Theorem:

A connected graph G is a tree $\Longleftrightarrow$ it is acyclic.

A tree with n vertices and m edges has $m = n-1$

(ie)

A tree with n vertices has (n-1) edges
$|E(G)| = |V(G)| - 1$

Proof:

Proof using induction on n

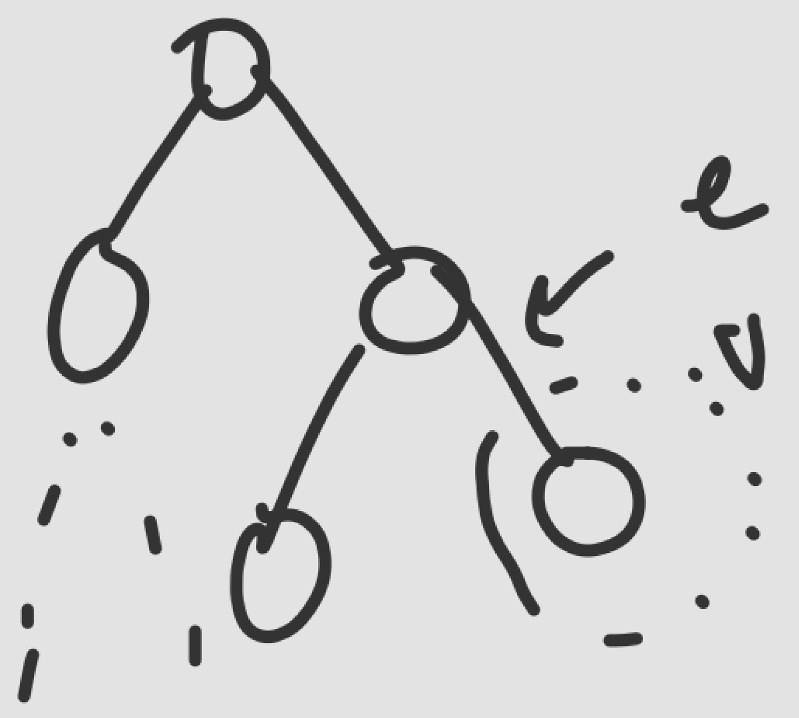
$n = 1$

$m = 0$ (n-1) holds

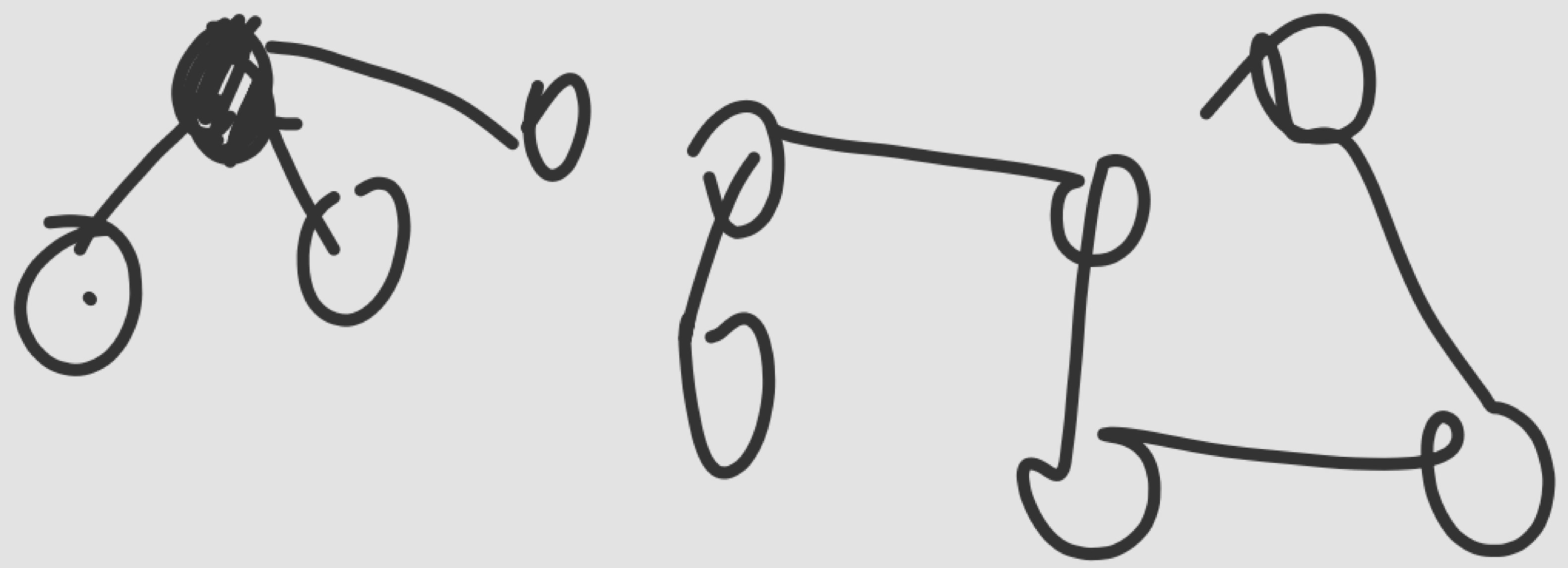
Induction hypothesis

It is true for a tree with n vertices. for $n > 1$

Let T be a tree with (n+1) vertices



Any tree with ≥ 2 vertices has atleast 2 leaves (deg 1)



If we remove vertex v with degree 1

The resulting tree T' has n vertices

& $(n-1)$ edges

$$T' = T - v$$

No of edges in T = no. of edges in T' + 1

$(n+1)$

$$= (n-1)+1$$

$$= n$$

Acyclic G

Connected components

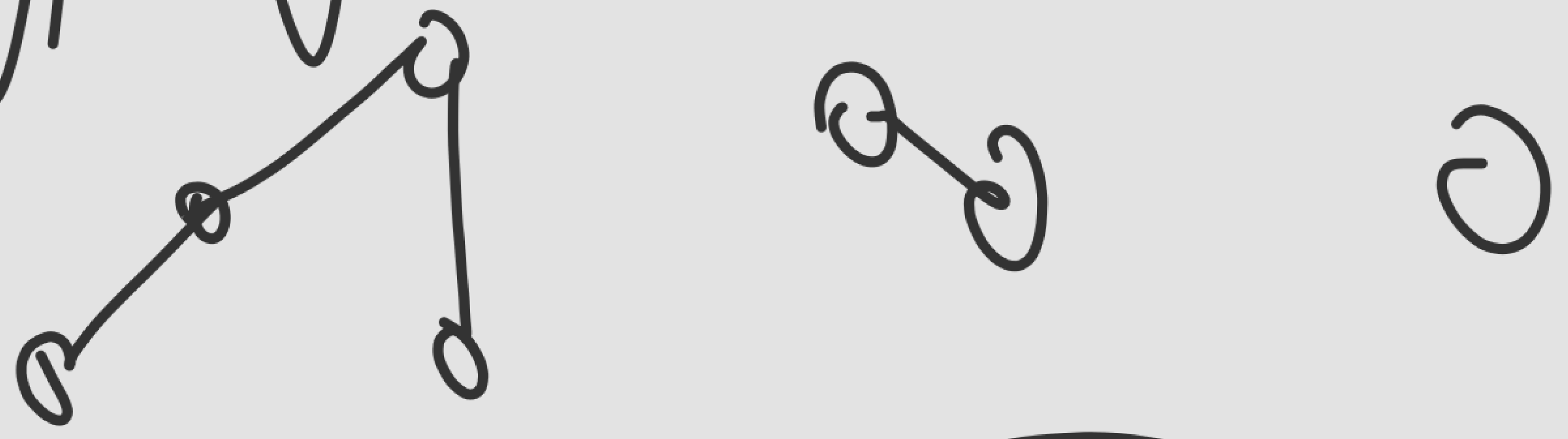
$$\cancel{\sum_{v_i \in V(G)} \deg(v_i)}$$

