A Sensitivity Analysis of Electrical Resistivity Tomography Array Types Using Analytical Element Modeling

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ABSTRACT

The analytic element method is used to investigate the spatial sensitivity of different electrical resistivity tomography (ERT) arrays. By defining the sensitivity of an array to a subsurface location we were able to generate maps showing the distribution of the sensitivity throughout the subsurface. This allows us to define regions of the subsurface where different ERT arrays are most and least sensitive. We compared the different arrays using the absolute value of the sensitivity and using its spatial distribution. Comparison is presented for three commonly used arrays (Wenner, Schlumberger, and double dipole) and for one atypical array (partially overlapping). Most common monitoring techniques use a single measurement to measure a property at a single location. The spatial distribution of the property is determined by interpolation of these measurements. In contrast, ERT is unique in that multiple measurements are interpreted simultaneously to create maps of spatially distributed soil properties. We define the spatial sensitivity of an ERT survey to each location on the basis of the sum of the sensitivities of the single arrays composing the survey to that location. With the goal of applying ERT for time-lapse measurements, we compared the spatial sensitivities of different surveys on a per measurement basis. Compared are three surveys based on the typical Wenner, Schlumberger, and double dipole arrays, one atypical survey based on the partially overlapping array, and one mixed survey built of arrays that have been shown to be optimal for a series of single perturbations. Results show the inferiority of the double dipole survey compared with other surveys. On a per measurement basis, there was almost no difference between the Wenner and the Schlumberger surveys. The atypical partially overlapping survey is superior to the typical arrays. Finally, we show that a survey composed of a mixture of array types is superior to all of the single array type surveys. By analyzing the spatial sensitivity of the single array, and most significantly the sensitivity of the ERT survey, we set the basis for quantitative measurement of subsurface properties using ERT, with applications to both static and transient hydrologic processes.

Electrical resistivity tomography is widely used for mapping shallow subsurface geological structure (Storz et al., 2000), solute distribution (Binley et al., 1996), water content (Zhou et al., 2001), and other environmental, hydrological, and engineering features (Dahlin, 2000). The method is based on the introduction of electrical current into the soil through two surface electrodes and the simultaneous measurement of the induced potential gradient with other surface electrodes. Each potential measurement gives insight into the electrical properties of the subsurface materials. The inversion of multiple measurements made with overlapping electrode arrays allows for the interpretation of the two-dimensional distribution of electrical conductivity in the subsurface. The electrical conductivity distribution can then be related to the volumetric water content, concentration of electrolytic solutes in the pore water, and the surface conductivity of the subsurface materials.

Each ERT measurement represents some average of the heterogeneous subsurface electrical conductivities in the shallow subsurface. Given that the current is applied and the potential is measured at the surface, all ERT arrays are more sensitive to the properties of shallow subsurface materials than to deeper material properties. In general, arrays with larger electrode separations are assumed to have sample areas that extend deeper beneath the ground surface, while the sample areas of smaller arrays are limited to the shallow subsurface. To form a more unique image of the subsurface electrical conductivity distribution, the apparent resistivity is measured through many electrode combinations and interpreted simultaneously. In general, it is felt that the use of a large number of arrays with different array sizes will lead to a more accurate representation of the spatial structure of the subsurface electrical conductivity.

With modern field equipment, many electrodes can be installed and measurements can be made automatically using multiple electrode arrays. As a result, many thousands of combinations of electrodes can be formed using a standard set of 21 electrodes. However, caution should be used in choosing the arrays comprising a survey to minimize soil charge time (Dahlin, 2000). The total time required to collect stacked measurements for each array is typically only about 15 s. However, the total measurement time can become impractical if too many arrays are used. Therefore, more efficient application of the ERT method requires the development of a quantitative method to define the most useful subset of ERT arrays to form an ERT survey.

The identification of an optimal ERT array set is even more critical when monitoring transient processes. The simultaneous inversion of multiple measurements is based on the underlying assumption that all measurements were made on an identical subsurface conductivity field. However, for many transient processes, such as water infiltration or solute transport, the subsurface electrical conductivity distribution can change rapidly with time. To use ERT to monitor these processes it is critical that all electrode arrays are measured within a time frame during which it is reasonable to assume that the conductivity field is unchanging. In practice, this time frame will be defined based on the expected rate of change of the subsurface electrical conductivity

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Abbreviations: ERT, electrical resistivity tomography; TDR, time domain reflectometry.

caused by the transient process under study. The optimal use of ERT for monitoring transient hydrologic processes requires a careful choice of the number and distribution of electrode arrays to achieve maximum spatial resolution and maximum accuracy in the time available for measurement.

We hypothesize that a quantitative approach to determining the optimal choice of ERT arrays to form an ERT survey can be designed based on the spatial sensitivities of ERT arrays. This general concept is not new. In fact, it is common practice to combine arrays of different electrode spacings to form an ERT survey, with the implicit assumption that the depth of penetration increases with increasing array width. This practice is based on definitions of the depth of penetration of arrays. For example, Spies (1989) described the depth of penetration of arrays as "the maximum depth at which a buried half-space can be detected by a measurement system at a particular frequency" (Spies, 1989). Similarly, Evjen (1938) and Roy and Apparao (1971) defined the depth of investigation as the depth at which a thin horizontal layer of ground contributes the maximum amount to the total measured signal at the ground surface. Using this definition, Roy and Apparao (1971) and Roy (1972) calculated the depth of investigation for many array types. These analyses are aimed primarily at determining the depth investigation of arrays, with no consideration of the distribution of array sensitivity throughout the subsurface. As a result, there is no consideration of the cumulative sensitivity distribution of the arrays that comprise a survey. We present a more complete method of analysis of the spatial distribution of ERT spatial sensitivity, leading to a more complete definition of the optimal set of arrays to meet specific monitoring objectives.

