
Language of the Universe

Math Fundamentals

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issue details

built: February 2, 2019 08:43:34 PM

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Preface

Math is the definitive, yet human-perceivable way of describing the universe. We can express any feature of any object in math terms. Even when no concrete numbers are taken, we can still use variables denoting features and create a model to depict real-world events. Furthermore, laws of math exist in the abstract as well, even without being put in context of real-world practical calculations. Some even say this is a proof of a divine design.

Whether or not this is true, mathematics has the potential to expand minds and inspire the creator which lies in all of us. Once you acquire the confidence to produce and perceive mathematical models in topics that concern you, you will experience a moment of empowerment and wisdom. Or what we often refer to as “a-ha moment”.

This alone is a good reason to study math, but there are more. Have an upcoming exam, aim to solve a more complex problem in your job, fill in knowledge gaps or just leverage your general culture. My hope is this book will make a good friend on your way of learning math.

About the book

The book covers chosen mathematical topics which I believe to be essential and lay a good foundation before more specialized literature. The book aims to build up your confidence in math by explaining wide variety of topics. Each chapter is self sufficient and can be looked up individually. Use the book as a structured mini-reference.

About the reader

The wide public and students will equally benefit of this reading. Whether you need a different aspect on material you met before or introduction to new stuff, I got you covered. Either way you get a gentle walk-through the topics we cover here.

Math is often seen to be hard. Lots of people say they “hate” it and believe they are just “not a math person”. While we all have our differences, most of those who give up on math fall victims of known bad practices in teaching, learning or both. Some give up if the “a-ha” moment doesn’t come when they expect it, when it’s just around the corner. It just breaks my heart. My biggest reward is to see such a wandering soul finding the path they’re looking for.

My promise to the reader is that everybody can learn math to the extent he needs. There’s no such thing as “not-a-math-person”. Your no less than “a-person” and persons can learn math. I know it because I’m a person myself and I did it. There’s nothing alien or cryptic about it. Just some concentrated effort on some well put material.

About the author

I have been developing software my whole consent life. I chased a career and then delved into entrepreneurship. All of my work until recently has been unrelated to math beyond basic arithmetics, equations and formulas.

With AI (Artificial Intelligence) and Data Science taking major role in the industry it become necessary to step up my math skills. I took individual classes and researched on my own. At first I was unsure, even scared. I was uncomfortable with math since I was a

kid. But in time I came to know math is as reachable as you are dedicated.

I have proved this to work for me in many areas like guitar playing, motorcycling and sports. Recently I discovered teaching to be a new passion of mine. I have been counseling friends and mentoring younger colleagues for a while now. This book is my first attempt to approach this matter directly. My sincere hope is to help my readers' succeed in their learning path and know I have a small part of it.

Learning and reading

We learn our whole life. Yet learning itself as a skill is often undervalued. The way you consume and make use of information determines the pace you will keep, volumes you will reach and time you will remember what you learned. This is why it's important to have at least some of idea of some good learning practices.

Read enough – critical volume

Human brain is good at storing abstract ideas rather than technical facts. But to extract a model from a set of facts, you need a certain minimum level of perceived information. Reading and writing itself is unnatural to humans as it presents a single line of information unlike drawing. It's said "a picture is worth a thousand words" for that reason – the picture contains a lot of facts available to the brain simultaneously. To obtain this with reading you to be prepared with patience. You just need to read a lot to gain that critical volume of facts for your brain to form conclusions, models and associations.

Read often – habit

By reading often you will get used to learn and get into learning mode faster and easier. Associations will build up in your mind that will connect with the material you're learning. It will create "context" for your mind to work in, which will increase your focus. Reading often gives you regular chances to get the grasp on the material.

Imagine if your being asked out of the sudden about a dear friend of yours who you haven't heard from for ages. You need to think about it for a while but you can recall. But if you're asked about a distant acquaintance who you barely know but see very often – you will recall the little you know about him immediately.

It's also a good habit and habits don't cost will power.

Also try sleeping over the material you've read also helps is move from the short-term to the long-term memory. This gives you time for the material to sink in.