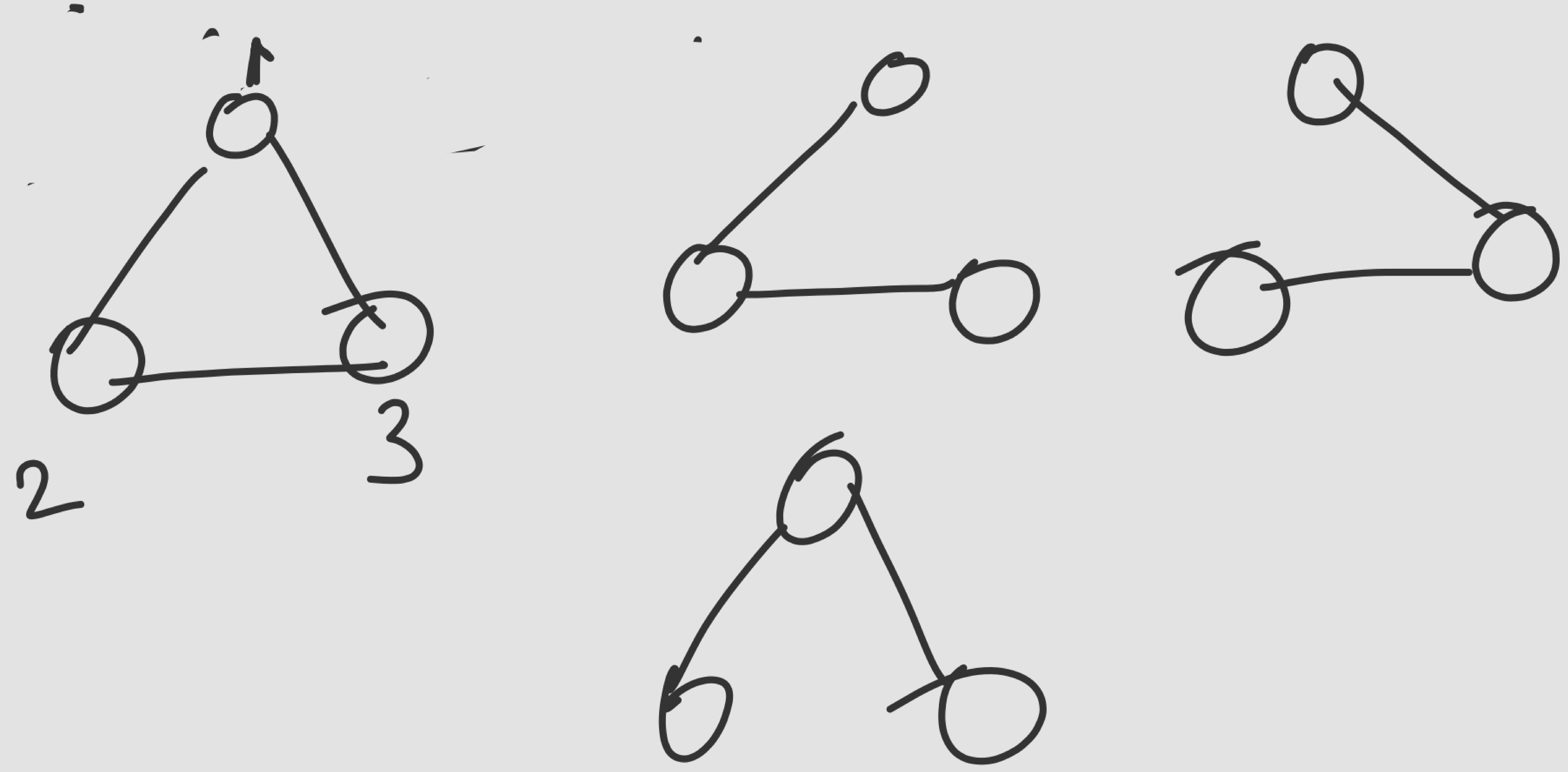
$$\cancel{\sum m_i}$$

$$\sum m_i = \sum_{k} (n_i - 1)$$

$$= \sum n_i - \sum 1$$

$$\boxed{m = n - k}$$





Complete Graph

n Vertices

n^{n-2}

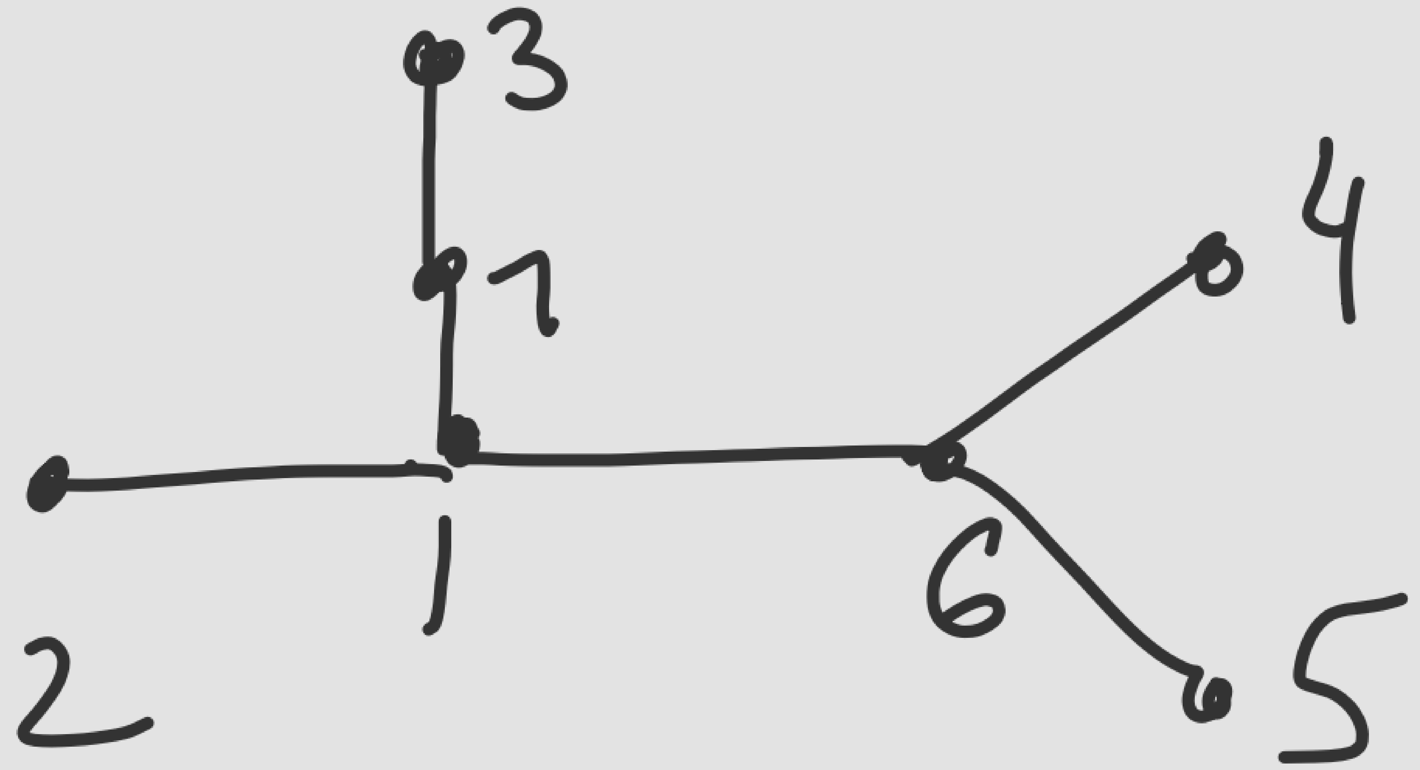
Prüfer Sequence

labelled Tree T on n vertices $\xleftrightarrow{\;1:1\;}$ $\underbrace{\sqcup\ \sqcup\ \sqcup\ \sqcup}_{\text{length } (n-2)}$