The objective of this investigation was to apply the analytic-element solution for a circular perturbation in a homogeneous field, which was presented by Furman et al. (2002b), to investigate the sensitivity of the electrical potential at the ground surface to electrical heterogeneities distributed throughout the subsurface. This allows for the definition of the spatial sensitivity of any given electrode array and the development of a quantitative measure of the contribution of that array to the cumulative response of an ERT survey. With this definition, we identify the basis on which to choose an optimal survey set, which minimizes survey time while maximizing the useful information for inferring subsurface properties. Development of an optimization method based on this description of sensitivity distribution may be used to improve the capabilities of ERT for quantitative monitoring of static and transient subsurface hydrologic processes.

THEORY

A unique description of the contribution of each ERT measurement to the definition of the distributed subsurface electrical conductivity requires an understanding of the spatial sensitivity of each ERT measurement. Understanding of the spatial sensitivity of indirect measurement methods is most advanced for time domain reflectometry (TDR). Baker and Lascano (1988) used direct measurements of the response of TDR probes to small perturbations to describe the spatial sensitivity of TDR probes. Knight (1992) developed an analytical expression defining the response of TDR probes to small changes from a uniform distribution of dielectric permittivity in the plane transverse to a TDR probe. Ferré et al. (1996) extended this to consider special cases of heterogeneous dielectric permittivity distributions. Knight et al. (1997) introduced numerical approaches to allow for fully heterogeneous distributions. Finally, Ferré et al. (1998, 2000) used these definitions to uniquely define the spatial sensitivities and sample areas of TDR probes. The approaches used in these investigations can be used to describe the spatial sensitivity of any instrument that measures the steady-state distribution of energy potentials at the point at which the energy is applied to the medium, such as TDR, a well used for a slug–bail test, or an air permeameter. However, the method is not directly amenable to the interpretation of a method for which energy is applied at one location and measured at another location, such as ERT. To date, the spatial sensitivity of ERT measurements has only been discussed to a limited extent (Loke, 2000). There has been no attempt to rigorously define the spatial sensitivity of individual ERT measurements, which is necessary to optimize the design of an ERT survey.

Solution for Flow through an Inhomogeneity

To investigate the sensitivity of an ERT system, one needs an efficient way to describe the spatial distribution of electric potential in the subsurface, and in particular at the ground surface, in response to the applied electric current and as a function of the subsurface geoelectric structure. An efficient approach is necessary because of the large number of four-electrode arrays that can be formed even with a limited number of electrodes. For example, with a system of 50 electrodes, almost 1.5 million different four-electrode arrays can be formed. The small current applied requires that the potential at the ground surface be calculated with high accuracy. This, together with the need for rapid solutions, makes using finite element or finite difference numerical methods less attractive.

Many models have been developed to predict the distribution of electrical potential in the subsurface in response to current applied at, or below, the surface. Of these models, most were developed for homogeneous domains (e.g., Keller and Frischknecht, 1966), layered strata (e.g., Vozoff, 1958), and dipping beds (e.g., De Gery and Kunetz, 1956). Keller and Frischknecht (1966) presented a model, partially based on the work of Vozoff (1960) for the potential at the surface due to applied current and an arbitrarily shaped inhomogeneity. Zhadanov and Keller (1994) presented an analytical solution, based on Siegel (1959), for the response of electrode array to a spherical inhomogeneity. The method presented captures many of the principles used for the analytic element solution presented by Barnes and Janković (1999), but uses Lagendre polynomials instead of the variable separated power series used by Barnes and Janković (1999). Seigel (1959) presented a series of dimensionless response curves for specific arrays. The solution presented by Zhadanov and Keller (1994) may well be suited for our purpose, but we chose to use the readily available two-dimensional solution through the analytic element method.

Based on Barnes and Janković (1999), Furman et al. (2002b) presented an analytic element based solution for the problem of steady-state flow in a semi-infinite vertical domain composed of cylindrical inhomogeneities in a otherwise homogeneous subsurface. This solution can be written as

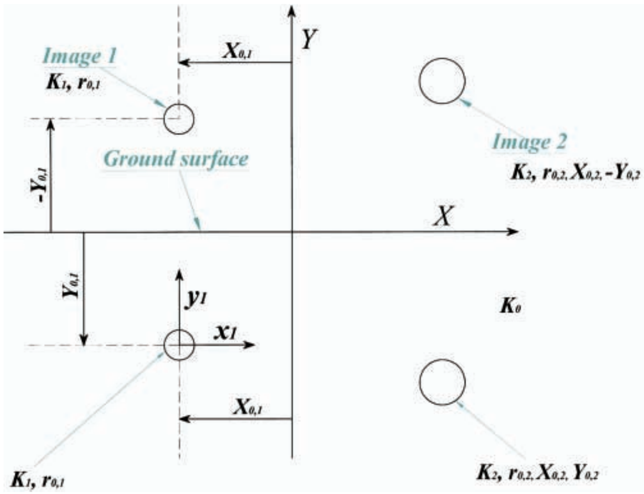

Fig. 1. Schematic diagram of solution domain including two heterogeneities and corresponding images.

