Master of Computer Application (MCA)

MCAC-301: Design and Analysis of algorithms

Unique Paper Code: 223401301

Semester: III

December 2021

Year of admission: 2020

Time: 3 Hours **Max. Marks: 70**

Instructions for the Students:

Attempt any 4 out of 6 questions. All questions carry equal marks.

1. a. Consider a variation of binary search algorithm; The instructor wants to search a number in a sorted array of size n by dividing it into two parts of size 2n/3 and n/3. Find the recurrence for the running time for the best and worst case scenario.

   b. Consider an algorithm A with run time O(n). What is the condition on A for it to be usable as the intermediate sort in Radix sort? Is it possible to use quick sort as intermediate sort. Justify your answer.

   c. Suppose you are given an array A having 8 integers {5, 3, 8, 2, 7, 1, 6, 4}. Draw a randomized binary search tree (BST) using the randomized Quicksort. Why ordinary quick sort is not good to draw a randomized BST. Justify your answer.

2. a. A naive approach to multiply 2 n-digit integers takes O(n^2) time.. Give an algorithm to do the same, in time strictly less than n^2 asymptotically.
   Show all the steps of the above algorithm to multiply 123456 and 654321.

   b. Given the following jobs, their deadlines and associated profits -

Jobs	J1	J2	J3	J4	J5	J6
Deadlines	5	3	3	2	4	2
Profits	200	180	190	300	120	100

   Write the optimal schedule that gives maximum profit? Are all the jobs completed in the optimal schedule? What is the maximum profit earned?

c. Suppose you are given an array of n integers. The students want to search the minimum and maximum elements in the array. Suggest a best searching algorithm for it and also compute the minimum number of comparisons (In worst case, the number of comparisons should be less than 2n-2).

3. a. The students from Algorithms class want to sort the inputs drawn from a uniform distribution [0, 1). As an instructor, how would you argue whether count sort would be a good choice or not. Justify your answer.

b. Consider a undirected graph with a vertex set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Entry Wij in the matrix W below is the weight of the edge {i,j}.

W =

0	7	3	∞	12	∞	∞	∞	∞
7	0	∞	4	∞	13	∞	∞	∞
3	∞	0	8	9	5	∞	∞	∞
∞	4	8	0	∞	15	∞	∞	∞
12	∞	9	∞	0	1	∞	∞	∞
∞	13	5	15	1	0	∞	∞	∞
∞	∞	∞	∞	∞	∞	0	2	6
∞	∞	∞	∞	∞	∞	2	0	10
∞	∞	∞	∞	∞	∞	6	10	0

Find the minimum spanning tree(MST) for the above adjacency matrix. Also mention whether Prim's algorithm or Kruskal's algorithm would be your first choice in above matrix W. Justify your choice. Suppose you increase the weight on each edge by five in the graph then comment whether the MST will remain the same or not. Justify your answer.

c. Let QA denote the Quick sort algorithm to sort integers in non-decreasing order using last element as pivot, C1 and C2 be the number of comparisons made by QA for the given inputs {12, 34, 45, 60, 72, 80} and {40, 10, 50, 30, 60, 20} respectively. What will be the values of C1 and C2?

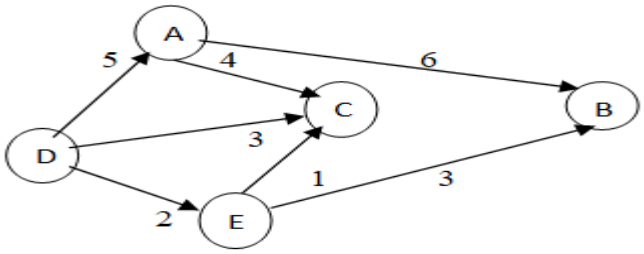
4 a. Consider an instance of subset sum in which w1 = 1, w2 = 2, w3 = 3, w4=5 and W = 9. Draw the table of opt(i, w) values computed by dynamic programming. For the subset sum problem, if we iteratively build a table for Opt(i, w), it takes O(n*W) time and space , namely the size of the table. What is the time and space complexity if we used recursion instead?

b. A thief enters a house for robbing it. He can carry a maximal weight of 10 kg into his bag. There are 4 items in the house with the following weights and values. What items should the thief take if he either takes the item completely or leaves it completely so as maximize the value?

Item	Weight (Kg)	Value ($)
Mirror	5	10
Silver nugget	4	40
Painting	6	30
Vase	3	60

Write the running time complexity of the algorithm you used to solve the above problem. Is the algorithm polynomial-time bound? Justify the answer.

5. a. Run the Dijkstra's algorithm on the network given in the figure starting from vertex s=D. Show the steps in running the algorithm, you do not need to draw the graph repeatedly, just write which is the next vertex picked and which labels get updated from what value to what value at each step.



Does it work for DC= -5? Justify.

b. Let $f(n) = \{n^4 \quad 0 < n < 2{,}000\ ; \quad n^{log(8)} \quad n \geq 2{,}000$
and
$g(n) = \{n^5 log n \quad 0 < n < 1000\ ; \quad n^{sqrt(4)} \quad n \geq 1000$

Is $f(n)$ is $O(g(n))$. If yes then find the value of constants C and n_o otherwise justify it.

c. Sorted list is a best case for insertion sort. Give another best case input for it and give arguments to justify your answer.

6. a. State True or False. Justify your Answer.
(a)Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial-time reducible to S and S is polynomial-time reducible to R. Then R is NP-hard.
(b) Assuming $P! = NP$, then $NP - hard\ ! =\ NP$.

b. Solve the recurrence relation $T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{2n}{5}\right) + 5n$ using the recursion tree method.

c. Compute the Prefix function for the following pattern using the KMP string matching algorithm.

$$abefabefgabefg$$