Negative Binomial Distribution

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	BINOMIAL	NEGATIVE BINOMIAL
1	Fixed number of n trials.	The number of trials, n is not fixed.
2	Each trial is independent.	Each trial is independent.
3	Only two outcomes are possible.	Only two outcomes are possible.
4	Probability of success (p) for each trial is constant.	Probability of success (p) for each trial is constant.
5	A random variable Y= the number of successes.	A random variable Y= the number of trials needed to make r successes.

Reference: Guru Ji Ki Discrete Variable wali Slide me Page No. 36

NEGATIVE BINOMIAL

Negative binomial distribution is a probability distribution of number of occurences of successes and failures in a sequence of independent trails before a specific number of success occurs.

Following are the key points to be noted about a negative binomial experiment.

- The experiment should be of x repeated trials.
- Each trail have two possible outcome, one for success, another for failure.
- Probability of success is same on every trial.
- Output of one trial is independent of output of another trail.
- Experiment should be carried out until r successes are observed, where r is mentioned beforehand.

Negative binomial distribution probability can be computed using following Formula :

$$f(x; r, P) = {}^{x-1}C_{r-1} \times P^r \times (1-P)^{x-r}$$

Where:

x = Total number of trials.

r = Number of occurences of success.

P = Probability of success on each occurence.

$1-P$ = Probability of failure on each occurence.

Exception And Variance :

$$p(X = x|p, r) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \text{x = r, r + 1, ...}$$

$$E[X] = \sum_{x=r}^{\infty} x \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

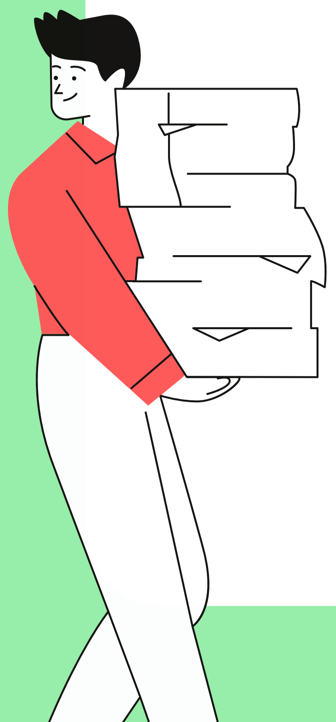
$$E[X] = \sum_{x=r}^{\infty} x \frac{(x-1)!}{(x-r)!\,(r-1)!} p^r (1-p)^{x-r}$$

Let, $k = x - r$

$$E[X] = \sum_{x=r}^{\infty} \frac{x!}{(x-r)!\,(r-1)!} p^r (1-p)^{x-r}$$

Let, $k = x - r$

$$E[X] = \sum_{k=0}^{\infty} \frac{(k+r)!}{k!\,(r-1)!} p^r (1-p)^k$$



Exception And Variance :

$$E[X] = r\sum_{k=0}^{\infty}\frac{(k+r)!}{k!\,r!}p^{r}(1-p)^{k}$$

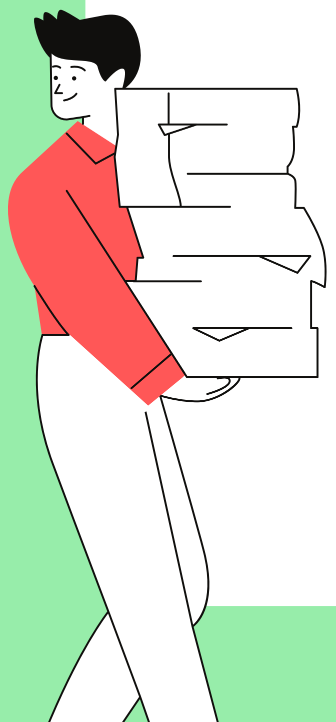
$$E[X] = rp^{r}\sum_{k=0}^{\infty}\binom{k+r}{k}(1-p)^{k} \qquad \binom{-\beta-1}{k} = (-1)^{k}\binom{k+\beta}{k}.$$