$$V = \frac{\Phi_B + \Phi_P + \Phi_I}{K} \quad [1]$$

$$\Phi_B = \frac{q}{4\pi}\ln\left[\frac{(x-s)^2 + y^2}{(x+s)^2 + y^2}\right] \quad [2]$$

$$\Phi_{P|I} = \sum_{n=0}^{N}\lambda\left(\frac{r}{r_0}\right)^{\lambda \cdot n}\left[A_n\cos(\theta n) + B_n\sin(\theta n)\right] \quad [3]$$

$$\lambda = \begin{cases} 1 & r \le r_0 \\ -1 & r > r_0 \end{cases} \quad [4]$$

where V is the electric potential (V); Φ is the flux potential (V Ω^{-1} m^{-1}), equal to the electric potential multiplied by the electrical conductivity, K (Ω^{-1} m^{-1}), which varies by location; x and y are the horizontal and vertical location coordinates; q is the source strength (A), which is equal to twice the current, I (A), applied to the soil through the source electrode; r and r_0 are the distances from the cylinder (or its image) center and the cylinder (or its image) radius, respectively (m); and θ is the angle from the horizontal. The subscript "B" represents the background (homogeneous) solution, "P" represents the perturbation solution, and "I" represents the image cylinder solution, which is needed to accurately simulate the zero conductivity of the above-surface air. Figure 1 illustrates the locations and properties of two cylinders and their two images. The flux potential is calculated for each inhomogeneity in the subsurface (marked P), and for each corresponding image inhomogeneity (marked "I"). A_n and B_n are coefficients to be determined by matching the electrical potential at M points on each of the cylinder interfaces.

Sensitivity of an ERT Array to a Conductivity Perturbation

To form the most efficient set of ERT arrays, we begin by defining the contribution that each array makes to a unique definition of the subsurface properties. One approach is to identify the set of arrays that has the greatest sensitivity to a single perturbation placed at a number of fixed locations in the subsurface. Another approach is to identify those arrays that have the greatest cumulative sensitivity within regions of the subsurface. Both approaches begin with the definition of a response function that is based on the response of an array to a perturbation in an otherwise homogeneous subsurface (Furman et al., 2002a). This response function can then be used directly, or it can be integrated across subsurface regions to define a relative sensitivity index.

We use a two-dimensional analytic solution for electrical flow through a vertical cross section from two surface line sources through a subsurface containing a single electrical heterogeneity that is circular in cross section. This solution defines the response of any ERT array to a small perturbation. The response function (R) is then defined as the difference between the apparent resistivity, ρ_a (Ω m) calculated for the array with, ρ_a, and without, ρ_a^H, the inclusion normalized by the applied current, I:

$$R = \frac{(\rho_a - \rho_a^H)I}{G} \quad [5]$$

The geometric factor, G (m), is specific to each ERT array (m). It is defined such that the ratio of the potential difference measured between the potential electrodes, ΔV, to the current applied through the current electrodes, I, is equal to the true resistivity of a homogeneous subsurface, ρ^H, (Telford et al., 1990).

$$\rho_a^H = \frac{\Delta V^H}{I}G \quad [6]$$

Note that a factor of 2π is embedded in G. If the same current is applied to this array in a heterogeneous medium, the apparent resistivity will be

$$\rho_a = \frac{\Delta V}{I}G \quad [7]$$

Combining Eq. [5] through [7] gives

$$R = \Delta V - \Delta V^H \quad [8]$$

R is therefore the change in the potential measured between the potential electrodes for a given array with a fixed applied current that is caused by a single, small subsurface perturbation. Due to the relative locations of current application and potential measurements, a region of increased electrical conductivity can cause either an increase or a decrease in the measured potential difference, depending on the perturbation location. To reduce the complications due to this polar response, we define the sensitivity, S (V), of an array to a single perturbation as the magnitude of the response:

$$S(\mathbf{P}) = |R| \quad [9]$$

where $\mathbf{P}$ is a vector describing the perturbation properties (i.e., location, size, and relative electrical conductivity).

RESULTS AND DISCUSSION

Single Array Sensitivity Maps

A map of the distribution of measurement sensitivity can be constructed based on the sensitivity of different arrays to single conductivity perturbations located throughout the subsurface. These maps show the distribution of the array sensitivity to the location of a perturbation. This procedure may be repeated using different perturbations (i.e., of different size or relative conductivity). The spatial sensitivity distribution is a property of the array and will not change with the perturbation properties; only the magnitude of the response function will change. The polarity of the response function will change if the polarity of the perturbation changes, but the sensitivity function will remain unchanged and posi-