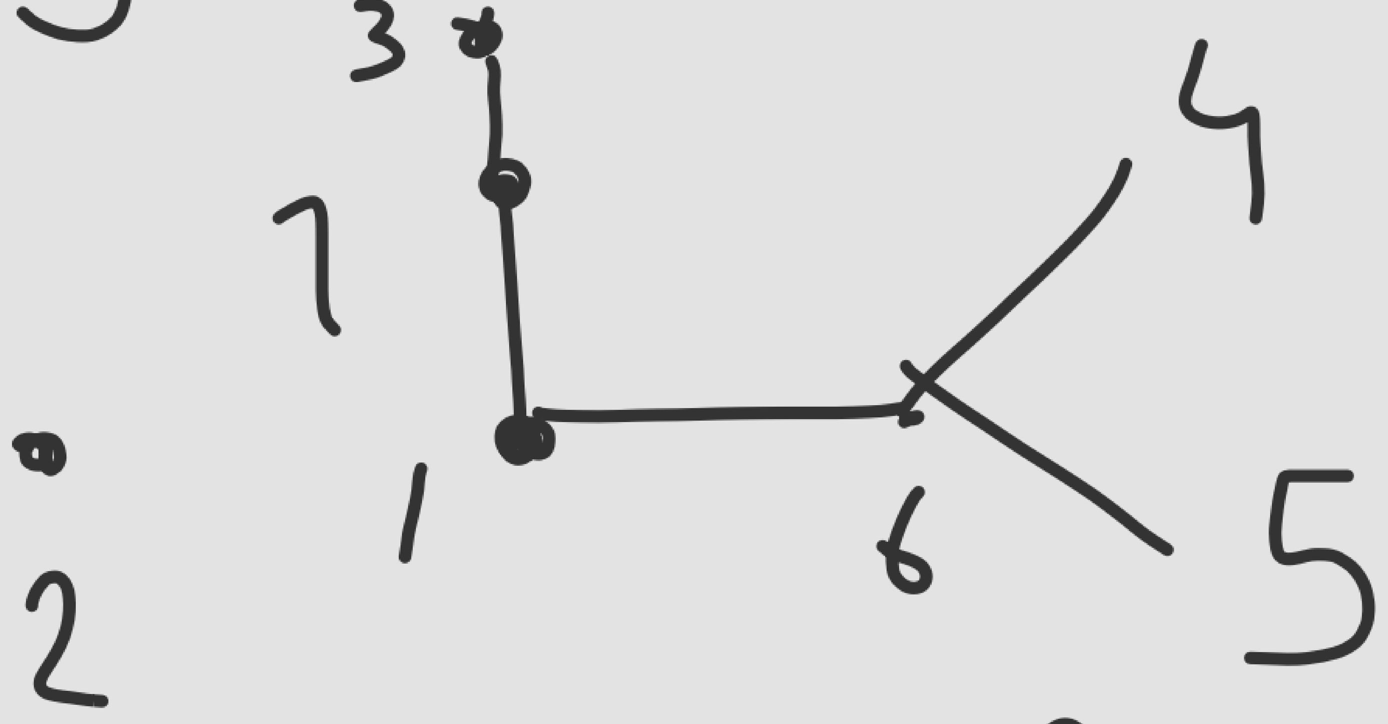
1:1 correspondence

$\underbrace{n \times n \times \ldots\ldots}_{(n-2) \text{ times}}$

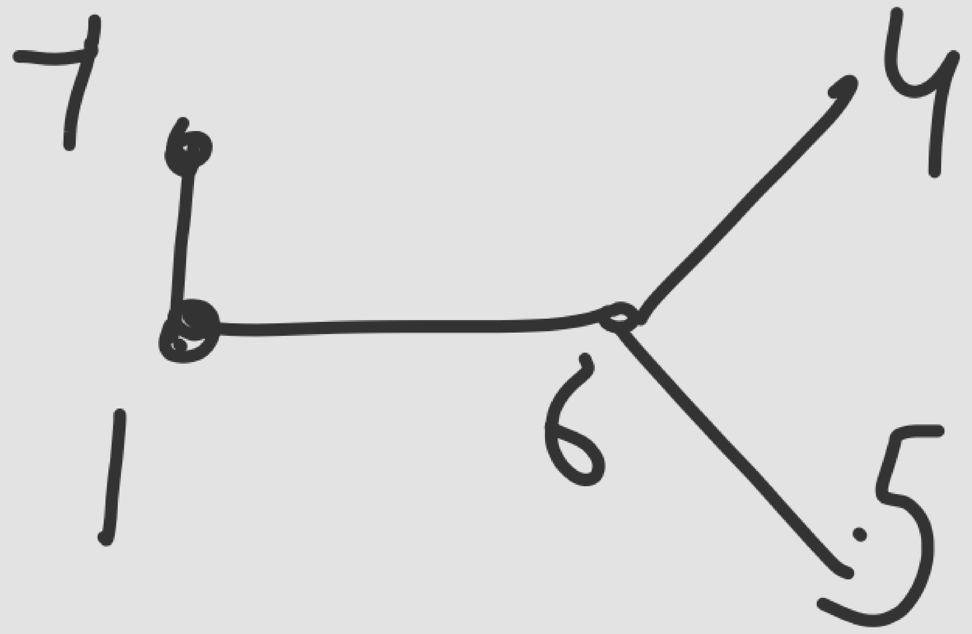
$= n^{n-2}$



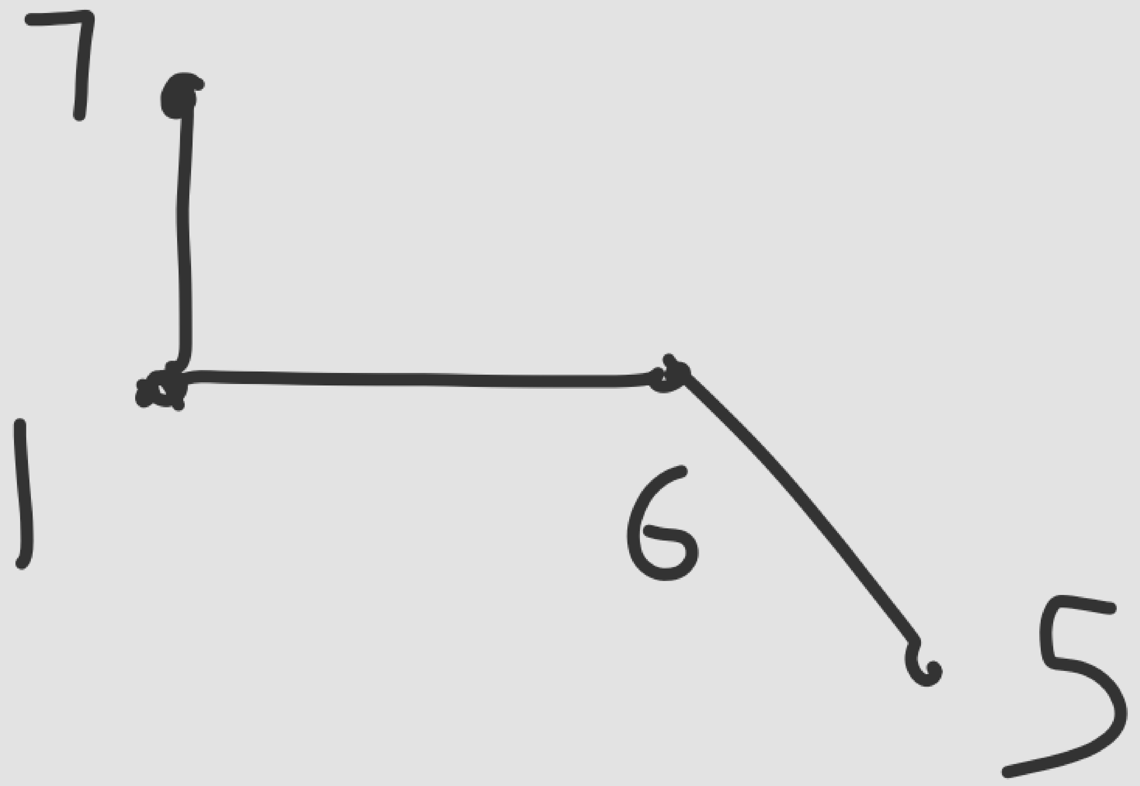
$S = (1)$



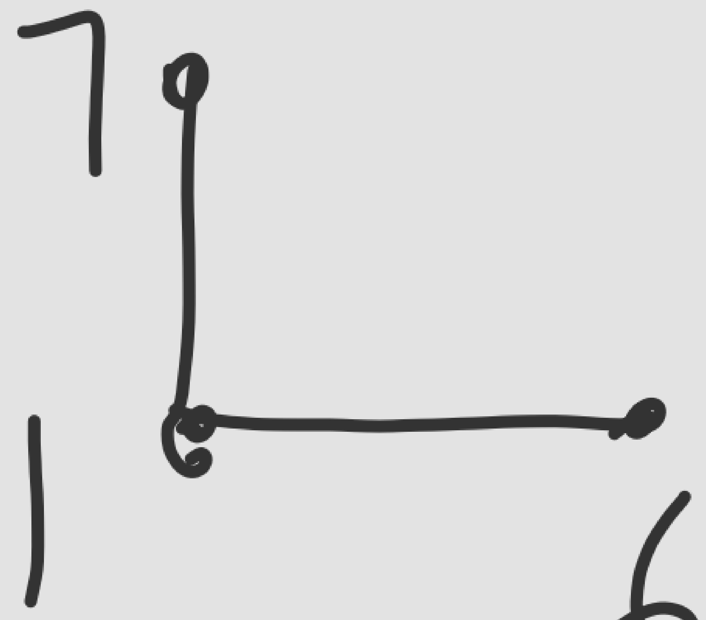
$S = (1,7)$



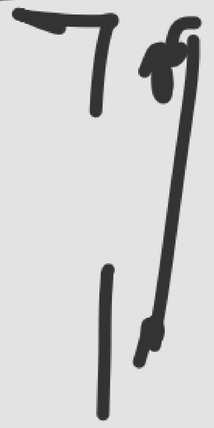
$S = (1,7,6)$



$S = (1,7,6,6)$



$\boxed{S = (1,7,6,6,1)}$



K_2 ← stop →

$S = (\cancel{7}, 7, 6, 6, 1)$

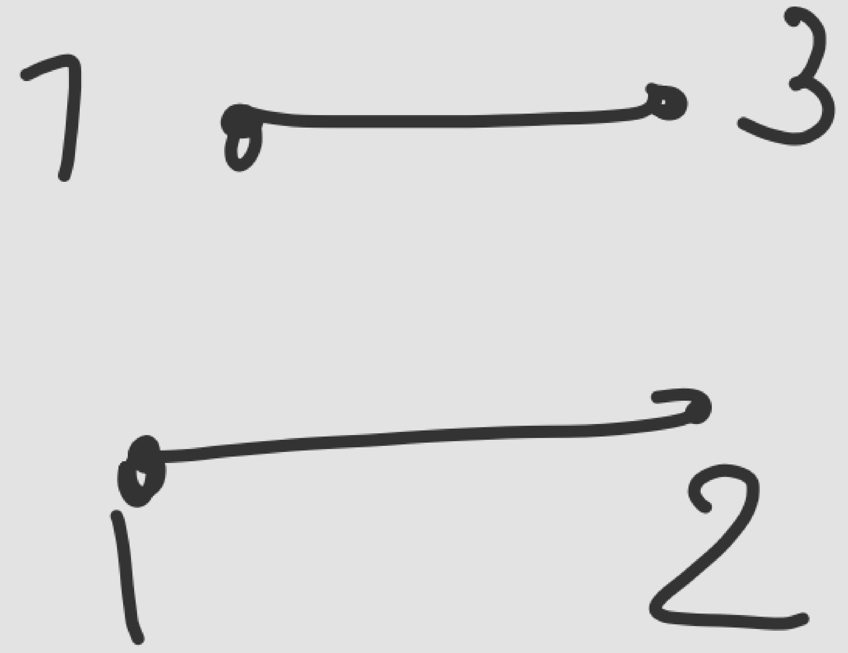
$L = (1, 2, 3, 4, \cancel{5}, 6, 7)$

$x = 2$

$y = 1$

$S = (7, 6, 6, 1)$

$L = ($



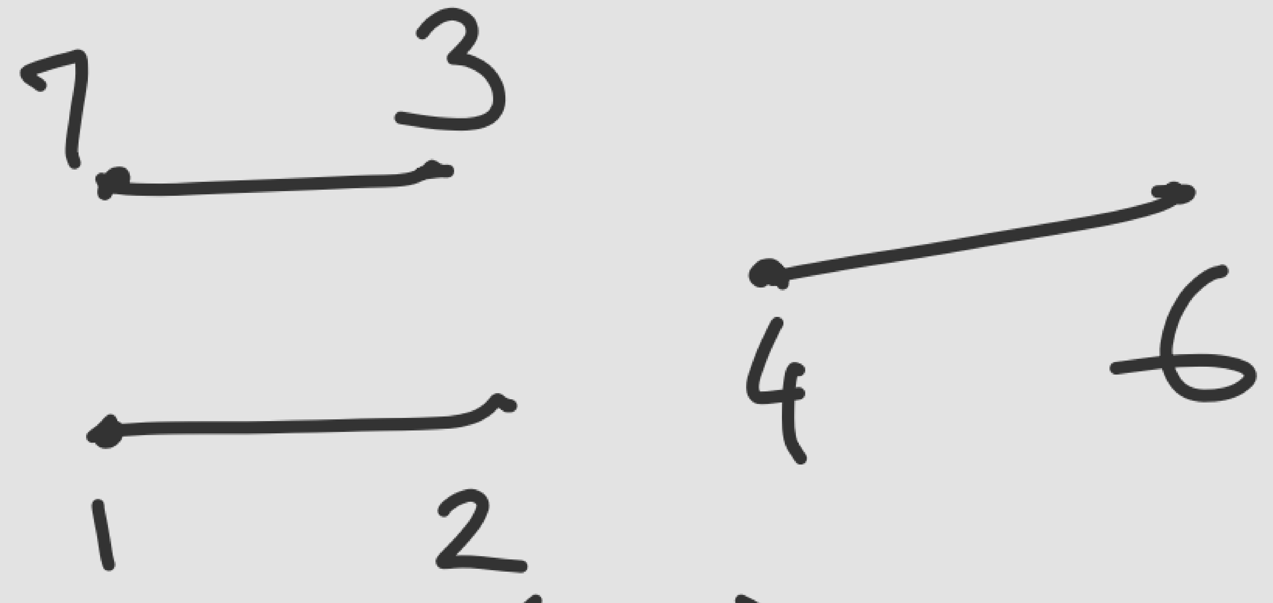
$\boxed{\begin{array}{l} S \to T \\ T \to S \end{array}}$