$$E[X] = rp^{r}\sum_{k=0}^{\infty}\binom{-(r+1)}{k}(-1)^{k}(1-p)^{k}$$

$$E[X] = rp^{r}\sum_{k=0}^{\infty}\binom{-(r+1)}{k}(-1)^{k}(1-p)^{k}$$

$$E[X] = rp^{r}\left(1-(1-p)\right)^{-(r+1)}$$

$$E[X] = rp^{r}(p)^{-(r+1)} = \frac{r}{p}$$



Exception And Variance :

$$Var[X] = \frac{r(1-p)}{p^2}$$

Example

Ms. Sharарti is a gajab ki professor. Her success rate of giving assignment is 70%. What is the probability that Ms. Shararti give her third assignment in her fifth class ?

Here,

probability of success, P is 0.70.

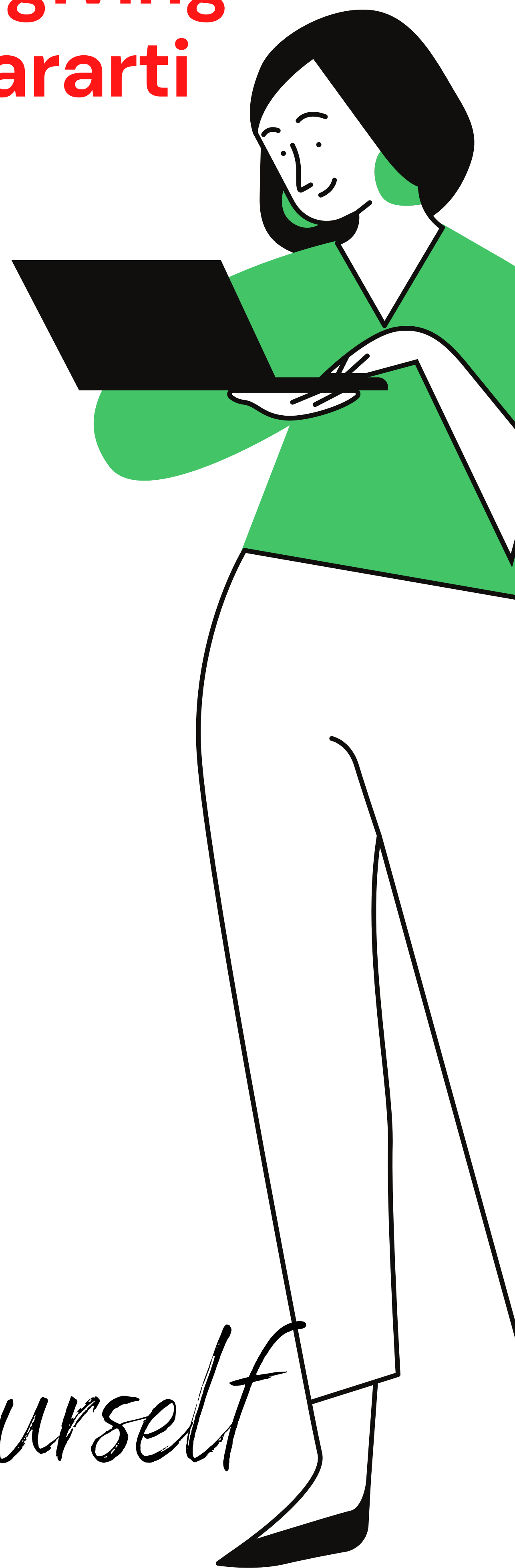
Number of trials, x is 5 and Number of successes, r is 3

$$f(x; r, P) = {}^{x-1}C_{r-1} \times P^r \times (1-P)^{x-r}$$

$$\implies f(5; 3, 0.7) = {}^4C_2 \times 0.7^3 \times 0.3^2$$

$$= 6 \times 0.343 \times 0.09$$

$$= 0.18522$$



Explore Yourself

Thank You!

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