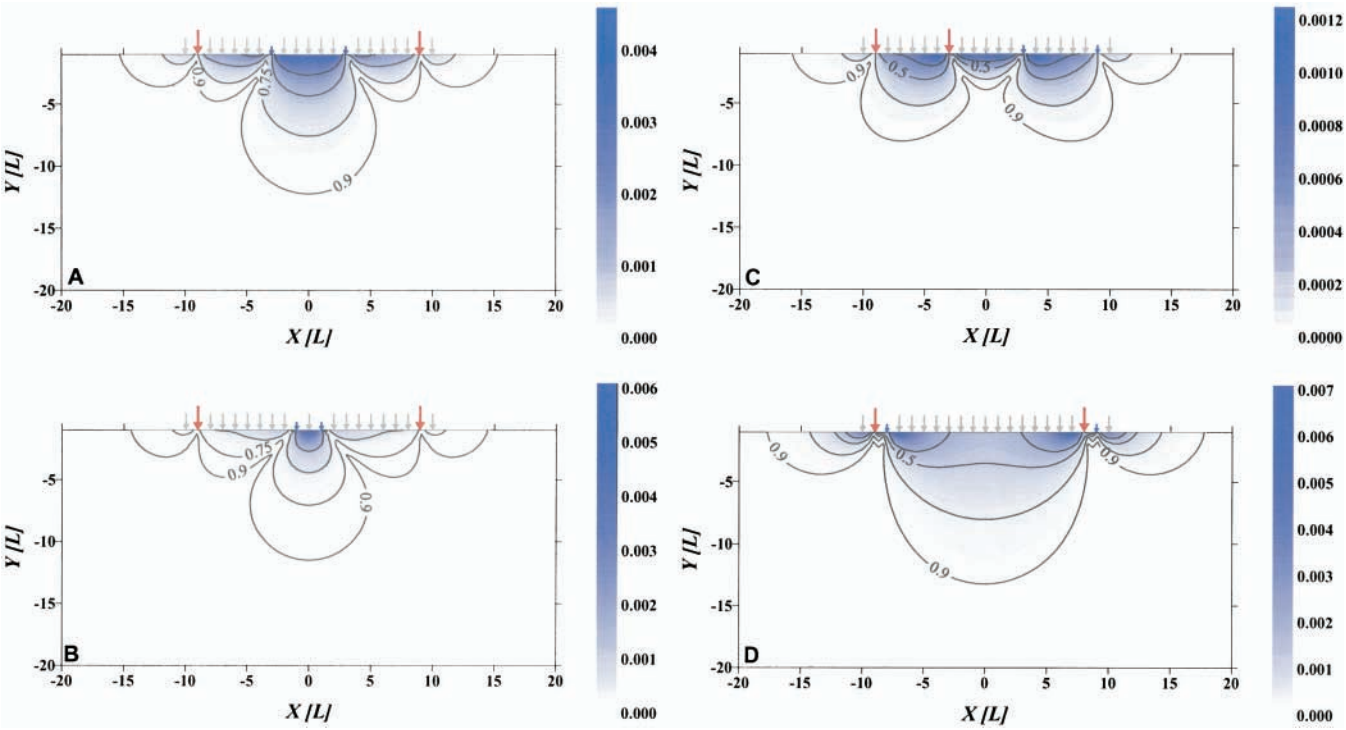

Fig. 2. Sensitivity maps for (A) Wenner, (B) Schlumberger, (C) double dipole, and (D) partially overlapping arrays. Contours of cumulative sensitivity relative to the total sensitivity are shown. All of the electrodes used in simulations for this study are shown as arrows. Long red arrows show current electrode locations and short blue arrows show potential electrodes for specific array for which sensitivity is shown. Note that the sensitivity scale differs among arrays.

tive. Throughout this paper we use a circular perturbation with a diameter of one (i.e., equal to the smallest electrode separation), and a relative electrical conductivity of two (i.e., twice the electrical conductivity of the background).

The cumulative sensitivity, CS, is calculated by

$$\mathrm{CS} = \frac{\sum_{i=1}^{J} S}{\sum_{i=1}^{N} S} \quad [10]$$

where i is a rank index (equal to one for the most sensitive perturbation and equal to N for the least sensitive one), and J is a summation limit.

Figure 2 shows example sensitivity maps for Wenner, Schlumberger, and double dipole arrays and for an atypical array that has partially overlapping current and potential electrode pairs. All 21 electrodes are shown in each figure for ease of comparison. The electrodes used for each array are identified with long red arrows for current electrodes and short blue arrows for potential electrodes. The contours are filled with colors to represent the smooth variation of the sensitivity function with location. While it is difficult to determine the value of sensitivity at any location on these maps, these absolute values are not as important as the distribution of sensitivity throughout the subsurface. To show this distribution more clearly, contours for CS = 25, 50, 75, and 90% are shown. These contours are defined such that, for example, the 25% contour includes the regions of highest sensitivity whose cumulative sensitivity is 25% of the total sensitivity of the array. In other words, only 10% of the measurement sensitivity of the array lies outside of the 90% sample area contour. We use the 90% contour to define the measurement area of an array. The lower percentile contours show the distribution of sensitivity within the sample area. The more closely spaced these contours, the more spatially concentrated the sensitivity within the sample area.