$S = (6, 6, 1)$

$L = (1, 4, 5, 6, 7)$

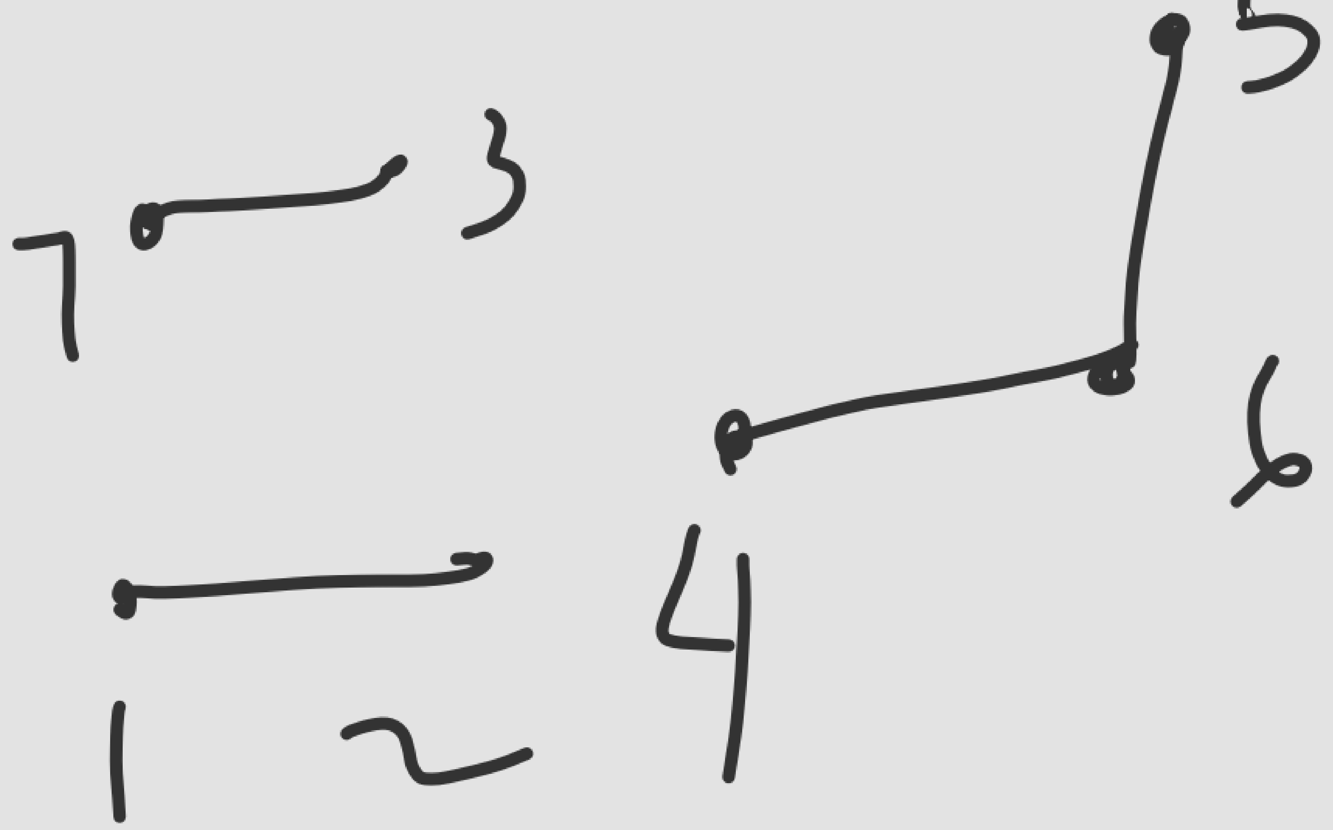
$x = 4$

$y = 6$



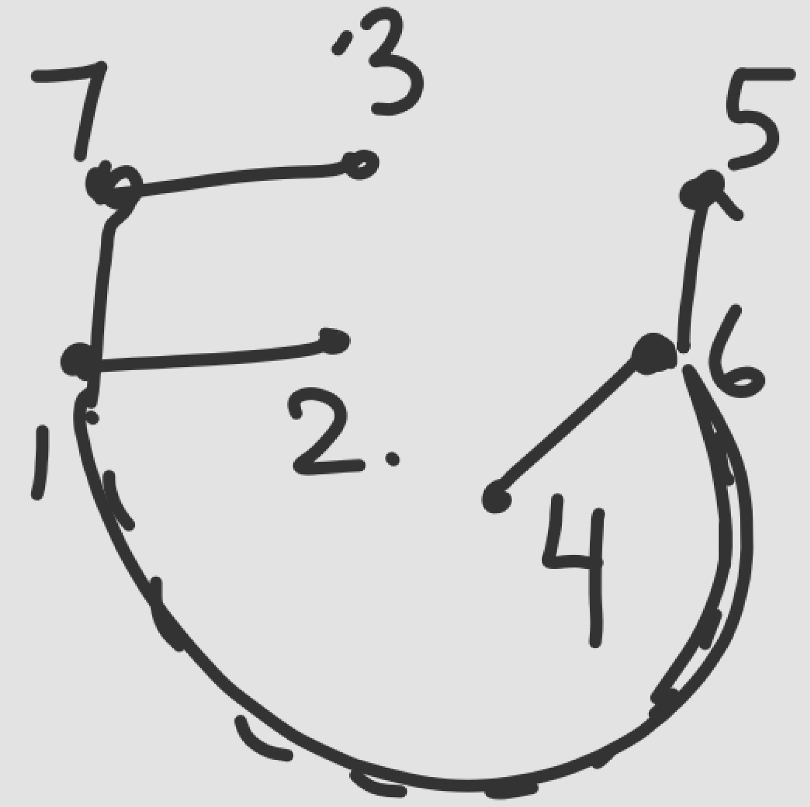
$S = (6, 1)$

$L = (1, 5, 6, 7)$



$S = (1)$

$L = (1, 6, 7)$



$S = (\ )$

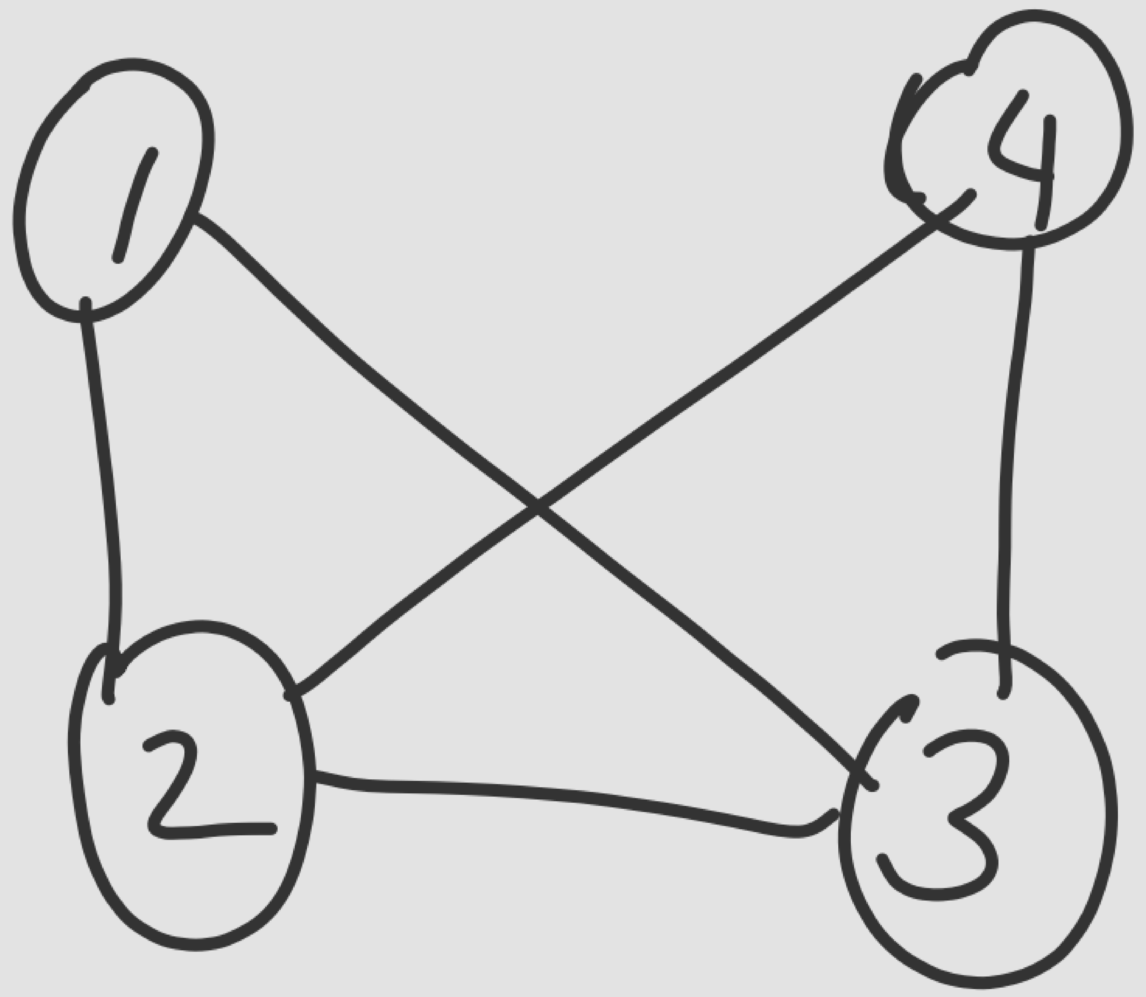
