$ yes
- ✓ $[X \to .\,]$ $\quad \therefore$ S/R conflict

$A_4 = V_0(X1) = goto(A_1, 1)$
- ✓ $I: [Y \to 1.\, Y]$ $\quad \varepsilon \overset{?}{\in} EFF_0(Y) = \{\varepsilon\}$ yes
- ✓ $[Y \to .\, 1Y]$ goto A_4 $\quad (1Y) = \{1\}$ yes
- ✓ $[Y \to .\,]$ $\quad \therefore$ S/R conflict

$A_5 = V_0(0X) = goto(A_2, X)$
- ✓ $I: [X \to 0X.\,]$ $\quad$ 1 item $\therefore$ cons.

$A_6 = V_0(xyx) = \text{goto}(A_3, x)$

✓ $I: [S \to xyx . y]$ $\quad \varepsilon \overset{?}{\in} EFF_0(y) = \{\varepsilon\}$ yes

✓ $[Y \to . \; 1\; Y]$ goto A_4 $\quad (1\,Y) = \{\varepsilon\}$ yes

✓ $[Y \to . \;]$ $\quad \therefore$ S/R conflict

$A_7 = V_0(x1Y) = \text{goto}(A_4, Y)$

✓ $I: [Y \to 1\,Y .]$ $\quad$ 1 item ∴ cons.

$A_8 = V_0(xyxy) = \text{goto}(A_6, Y)$

✓ $I: [S \to xyxy .]$ $\quad$ 1 item ∴ cons.

Parser:

	ε	S	x	y	o	1
A_0	s, r3		A_1		A_2	
A_1	s, r5			A_3		A_4
A_2	s, r3		A_5		A_2	
A_3	s, r3		A_6		A_2	
A_4	s, r5			A_7		A_4
A_5	r2					
A_6	s, r5			A_8		A_4
A_7	r4					
A_8	r1 (acc)					

conflict

1. (b) $S \to xyxy$

$X \to oX \mid \varepsilon$

$Y \to 1Y \mid \varepsilon$

LR(1) tables and prove it's not LR(1).

$A_0 = V_1(\varepsilon)$

I: $[S \to .\, xyxy, \varepsilon]$ $\quad 1 \mid o \mid \varepsilon \overset{?}{\in} EFF_1(xyxy) = \{o\}$ yes

$[X \to .\, oX, 1 \mid o \mid \varepsilon]$ $\quad oX1 = \{o\}$ yes

$[X \to .\,, 1 \mid o \mid \varepsilon]$ $\quad oXo = \{o\}$ yes

$oX\varepsilon = \{o\}$ yes

∴ S/R conflict

∴ G_1 is <u>not</u> LR(1).

2 ⓐ (0) $S' \to S$

(1) $S \to i\,E\,t\,S$

(2) $S \to i\,E\,t\,S\,e\,S$ LR(1) tables and parser

(3) $S \to a$

(4) $E \to b$

$A_0 = V_1(\varepsilon)$

✓ I: [S' → . S, ε]

✓ [S → . i E t S , ε] all shift

✓ [S → . i E t S e S, ε] ∴ cons.

✓ [S → . a , ε]

$A_1 = V_1(S) = goto(A_0, S)$

✓ I: [S' → S. , ε] 1 item ∴ cons.

$A_2 = V_1(i) = goto(A_0, i)$

✓ I: [S → i. E t S , ε] all shift

✓ I: [S → i. E t S e S , ε] ∴ cons.

✓ [E → . b, t]

$A_3 = V_1(a) = goto(A_0, a)$

✓ I: [S → a. , ε] 1 item ∴ cons.

$A_4 = V_1(iE) = goto(A_2, E)$

✓ I: [S → iE. t S, ε] all shift ∴ cons.

✓ I: [S → iE. t S e S, ε]

$A_5 = V_1(ib) = goto(A_2, b)$

✓ I: [E → b. , t] 1 item ∴ cons.

$A_6 = V_1(iEt) = goto(A_4, t)$

✓ I: [S → iEt. S, ε] all shift ∴ cons.

✓ I: [S → iEt. S e S, ε]

✓ [S → . i E t S , ε/e]

✓ [S → . i E t S e S, ε/e]

✓ [S → . a , ε/e]

$A_7 = V_1(iEtS) = goto(A_6, S)$

✓ $I: [S \to iEtS., \varepsilon]$ $\quad \varepsilon \notin EFF_1(eS\varepsilon) = \{e\}$ X

✓ $I: [S \to iEtS.eS, \varepsilon]$ $\quad$ ∴ cons.