Although the sensitivity distribution is similar for all of the arrays, some significant differences exist. The major difference is the magnitude of the sensitivity. The sensitivity of the double dipole array is much lower than those of the other arrays. The partially overlapping array shows the highest sensitivity. These results apply not only to the four specific arrays analyzed. Rather, they are based on observations of simulations of many different arrays of each array type. We have shown results for arrays with similar dimensions to allow for the most direct visual comparisons. Table 1 presents the percentage of the subsurface area enclosed within the 25, 50, 75, and 90% sample areas (i.e., the CS = 25, 50, 75, and 90% contours) for each array shown in Fig. 2. Note that in addition to having a higher total sensitivity,

Table 1. Percentage of area needed to cover percentile of the total sensitivity for arrays shown in Fig. 2.

% of total sensitivity	25%	50%	75%	90%
	%			
Wenner	1.1	3.2	8.6	20.4
Schlumberger	0.5	2.3	6.9	17.6
Double dipole	1.1	3.1	7.6	17.9
Partially overlapping	1.9	5.9	13.8	28.1

the partially overlapping array has larger sample areas than the Wenner, Schlumberger, and double dipole array types.

Time Constrained Survey Configuration

Because ERT measurements are rapid and nondestructive, the method can be ideal for monitoring transient hydrologic processes. However, this application introduces another constraint on the identification of an optimal survey design. That is, the total number of measurements may be limited. The simplest approach to reducing the number of arrays used in a survey, based on the sensitivity analysis presented in Furman et al. (2002b), is to reduce the discretization of the subsurface, thereby reducing the number of single perturbations used to identify the optimal array set. Another approach, which is commonly employed in designing traditional ERT surveys, is to arbitrarily reduce the electrode arrays considered for inclusion (e.g., only considering Wenner arrays). However, the subset of arrays identified in this manner will, of necessity, be less sensitive at many points than the set that includes all array types. A third approach is to determine which array type gives the most evenly distributed, highest sensitivity on a per measurement basis. Although the arrays identified using this approach will not match the maximum sensitivity at each point in the subsurface, this approach can be used to design a survey that gives a higher overall sensitivity using a fixed, smaller number of measurements. This approach also has the advantage of assessing the distribution of sensitivity of the entire survey, rather than only considering the sensitivity of individual arrays.

None of these approaches is best for all applications. In this study, we compare the results of optimizing ERT arrays by identifying the locally optimal arrays for a reduced number of subsurface points to the traditional method of limiting the survey to include only a single array type.

Survey Sensitivity Maps

The analysis leading to Fig. 2 shows how sensitivity maps can be generated for a single array. To compare different surveys, we need a way to describe the cumulative spatial sensitivity of all of the arrays comprising an ERT survey. To create such a map, we average the sensitivity at each location (X, Y), over all of the arrays that comprise a single survey. For example, if a Wenner survey is composed of 63 arrays, the sensitivity at point (X, Y) on all the 63 sensitivity maps will be summed and divided by 63.

$$\overline{S}(X, Y) = \frac{1}{N_A} \sum_{i=1}^{N_A} S_i\,(X, Y) \qquad [11]$$

where "i" in this case is index for the single array, and N_A is the total number of arrays in a single survey. Cumulative sensitivity contours are calculated for the resulting map as was done for the single arrays shown on Fig. 2.

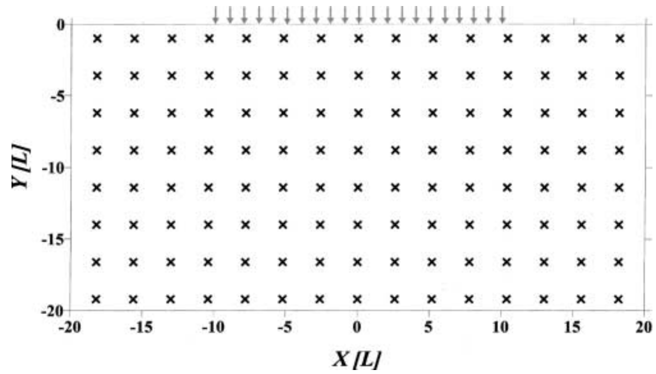

Fig. 3. Location of perturbations used to create survey set based on the locally optimal array. All of the electrodes used in simulations for this study are shown as arrows.

Designing a Survey Based on Sensitivities to a Single Perturbations

Furman et al. (2002b) presented a methodology for determining which array is most sensitive to a single conductivity perturbation. As a first approximation, this method may be used to identify the most efficient set of ERT arrays to sample the subsurface with well-distributed sensitivity. Specifically, the subsurface is divided into a finite number of cells. Then, the array that is most sensitive to a perturbation in each cell is found. These arrays are combined to form the ERT survey. For the case presented by Furman et al. (2002b), the subsurface was divided into 820 cells. All four-electrode combinations that could be formed using 21 electrodes were analyzed. Thus, the maximum number of arrays used (820) was a small fraction of the total number of possible arrays (35 910). In practice, some arrays showed the highest sensitivity to multiple perturbation locations, reducing the number of arrays used to 262. However, even this number of measurements may be too large for monitoring some transient events.

The case shown by Furman et al. (2002b) was chosen to create detailed coverage of the entire subsurface. Here, we restrict the number of perturbations used to match an arbitrary time requirement. For this example we assume that the monitored process requires that a survey be completed every 30 min. If each measurement takes 15 s, this allows for a maximum of 120 measurements per survey. We now locate 120 perturbations evenly through the investigated domain (Fig. 3) and use the locally optimal array approach of Furman et al. (2002b).