$L = (1, 7)$

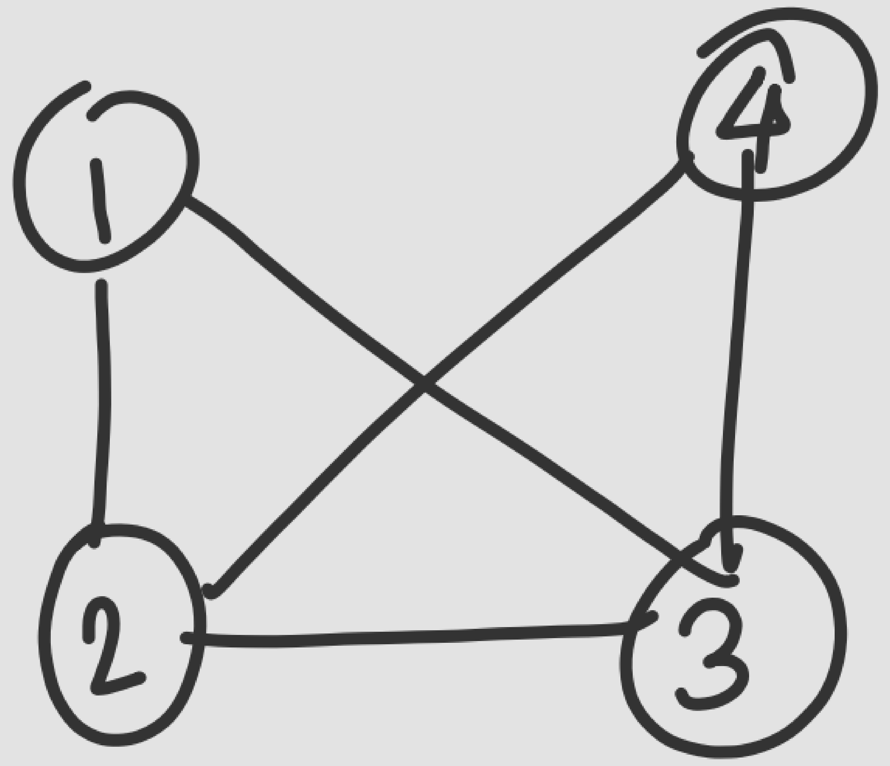
of Spanning Trees

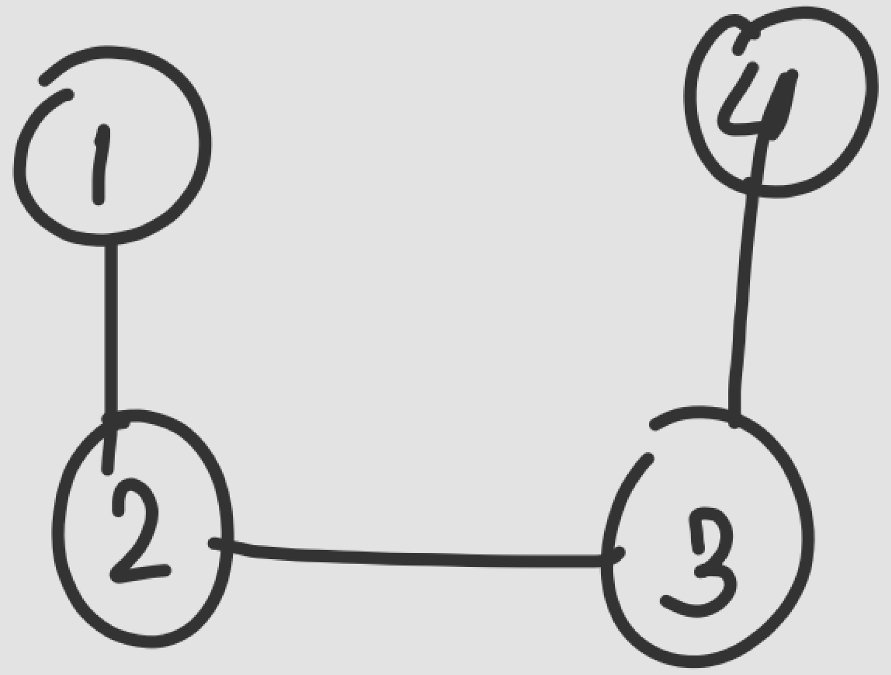
Complete

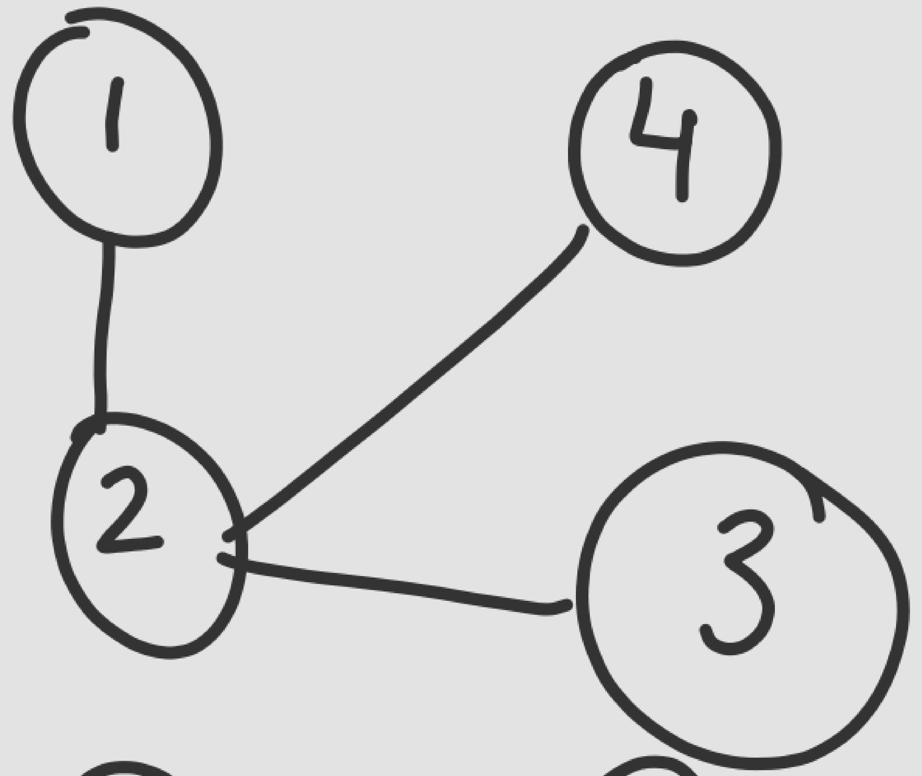
n^{n-2}

not complete




1
4
2
3


1
4
2
3


1
4
2
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