$A_8 = V_1(iEti) = goto(A_6, i)$

✓ $I: [S \to i.EtS, \varepsilon/e]$ $\quad$ all shift ∴ cons.

✓ $I: [S \to i.EtSeS, \varepsilon/e]$

✓ $[E \to .b, t] \to goto\ A_5$

$A_9 = V_1(iEta) = goto(A_6, a)$

✓ $I: [S \to a., \varepsilon/e]$ $\quad$ 1 item ∴ cons.

$A_{10} = V_1(iEtSe) = goto(A_7, e)$

✓ $I: [S \to iEtSe.S, \varepsilon]$ $\quad$ all shift ∴ cons.

✓ $[S \to .iEtS, \varepsilon]$

✓ $[S \to .iEtSeS, \varepsilon] \to goto\ A_2$

✓ $[S \to .a, \varepsilon] \to goto\ A_3$

$A_{11} = V_1(iEtiE) = goto(A_8, E)$

✓ $I: [S \to iE.tS, \varepsilon/e]$ $\quad$ all shift ∴ cons.

✓ $I: [S \to iE.tSeS, \varepsilon/e]$

$A_{12} = V_1(iEtSeS) = goto(A_{10}, S)$

✓ $I: [S \to iEtSeS., \varepsilon]$ $\quad$ 1 item ∴ cons.

$A_{13} = V_1(iEtiEt) = goto(A_{11}, t)$

✓ $I: [S \to iEt.S, \varepsilon/e]$ $\quad$ all shift ∴ cons.

✓ $I: [S \to iEt.SeS, \varepsilon/e]$

✓ $[S \to .iEtS, \varepsilon/e]$

✓ $[S \to .iEtSeS, \varepsilon/e] \to goto\ A_8$

✓ $[S \to .a, \varepsilon/e] \to goto\ A_9$

$A_{14} = V_1(iEtiEtS) = goto(A_{13}, S)$

✓ $I: [S \to iEtS.\ ,\ \varepsilon/e]$ $\varepsilon/e \notin EFF_1(eS\varepsilon) = \{e\}$ yes

✓ $I: [S \to iEtS.\ eS,\ \varepsilon/e]$ $(eSe) = \{e\}$ yes

∴ conflict

$A_{15} = V_1(iEtiEtSe) = goto(A_{14}, e)$

✓ $I: [S \to iEtSe.\ S,\ \varepsilon/e]$ all shift ∴ cons.

✓ $[S \to .\ iEtS\ ,\ \varepsilon/e]$ ⟩⟶ goto A_8

✓ $[S \to .\ iEtSeS\ ,\ \varepsilon/e]$

✓ $[S \to .\ a\ ,\ \varepsilon/e]$ ⟶ goto A_9

$A_{16} = V_1(iEtiEtSeS) = goto(A_{15}, S)$

✓ $I: [S \to iEtSeS.\ ,\ \varepsilon/e]$ 1 item ∴ cons.

Conflict in A_{14}, ∴ G_2 is <u>not</u> $LR(1)$

	i	t	e	a	b	ε	S	E	i	t	e	a	b
A_0	S			S			A_1		A_2			A_3	
A_1						r,0 (acc)							
A_2					S			A_4					A_5
A_3						r,3							
A_4		S								A_6			
A_5		r,4											
A_6	S			S			A_7		A_8			A_9	
A_7			S			r,1					A_{10}		
A_8					S			A_{11}					A_5
A_9			r,3			r,3							
A_{10}	S			S			A_{12}		A_2			A_3	
A_{11}		S								A_{13}			
A_{12}						r,2							
A_{13}	S			S			A_{14}		A_8			A_9	
A_{14}			r,1 S			r,1					A_{15}		
A_{15}	S			S			A_{16}		A_8			A_9	
A_{16}			r,2			r,2							

conflict.

2(b)

Stack	input	output
A_0	i b t a e a	ε
$\vdash^{S}$ $A_0 i A_2$	b t a e a	"
$\vdash^{S}$ $A_0 i A_2 b A_5$	t a e a	"
$\vdash^{r,4}$ $A_0 i A_2 E A_4$	"	4
$\vdash^{S}$ $A_0 i A_2 E A_4 t A_6$	a e a	"
$\vdash^{S}$ $A_0 i A_2 E A_4 t A_6 a A_9$	e a	"
$\vdash^{r,3}$ $A_0 i A_2 E A_4 t A_6 S A_7$	"	4, 3
$\vdash^{S}$ $A_0 i A_2 E A_4 t A_6 S A_7 e A_{10}$	a	"
$\vdash^{S}$ $A_0 i A_2 E A_4 t A_6 S A_7 e A_{10} a A_3$	ε	"
$\vdash^{r,3}$ $A_0 i A_2 E A_4 t A_6 S A_7 e A_{10} S A_{12}$	"	4, 3, 3
$\vdash^{r,2}$ $A_0 S A_1$	"	4, 3, 3, 2
$\vdash^{r,0}$ $A_0 S'$	"	4, 3, 3, 2, 0