In practice, 120 perturbations identify fewer than the optimal 120 arrays, as shown above. In our case, only 49 arrays were identified because some arrays are the optimum choice for more than one heterogeneity location. More perturbation locations could be added to identify 120 optimal arrays. But, this introduces an arbitrary choice into the survey design. Therefore, we use only the 49 arrays identified for further analysis. We then form a cumulative sensitivity for the survey by summing the sensitivities of all 49 arrays at each point in the subsurface. This sum is then normalized by the total number of arrays (i.e., 49 in this case). The result (Fig. 4) is a map of the average sensitivity distribution of this survey on a per measurement basis.

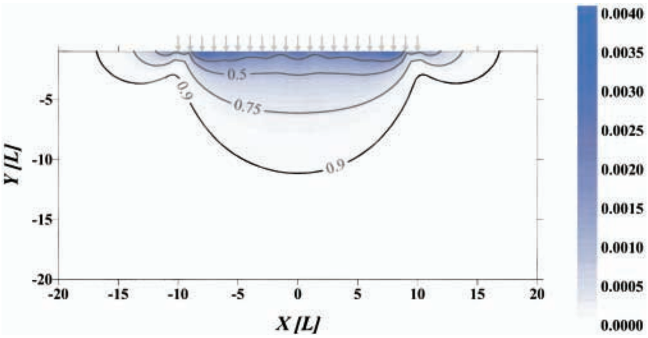

Fig. 4. Normalized cumulative sensitivity map for locally optimal survey set. Contours of percentile contribution to the sensitivity are shown. All electrodes used are shown.

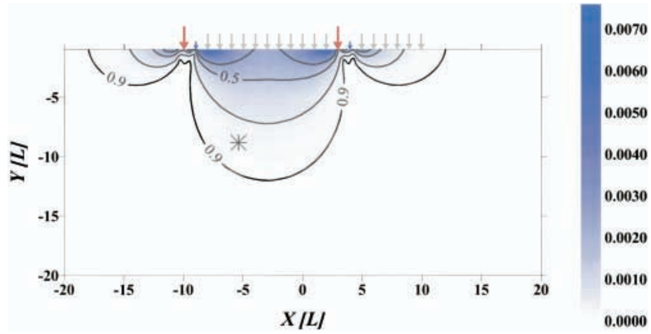

Fig. 5. Sensitivity map for partially overlapping arrays that is the most sensitive to the location marked by *. Contours of cumulative sensitivity relative to the total sensitivity are shown. All of the electrodes used in simulations for this study are shown as arrows. Long red arrows show current electrode locations, and short blue arrows show potential electrodes for specific array for which sensitivity is shown.

Other Survey Types

While the local optimization approach is appealing because it is conceptually simple and because it ensures that each location will be sampled with some sensitivity, it is not obvious that this approach will identify the arrays that lead to a survey with the highest or with the most evenly distributed sensitivity. This is because each array is not sensitive to only one location; it has some sensitivity distribution. For example, Figure 5 shows the sensitivity of a single array that was identified as most sensitive to a single perturbation located at $(X, Y) = (-5.2, -8.8)$ (marked by *). The array sensitivity at that point is 2.72×10^{-4} V. But, the same array has a sensitivity of 7.55×10^{-3} V to the point $(X, Y) = (2.2, -1.0)$. Therefore, while this array was chosen based on consideration of its sensitivity at only one location, it may have a much greater impact on the average sensitivity at other locations in the subsurface. Given that ERT measurements are processed simultaneously to generate a map of electrical conductivity, it would seem that a better approach to identifying the optimal array set would be based on considerations of the cumulative survey sensitivity distribution. A simple criterion for identifying the optimal survey is that which gives the highest cumulative sensitivity per measurement and the largest sample areas, generally leading to a more well distributed spatial sensitivity.

To demonstrate the cumulative sensitivity approach to designing an ERT survey, we consider ERT surveys comprised of only one array type. To compare the cumulative sensitivity of different array types, we begin by summing the sensitivity of many single arrays for each type. Different numbers of unique arrays can be made with a fixed number of electrodes for different array types. For example, using the 21 electrodes shown on previous figures, 63 Wenner, 171 partially overlapping, 172 Schlumberger, and 297 double dipole arrays can be constructed. To avoid biasing the results by arbitrarily choosing among arrays, we used all of the possible arrays of a given type and then normalized the cumulative sensitivity by the number of arrays used. The result (Fig. 6) shows the average spatial sensitivity of each survey type on a per measurement basis. Table 2 lists the percentage of the subsurface area within each sample area contour. Table 2 also includes the same parameters obtained from the locally optimal survey, as shown in Fig. 4, for comparison.

The Wenner and the Schlumberger type surveys show almost identical sensitivity distributions. This is not entirely unexpected, as almost one-third of the Schlumberger arrays are also Wenner arrays. On a per measurement basis, the Wenner survey seems slightly superior to the Schlumberger survey. Therefore, of the two, the Wenner survey should be preferred when survey time is a limitation.