Take notes – mind associations

Take notes to formalize associations you created while learning. They will get you in context and will walk you through the whole material they cover very fast. While they make sense mostly to you, your notes should be what makes most sense to you at all.

Mind that taking notes is a skill you build with practice. In time you will develop better feeling for what helps you recall later on. Even brief and chaotic notes can be very telling and even more so when reorganized after the learning session is over.

Re-read material – consolidate

Reading again something you've already covered, even briefly, strengthens the associations you built with the material at a very low cost. Re-reading books (or even re-watching movies) will expose you in different mood and state to the same material and give you opportunity to extract conclusions beyond the facts.

This also helps with material you don't yet understand. Just running your eyes through the pages every now and then will give you an opportunity to notice different things each time. First, you'll remember the layout, then the logical structure, then basic facts and so on until you "click" and you will progress before you know it. Don't necessarily set your goal to get everything this time. Aim to get "something". This way you build up knowledge each time, you don't burnout from struggling, you don't feel guilty for not progressing as much as you wished and you feel safe and stimulated to try again.

Introduction

On notation (find a suitable place for the topic)

The first thing that scares people off of math is weird looking symbols, the notation.

Notation is necessary because it brings your ideas unambiguously to any reader. Imagine if you there were no unified terminology and you had to describe common facts in your words:

A figure which consists of four lines, each connected with next one in either end, in a such a way there are two pairs of parallel lines and each pair of connected lines builds a 90° angle.

Parallel means ...

The above text describes a *square*.

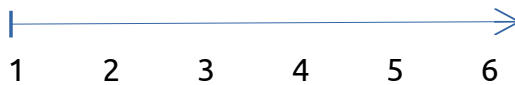
Notation also allows to squash complex equations and formulas into meaningful letters and signs which uniquely identify the content and allow you to consider them in the abstract.

Notation is well worth your effort in learning and getting used with it. It's the human dialect of Language of the Universe.

Basic Algebra

Numbers

We will start by describing numbers and operations we can do with them. You should be familiar with the number line:



Those are the numbers we use to count things in nature. 1 motorcycle, 100 000 000 dollars. That's why we call them *natural numbers* and denote them like so:

$$N = \{ 1, 2, 3, 4, 5, 6, 7, \dots \}$$

Figure 1: Natural Numbers

When you own stuff you can express it with natural numbers.

To express that you have none of something you need the zero. When you add zero to natural numbers you get the *whole numbers*:

$$W = \{ 0, 1, 2, 3, 4, 5, 6, \dots \}$$

If you borrow a book from your friend and you lose it then you not only have none of that book (*0 books*), but you owe the one you lost. You have *-1 books*. To denote this we use the *integers*:

$$Z = \{ \dots, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, \dots \}$$

Add fractions and you get the *rational numbers*. Note fractions should be computable. I.e. $\frac{22}{7}$ is good, but 3,1428571428... is not.

$$Q = \{ \dots -2, -1, -\frac{1}{7}, 0, 1, 1.75, 2, \frac{22}{7} \dots \}$$

Most common set we use are the *real numbers*. They include all of the above plus numbers with endless decimal part like Pi (approximately equal to 3.14):

$$R = \{ \dots, -1, 0, 1, \sqrt{(2)}, \pi, 4, \dots \}$$

Variables

Equations

Inequality

Fractions

Algebraic rules

Functions

The Cartesian plane

Quadratic equations

Exponents

Logarithms

Polynomials

Trigonometry

Complex numbers

Basic Geometry

Triangle

Square

Cube

Circle

Ellipse

Hyperbola

Sphere

Cone

Pyramid

Linear Algebra

Linearity

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Popular Math Topics

Probability theory

Number Theory

Graph Theory

Combinatorics

Constants and Algorithms

Pi

Fibonacci Chain

Golden Ratio

Useful Everyday Math

In everyday life people often say “math” when they really mean “arithmetics”. We can make us of much more than arithmetics to reveal answers to important questions and make decisions.

Loans and Investments

Car Fuel Efficiency

Triangulation

Linear Predictions

Artificial Intelligence

Problems

Appendixes

Math Notation

Set Notation

Vector and Matrix Notation

Complex Numbers Notation

Calculus Notation