Master of Computer Application (MCA)

MCAC-102: Discrete Mathematics

Unique Paper Code: 223401102

Semester: I

March 2021

Year of admission: 2020

Time: 3 Hours **Max. Marks: 70**

Instructions for the Students:

Attempt any 4 out of 6 questions. All questions carry equal marks.

1. a. Consider a class of "Discrete Mathematics" having n students. The instructor has given a major exam to the class and wishes to evaluate the class with grades O, A, B, C, D and F. The instructor is happy if there are at least eight students receiving the same grade. What is the minimum number of students required in the class to fulfill the above condition?

   b. Find the coefficient of x^{1986} in the generating function $G(x) = \frac{1}{1+4x}$

   c. Let $f(n) = \begin{cases} n^3 & 0 < n < 5{,}000 \\ n^2 & n \geq 5{,}000 \end{cases}$ and $g(n) = \begin{cases} n & 0 < n < 50 \\ n^3 & n > 50 \end{cases}$

   Is $f(n)$ is $O(g(n))$. If yes then find the value of constants C and n_o otherwise justify it.

2. a. In a "data mining" class having 20 students, the teacher wants to analyze the result at the end of the semester. She prepares the following tally of marks obtained by the students:

< 40	5
>= 40 and < 50	3
>= 50 and < 60	6
>=60 and < 70	4
>=70 and < 80	2

   (i) While sending the data over the network, the tally is compressed so that minimum number of bits are sent over the network. Use the Huffman's algorithm to generate the optimal binary prefix codes.

(ii) With the help of above table decode the message and find the minimum cost (in terms of bits) for the pattern 10110110000011011.

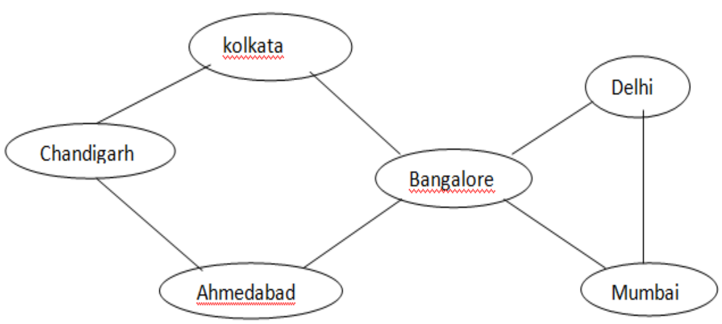
b. Let $Q(x, y)$ denotes $2x + y = 7$
where the domain of variables x and y consists of all integers.
Is it true i) $\forall x \exists y (Q(x, y))$ ii) $\exists x \forall y (Q(x, y))$. Justify?

c. Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5\ and\ a_2 = 15$.

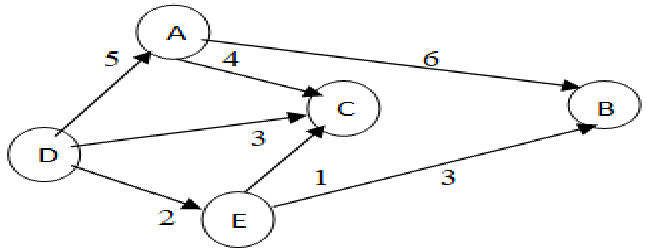
3. a. Arrange the following functions in increasing order of their rate of growth
i) $n^{7/4}$ ii)$nlgn$ iii) n^{lgn} iv) $\sqrt{n}$

b. Use mathematical induction to prove that
$$1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

c. A company has its head office in Delhi and branch office in various cities of the country. The offices are connected by the following air network (an edge shows a direct flight between the two cities at its end points). A salesman needs to visit all the branch offices of the company, starting from and returning to its head office. Is it possible for the salesman to plan his tour so that no city is visited more than once? Give the tour if "yes" justify your answer otherwise. Is it possible for him to plan his tour if he is allowed to visit a city more than once? Give the tour if "yes" justify your answer otherwise.



4 a. Run the Dijkstra's algorithm on the network given in the figure starting from vertex s=D. Show the steps in running the algorithm, you do not need to draw the graph repeatedly, just write which is the next vertex picked and which labels get updated from what value to what value at each step.



Does it work for CD= -5? Justify.

b. Negate the following statement $\forall x \exists y \big(P(x,y) \wedge Q(y)\big)$

c. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 5x + 3$ *and* $g(x) = 3x + 5$ what is the composition of f and g (*fog*) , and g and f (*gof*)?

5. a. Consider the following district map of Delhi. Is it four colorable? If yes, assign the color to each region else give the chromatic number and it's coloring.

b. Obtain the conjuctive normal form and disjunctive normal form for the following statement $(Q \rightarrow P) \wedge (\sim P \wedge Q)$

c. Draw the HASSE diagram and determine whether the POSETs ({1,2,3,4,6,8,12},|) and ({1,3,9,27,81},|) are lattices. Justify the answer.

6. a. Show that K_6 is a nonplanar graph.

b. Using the generation function, find a_n in which $a_0 = 2$ *and* $a_{n+1} = 3a_n, \forall\, n \geq 0$

c. Solve the recurrence relation $2T\left(\frac{n}{2}\right) + \sqrt{n}$ using the master's theorem.