The double dipole survey shows smaller sample areas than the other arrays. Its sensitivity distribution is relatively shallow and the absolute values of the sensitivity are orders of magnitude smaller than those of the other surveys. The atypical partially overlapping survey type shows the largest sample area and the highest average sensitivity (value of 1.3×10^{-4} V compared with 7.06×10^{-5}, 6.19×10^{-5}, and 1.33×10^{-5} V for Wenner, Schlumberger, and double dipole surveys, respectively). These results suggest that, on average, a survey comprised of only partially overlapping arrays is preferred to surveys comprised of Wenner, Schlumberger, or double dipole arrays. These results support the conclusions of Furman et al. (2002b), who found that the partially overlapping arrays were preferred, based on their maximum sensitivity to individual subsurface perturbations, for most regions of the subsurface.

Further examination shows that the survey configuration based on the locally optimal approach shows higher average sensitivity values (7.48×10^{-3}, almost twice that of the partially overlapping survey) and larger sample areas than the single array type surveys. This demonstrates that the inclusion of multiple array types was more important than the analysis of survey sensitivity in identifying the optimal survey design.

CONCLUSIONS

The use of analytic element methods allows for the rapid, accurate analysis of the sensitivity of many ERT arrays to single subsurface electrical conductivity perturbations. On the basis of these analyses, Furman et al. (2002a) defined the sensitivity of ERT arrays to single perturbations and used this definition to identify a set of arrays that gives the greatest sensitivity to point loca-

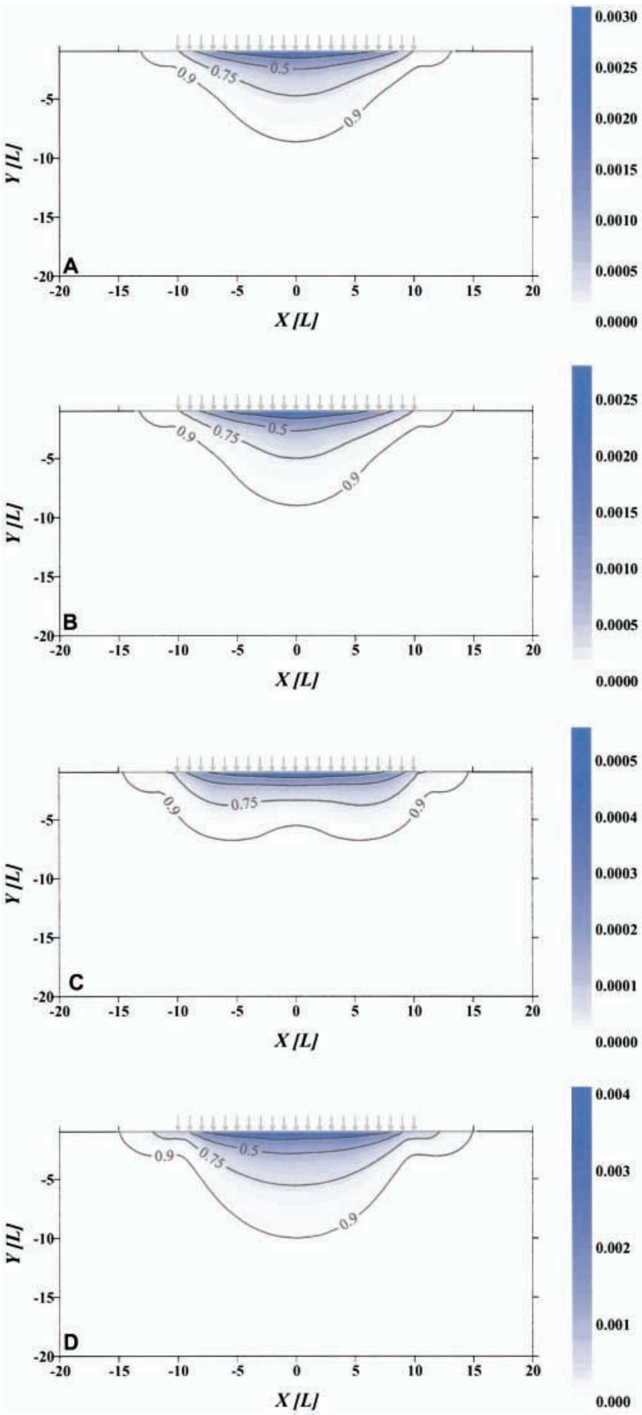

Fig. 6. Normalized survey sensitivity maps for (A) Wenner, (B) Schlumberger, (C) double dipole, and (D) partially overlapping arrays. Contours of cumulative sensitivity relative to the total sensitivity are shown. All of the electrodes used in simulations for this study are shown as arrows. Note that the sensitivity scale differs among arrays.

tions within the subsurface. We extended this analysis to define the spatial sampling areas and sensitivity distributions of three common array types (Wenner, Schlumberger, and double dipole) and an atypical partially overlapping array. The results show that the double dipole array has small sample areas with sensitivity concentrated near the ground surface. The Wenner and

Table 2. Percentage of area needed to cover percentile of the total sensitivity for survey sets shown in Fig. 4 and 6.

% of total sensitivity	25%	50%	75%	90%
		%		
Wenner	0.8	2.3	6.1	14.6
Schlumberger	0.8	2.4	6.3	15.0
Double dipole	0.9	2.5	6.4	15.8
Partially overlapping	1.2	3.3	8.5	19.4
Local optimum	1.6	4.5	11.4	24.8

Schlumberger arrays have very similar sample areas and sensitivities that are more sensitive to deeper materials than the double dipole array. The partially overlapping array has a sensitivity distribution that is similar to the Wenner and Schlumberger array types, but shows higher absolute sensitivities.