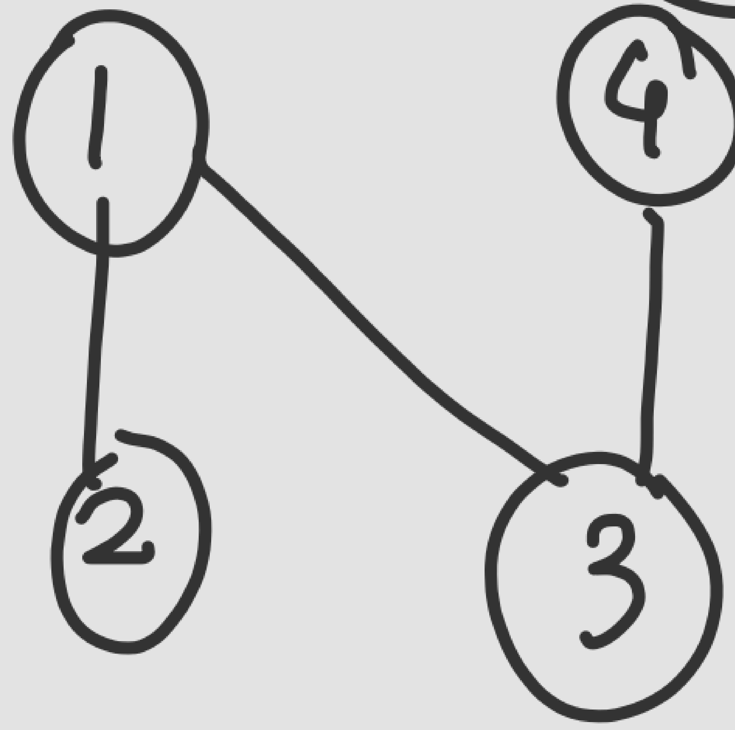
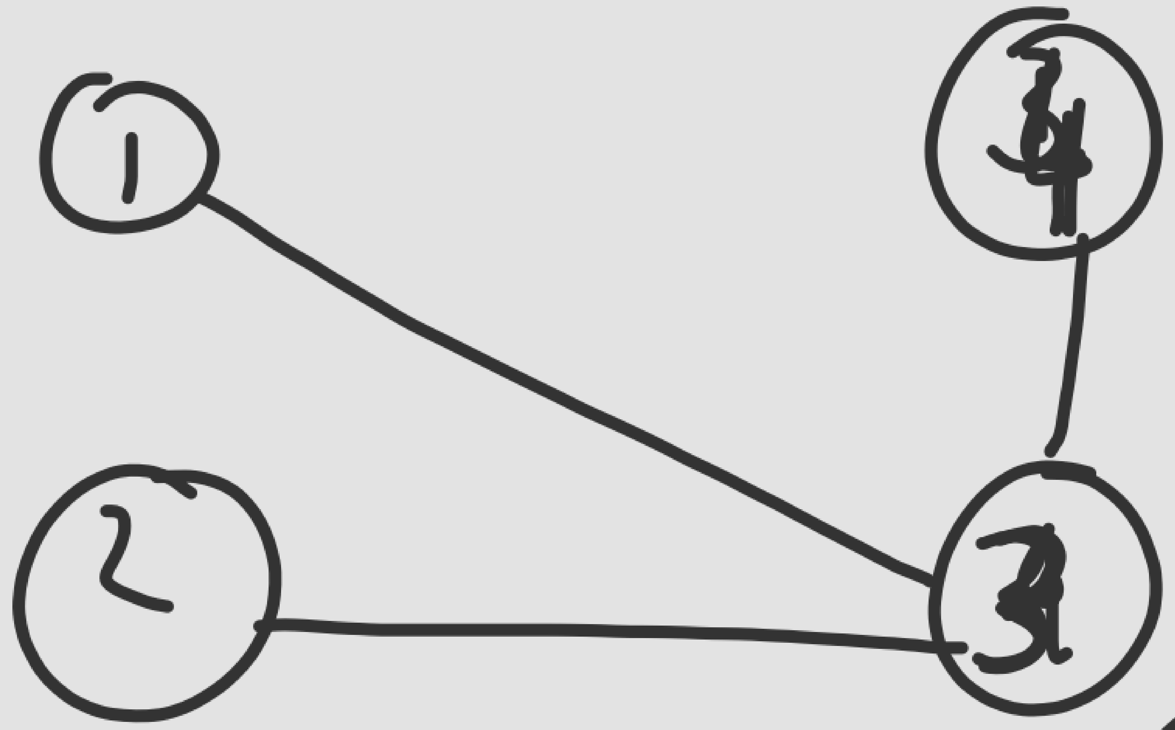
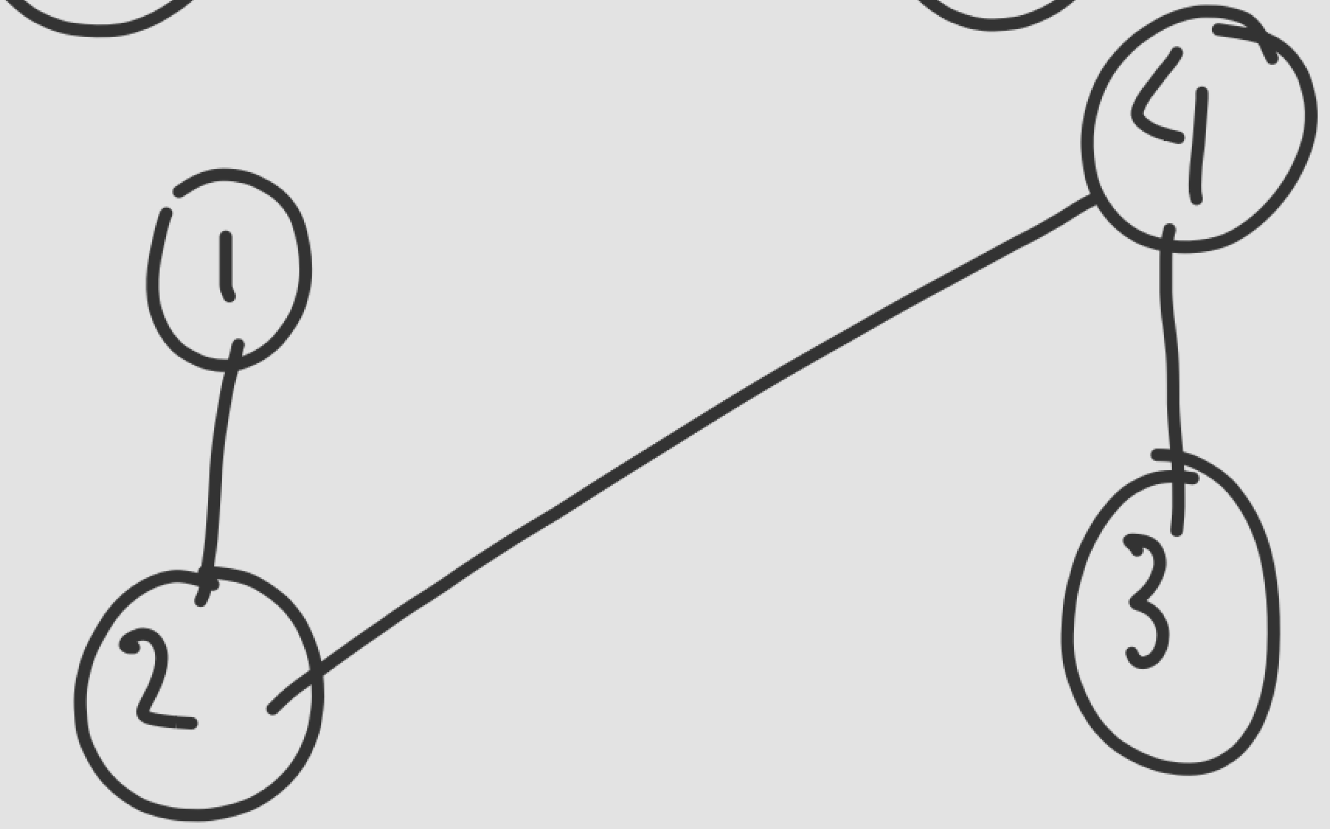
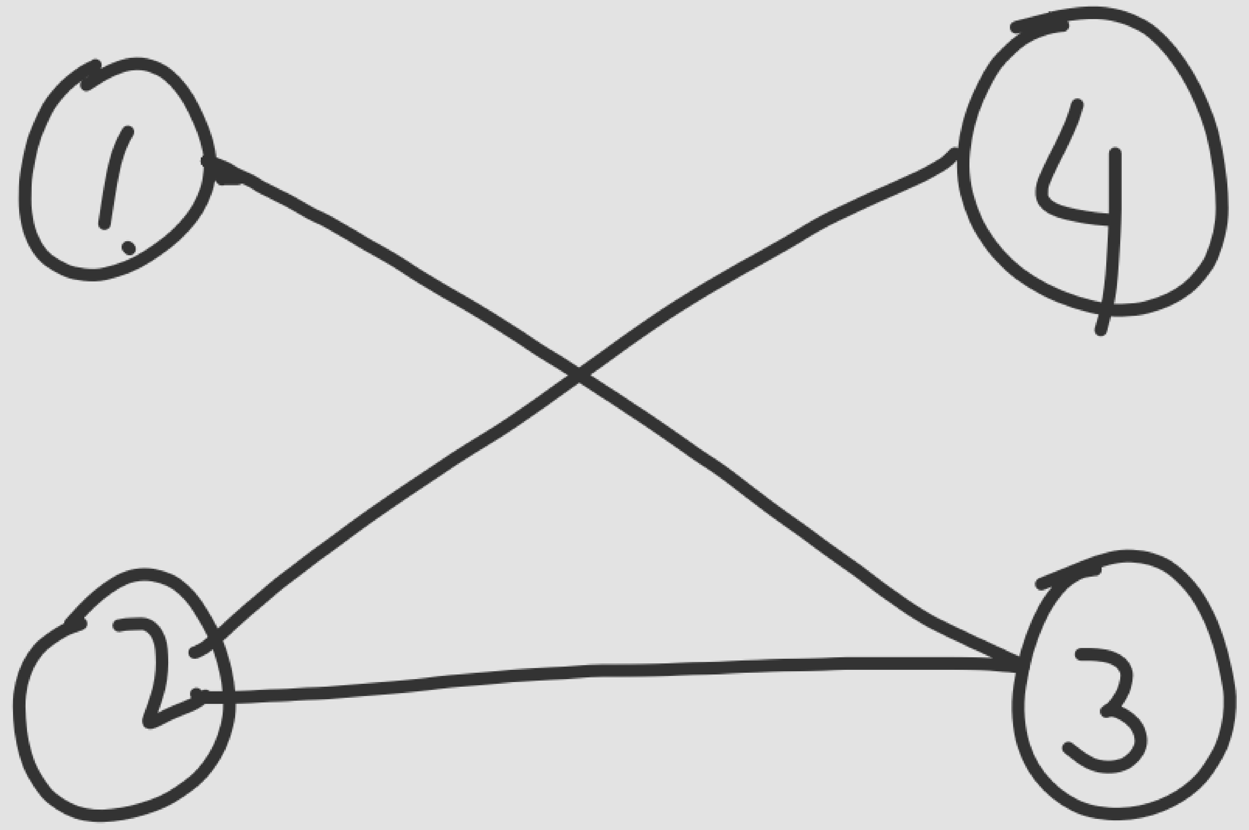
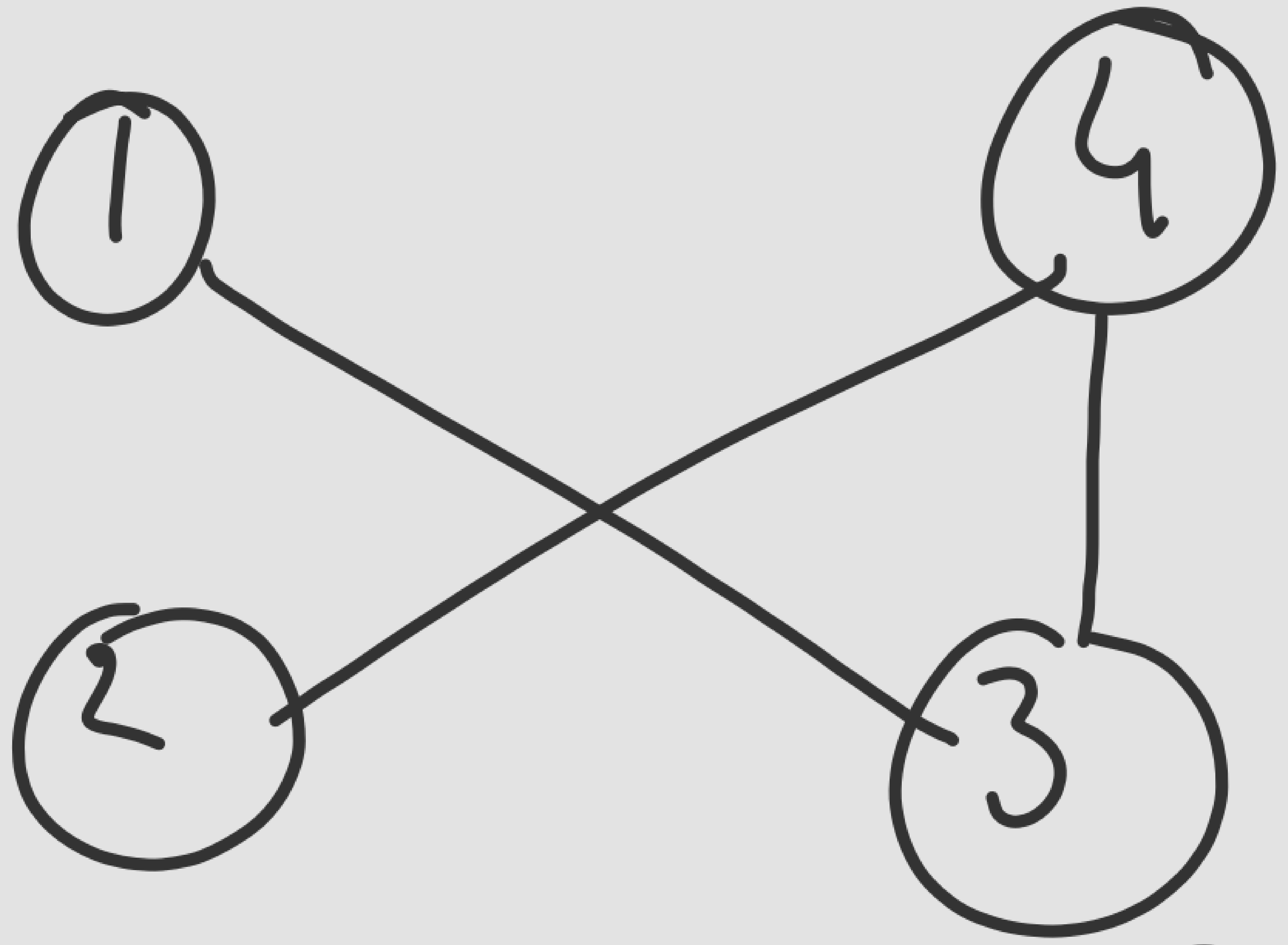
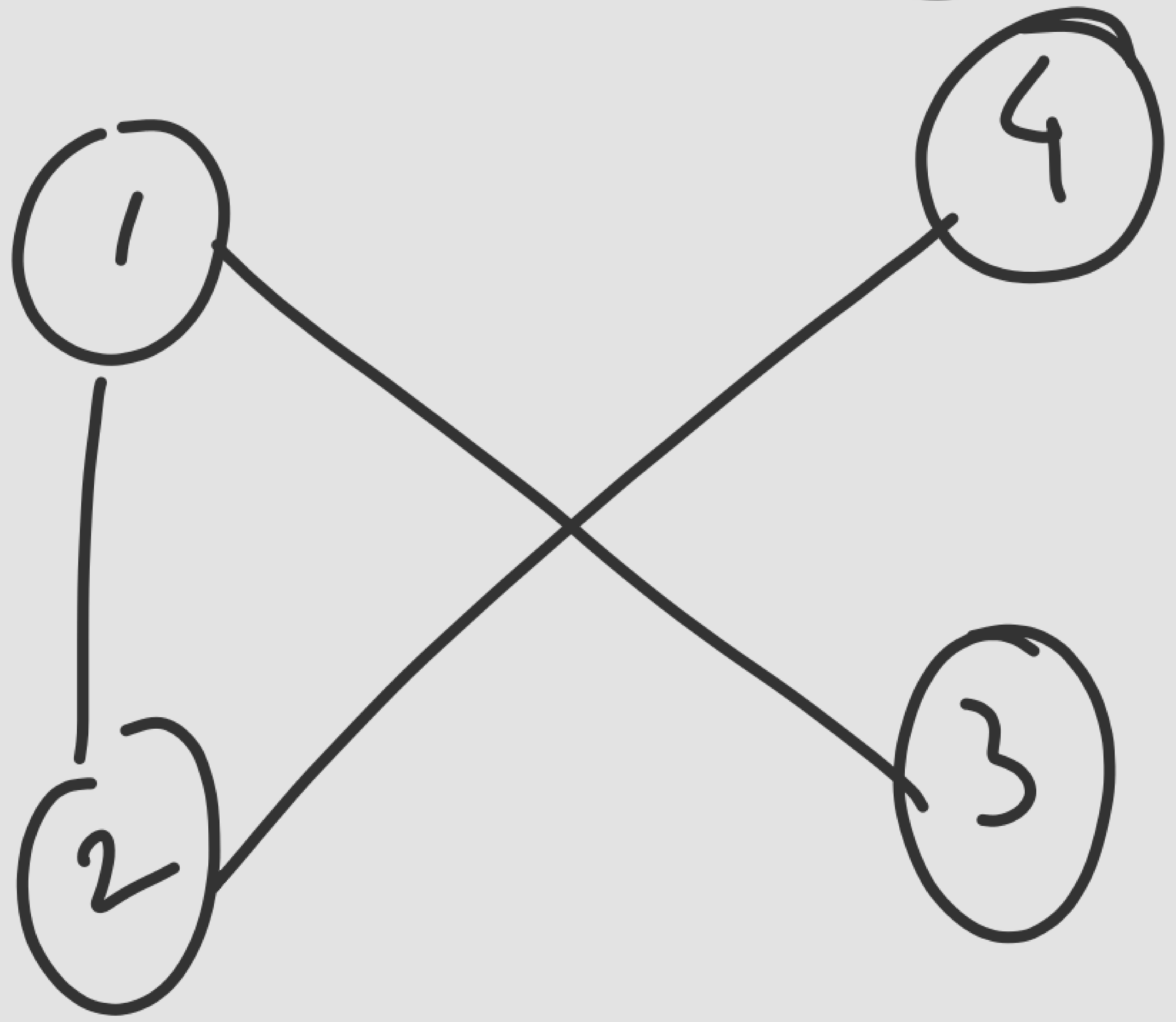