⊢ accept and output 0, 2, 3, 3, 4

(0) $S' \to S$

3. $S \to {}^{(1)}aSbS \mid {}^{(2)}bSSb \mid {}^{(3)}aSaS \mid {}^{(4)}bSa \mid {}^{(5)}a \mid {}^{(6)}b$

LR(2) tables:

$A_0 = V_2(\varepsilon)$

✓ I: $[S' \to .\, S, \varepsilon]$
✓ $[S \to .\, aSbS, \varepsilon]$
✓ $[S \to .\, bSSb, \varepsilon]$
✓ $[S \to .\, aSaS, \varepsilon]$
✓ $[S \to .\, bSa, \varepsilon]$
✓ $[S \to .\, a, \varepsilon]$
✓ $[S \to .\, b, \varepsilon]$

all shift items ∴ cons.

$A_1 = V_2(S) = \text{goto}(A_0, S)$

✓ I: $[S' \to S., \varepsilon]$ 1 item ∴ cons.

$A_2 = V_2(a) = \text{goto}(A_0, a)$

✓ I: $[S \to a.\, SbS, \varepsilon]$
✓ I: $[S \to a.\, SaS, \varepsilon]$
✓ I: $[S \to a.\ , \varepsilon]$
✓ $[S \to .\, aSbS, ba \mid bbl aa \mid ab]$
$[S \to .\, bSSb, \quad " \quad]$
✓ $[S \to .\, aSaS, \quad " \quad]$
$[S \to .\, bSa, \quad " \quad]$
✓ $[S \to .\, a, \quad " \quad]$
$[S \to .\, b, \quad " \quad]$

$\varepsilon \notin EFF_2(SbS\varepsilon)$ ×
$SaS\varepsilon$ ×
$aSbS \oplus_2 ba \mid bbl aa$ ×
$bSSb \oplus_2$ " ×
$aSaS \oplus_2$ " ×
$bSa \oplus_2$ " ×
$a \oplus_2$ " ×
$b \oplus_2$ " ×

∴ cons.

$A_3 = V_2(b) = goto(A_0, b)$

I: [S → b.SSb, ε]
I: [S → b.Sa, ε]
I: [S → b., ε]
[S → .aSbS, aa|ab|ba|bb|a]
[S → .bSSb, "]
[S → .aSaS, "]
[S → .bSa, "]
[S → .a, "]
[S → .b, "]

$\varepsilon \notin EFF_2(SSb\varepsilon)$ ✗
$Sa\varepsilon$ ✗
$aSbS \oplus_2 (aa|ab|ba|bb|a)$ ✗
$bSSb \oplus_2$ " ✗
$aSaS \oplus_2$ " ✗
$bSa \oplus_2$ " ✗
$a \oplus_2$ " ✗
$b \oplus_2$ " ✗

∴ cons.

$A_4 = V_2(aS) = goto(A_2, S)$

I: [S → aS.bS, ε]
I: [S → aS.aS, ε]

all shift items ∴ cons.

$A_5 = V_2(aa) = goto(A_2, a)$

I: [S → a.SbS, ba|bb|aa|ab]
I: [S → a.SaS, "]
I: [S → a., "]
[S → .aSbS, ba|bb|aa|ab]
[S → .bSSb, "]
[S → .aSaS, "]
[S → .bSa, "]
[S → .a, "]
[S → .b, "]

ba|bb|aa|ab

$\notin EFF_2(SbS \oplus_2 (ba|bb|aa|ab))$ ✓
$SaS \oplus_2$ " ✓
$aSbS$ " ✓
$bSSb$ " ✓
$aSaS$ " ✓
bSa " ✓
a " ✓
b " ✓

∴ S/R conflict
∴ G_3 is not $LR(2)$

LR(2) parser

	aa	ab	ba	bb	a	b	ε	S	a	b
A_0	S	S	S	S	S	S		A_1	A_2	A_3
A_1							r,0(acc)			
A_2	S	S	S	S			r,5	A_4	A_5	
A_3	S	S	S	S			r,6			
A_4	S	S	S	S						
A_5	r,5 S	r,5 S	r,5 S	r,5 S						

Conflict