Unlike point measurement methods, the sensitivity of the ERT method should be defined for a whole survey that is composed of measurements made with many individual ERT arrays. This analysis is extended to consider the specific sensitivity of surveys comprised of only Wenner, Schlumberger, double dipole, or partially overlapping arrays, as well as to the locally optimal survey set as defined by Furman et al. (2002b). The results show that of the surveys composed of only one array type, the survey composed of partially overlapping arrays gives higher, more evenly distributed sensitivity than those formed with the other, more typical array types. This supports and extends the conclusion of Furman et al. (2002a), who identified the partially overlapping array as preferable based on its maximum sensitivity to a larger number of individual subsurface locations. Furthermore, for the case examined, the locally optimal approach to designing a survey was found to be superior to the traditional approach of limiting a survey to one array type (e.g., Wenner). While it is not known whether this approach is best for all time-limited ERT applications, it serves as a very good starting point for identifying a global approach to survey optimization for monitoring transient hydrologic processes with ERT.

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REFERENCES

Baker, J.M., and R.J. Lascano. 1988. The spatial sensitivity of time domain reflectometry. Soil Sci. 147:378–384.

Barnes, R., and I. Janković. 1999. Two-dimensional flow through large number of circular inhomogeneities. J. Hydrol. 226:204–210.

Binley, A., S.H. Poulter, and B. Shaw. 1996. Examination of solute transport in an undisturbed soil column using electrical resistivity tomography. Water Resour. Res. 37:763–769.

Dahlin, T. 2000. Short note on electrode charge-up effects in DC data acquisition using multi-electrode array. Geophys. Prospect. 48: 181–187.

De Gery, J.C., and G. Kunetz. 1956. Potential and apparent resistivity over dipping beds. Geophysics 21:780–793.

Evjen, H.M. 1938. Depth factor and resolving power of electrical measurements. Geophysics 3:78–95.

Ferré, P.A., J.H. Knight, D.L. Rudolph, and R.G. Kachanoski. 2000.

A numerically based analysis of the sensitivity of conventional and alternative time domain reflectometry probes. Water Resour. Res. 36:2461–2468.

Ferré, P.A., J.H. Knight, D.L. Rudolph, and R.G. Kachanoski, 1998. The sample area of conventional and alternative time domain reflectometry probes. Water Resour. Res. 34:2971–2979.

Ferré, P.A., D.L. Rudolph, and R.G. Kachanoski. 1996. Spatial averaging of water content by time domain reflectometry: Implications for twin rod probes with and without dielectric coatings. Water Resour. Res. 32:271–279.

Furman, A., A.W. Warrick, T.P.A. Ferré. 2002a. Using numerical perturbation analysis to optimize electrode arrays for rapid subsurface monitoring. SAGEEP 2002. Las Vegas, NV. 10–14 Feb. 2002. EEGS, Denver, CO.

Furman, A., A.W. Warrick, T.P.A. Ferré. 2002b. Electrical potential distributions in response to applied current in a heterogeneous subsurface: Solution for circular inclusions. Available at www.vadosezonejournal.org. Vadose Zone J. 1:273–280.

Keller, G.V., and F.C. Frischknecht. 1966. Electrical methods in geophysical prospecting. Pergamon Press, New York.

Knight, J.H. 1992. Sensitivity of time domain reflectometry measurements to lateral variations in soil water content. Water Resour. Res. 28:2345–2352.

Knight, J.H., P.A. Ferré, D.L. Rudolph, and R.G. Kachanoski. 1997. The response of a time domain reflectometry probe with fluid-filled gaps around the rods. Water Resour. Res. 33:1455–1460.

Loke, M.H. 2000. Electrical imaging surveys for environmental and engineering studies: A practical guide to 2-D and 3-D surveys. Available at http://www.agiusa.com/index.shtml (verified 13 May 2003).

Roy, A. 1972. Depth of investigation in Wenner, three-electrode, and dipole-dipole DC resistivity methods. Geophys. Prospect. 20:329–340.

Roy, A., and A. Apparao. 1971. Depth of investigation in direct current methods. Geophysics 36:943–959.

Seigel, H.O. 1959. Mathematical formulation and type curves for induced polarization. Geophysics 29:547–565.

Spies, B.R. 1989. Depth of investigation in electromagnetic sounding methods. Geophysics 54:872–888.

Storz, H., W. Storz, and F. Jacobs. 2000. Electrical resistivity tomography to investigate geoelectrical structure of the earth's upper crust. Geophys. Prospect. 48:455–471.

Telford, W.M, L.P. Geldart, and R.E. Sheriff. 1990. Applied geophysics. 2nd ed. Cambridge Univ. Press, Cambridge, UK.

Vozoff, K. 1958. Numerical resistivity analysis: Horizontal layers. Geophysics 23:536–566.

Vozoff, K. 1960. Numerical resistivity interpretation: General inhomogeneity. Geophysics 25:1184–1194.

Zhadanov, M.S., and G.V. Keller. 1994. The geoelectrical methods in geophysical exploration. Elsevier, New York.

Zhou, Q.Y., J. Shimada, and A. Sato. 2001. Three-dimensional spatial and temporal monitoring of soil water content using electrical resistivity tomography. Water Resour. Res. 37:273–285.