1
4
2
3


1
2


4
1
3
2


1
4
2
3


1
4
2
3


4
1
3
2

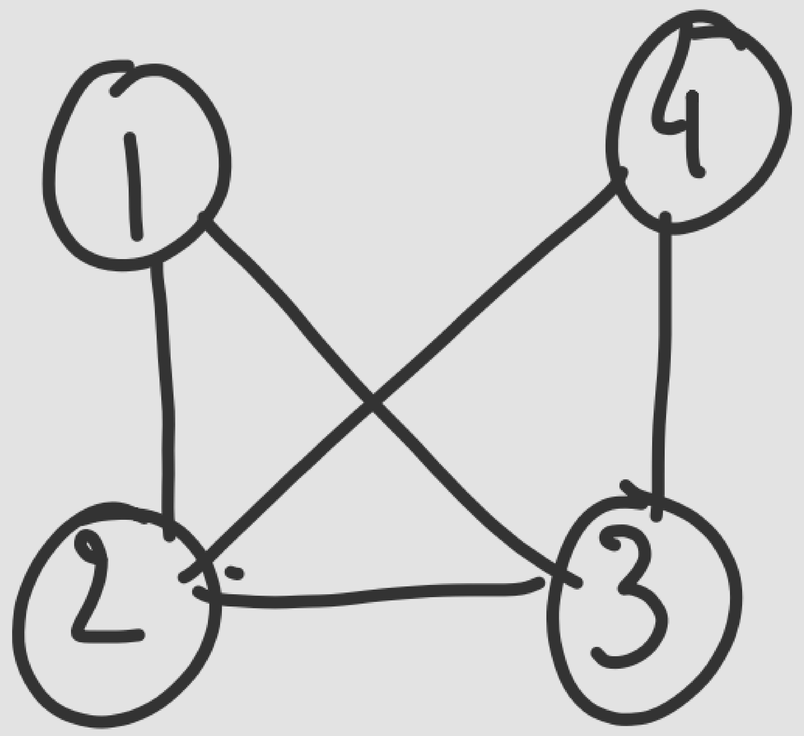
KIRCHOFF'S MATRIX THEOREM

→ running time is polynomial

→ Degree Matrix

→ Adjacency Matrix

Degree − Adj = laplacian



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

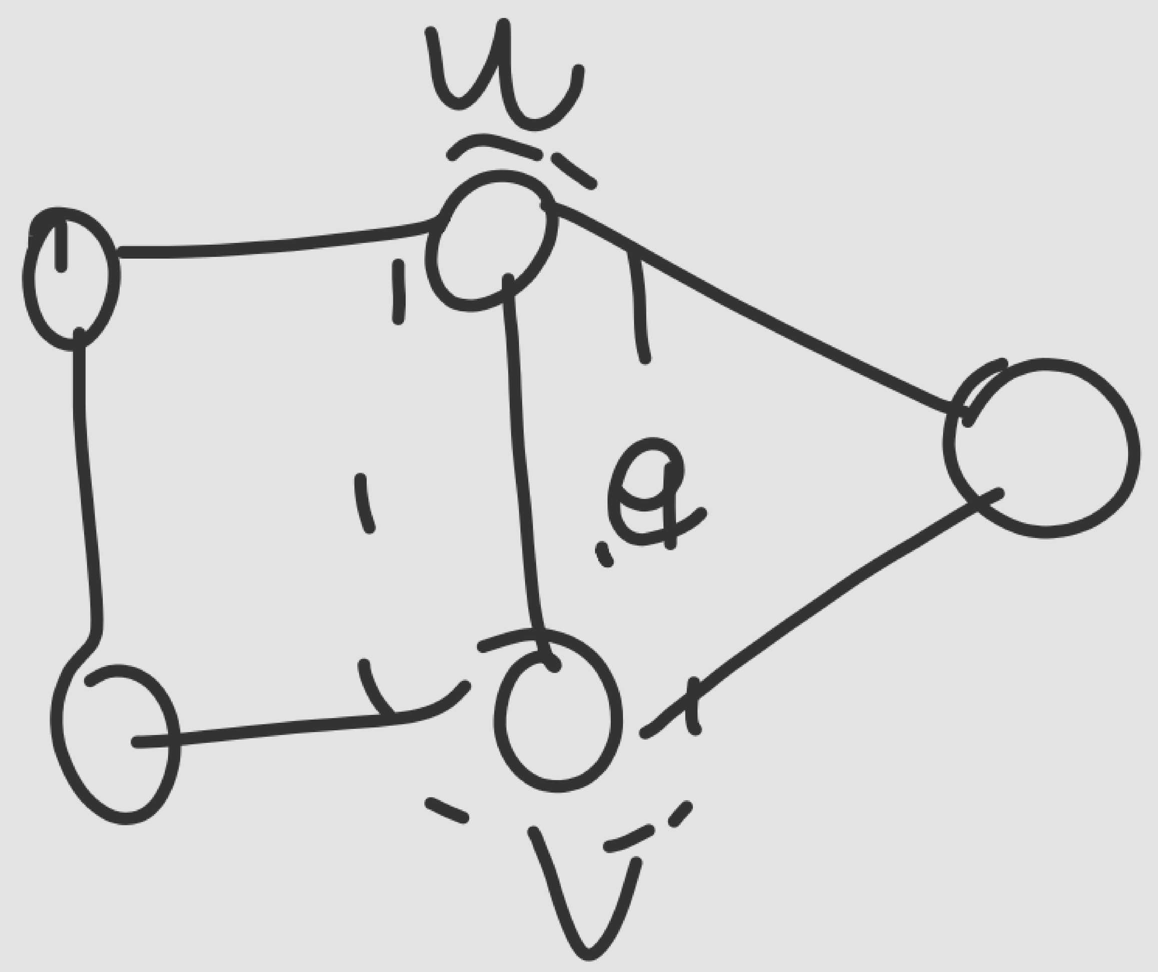
$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

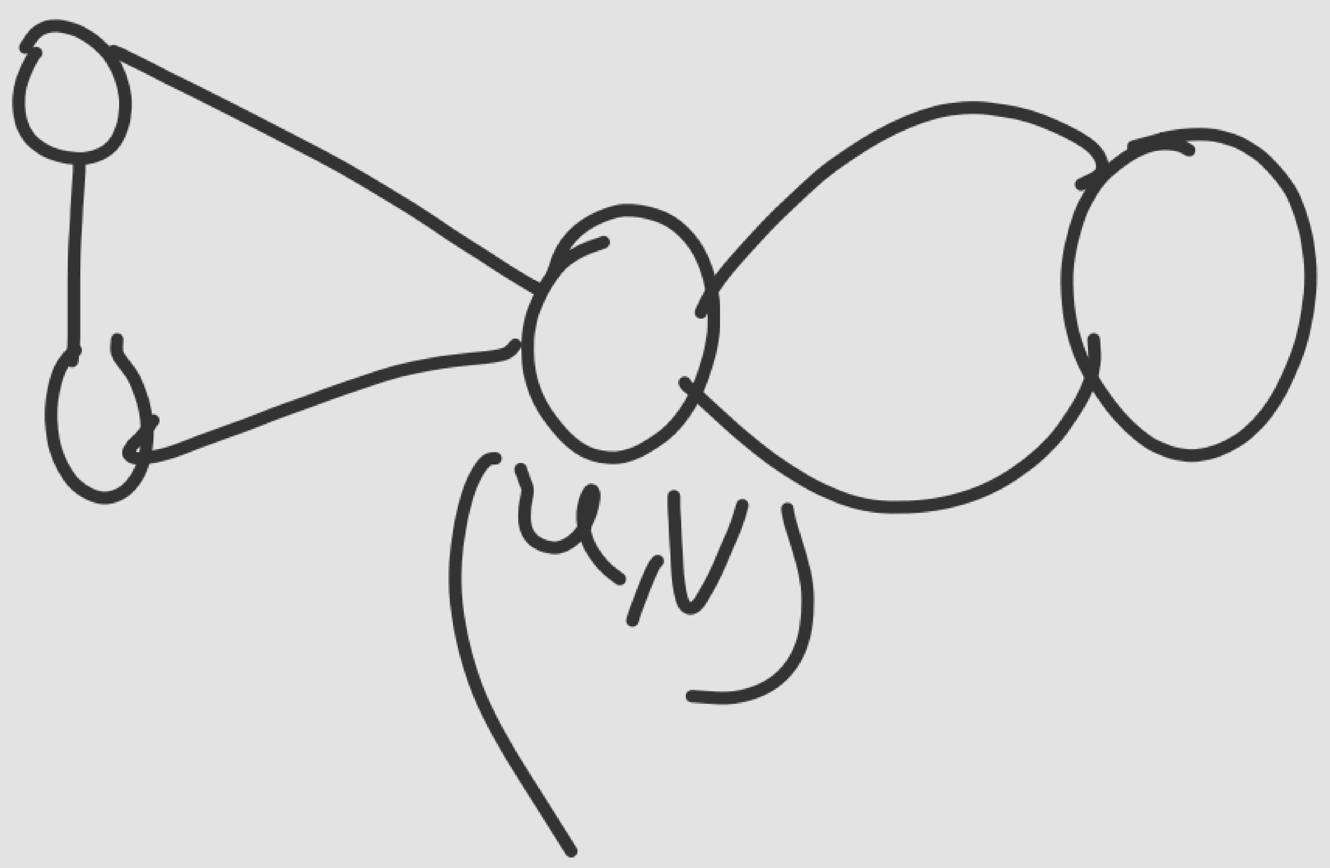
$$3(6-1) - (-1)(-2-1) - 1(-3-1)$$

$$15 - 3 + 4$$

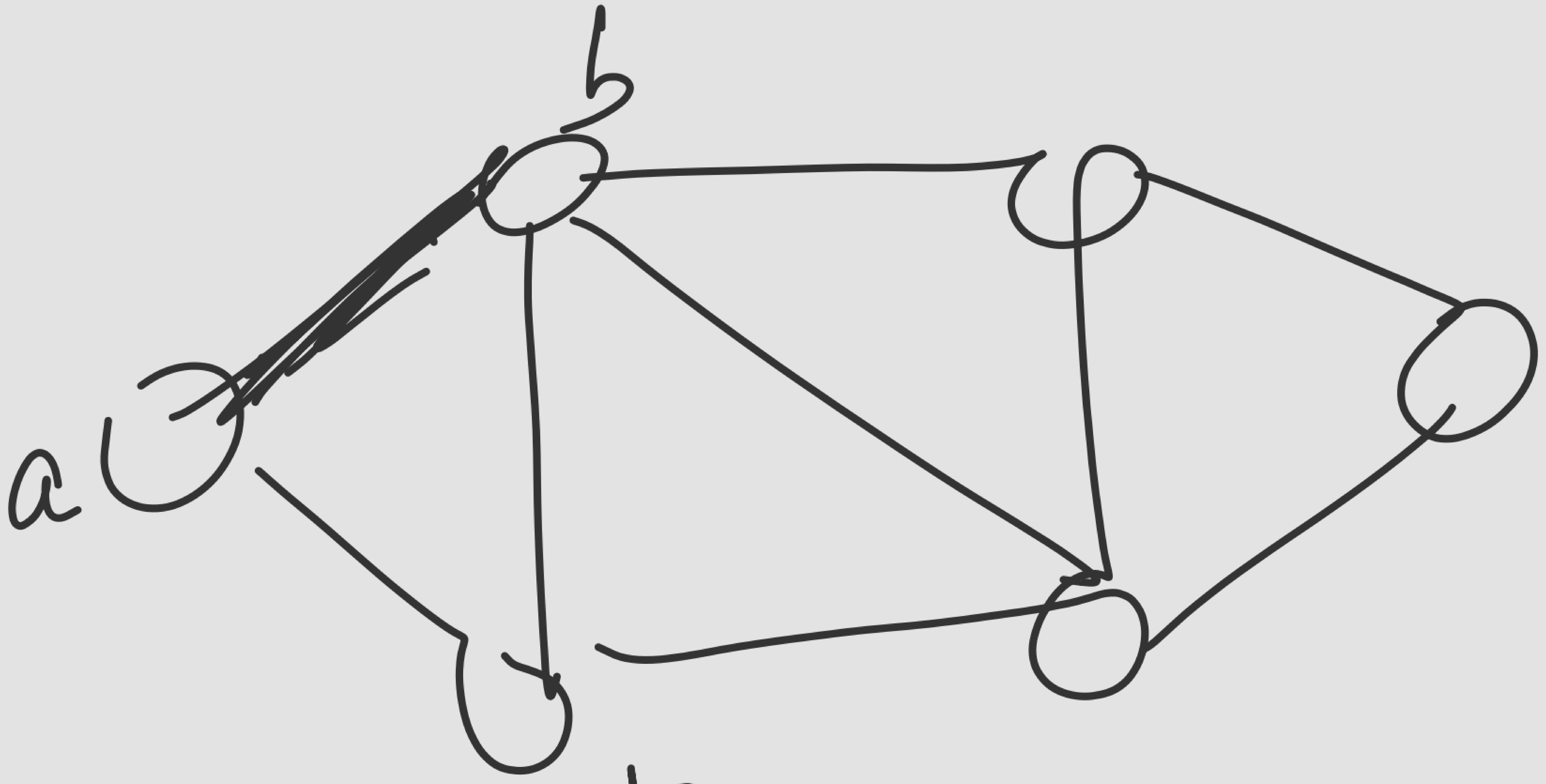
Contraction-Deletion

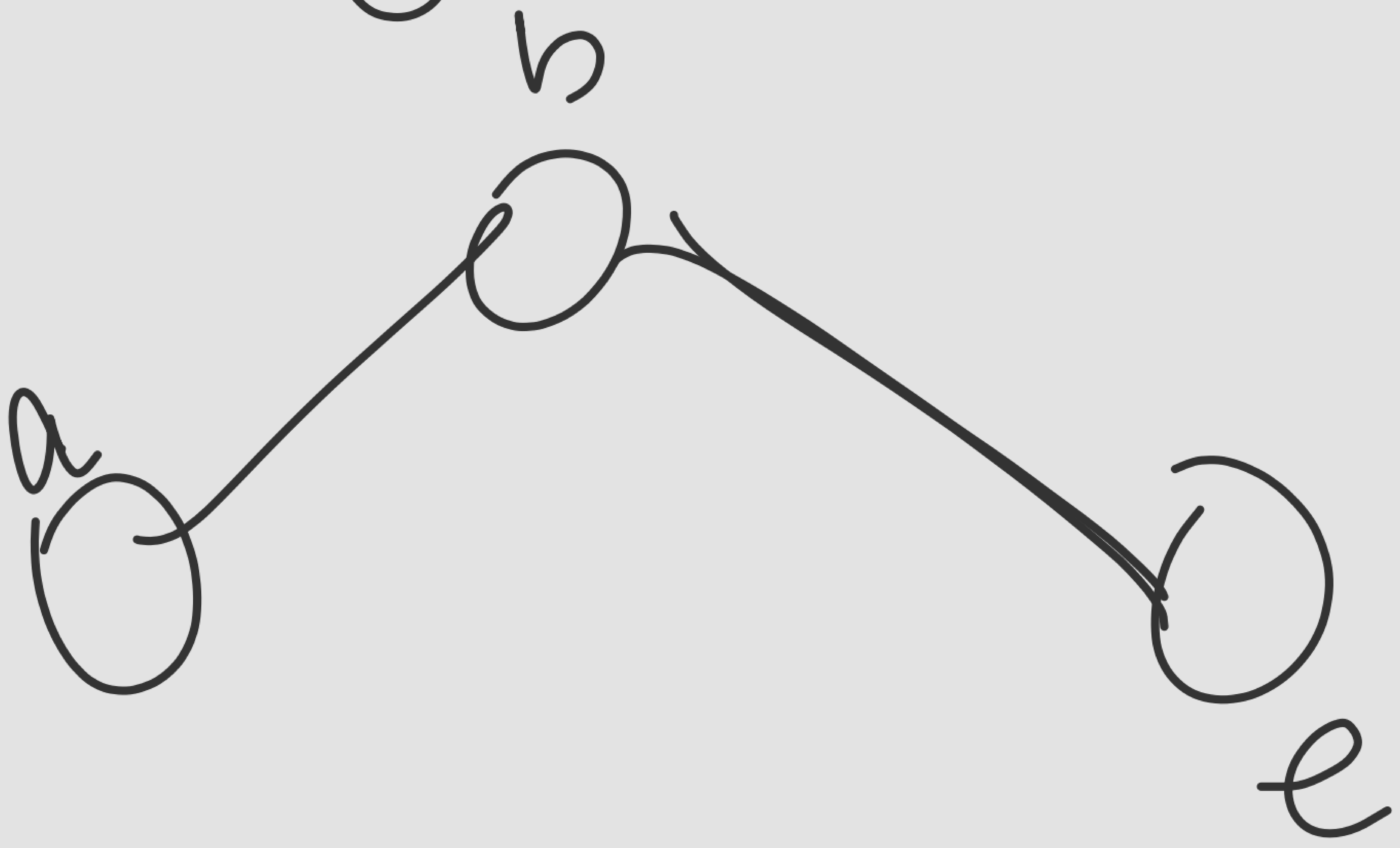
→ exponential

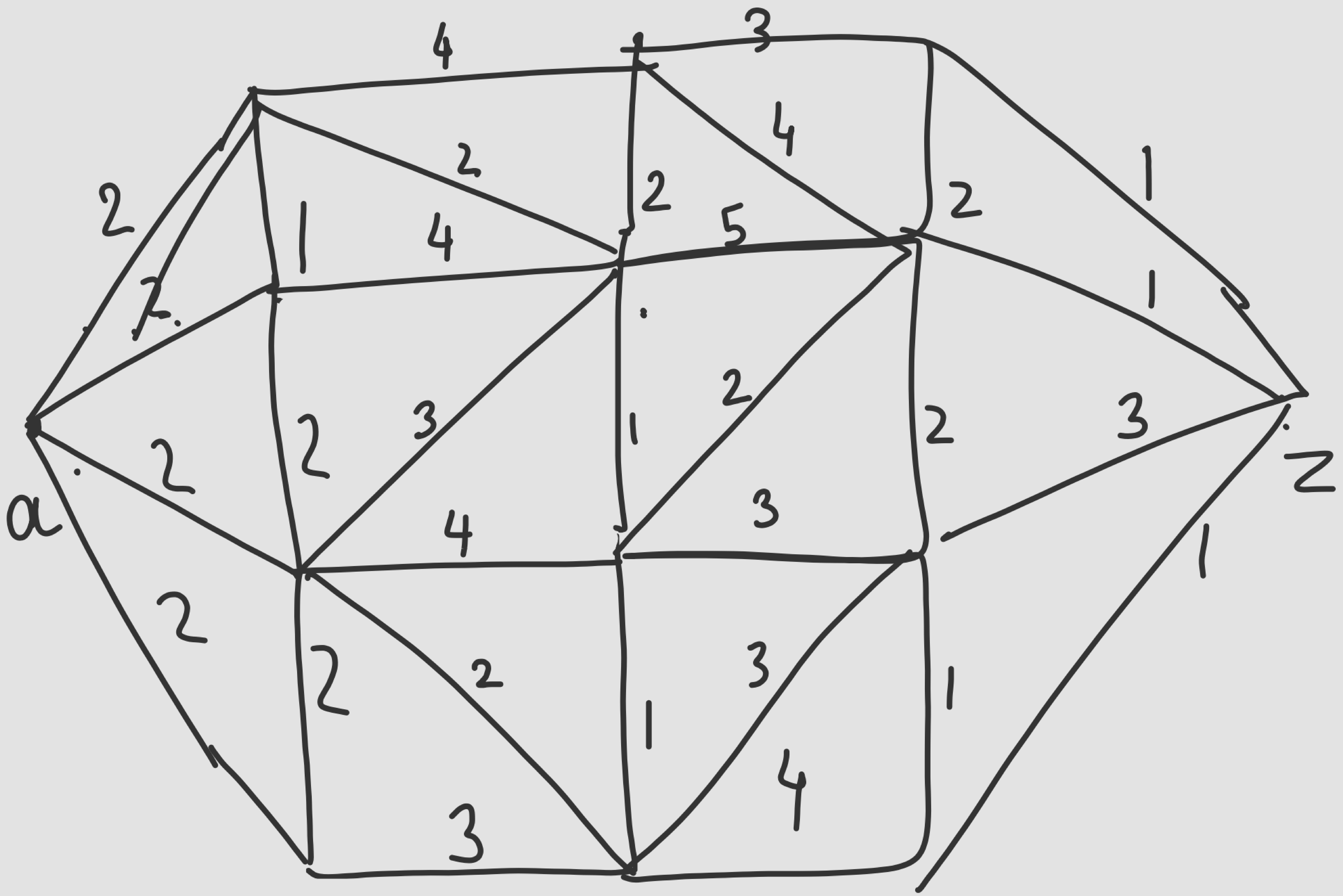




} deleted e
} contracted (u,v)







Shortest path a-z