Critical Design Review

CVT Simulation

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Project Highlights

- Derived governing equations for CVT system and vehicle dynamics, using practical simplifying assumptions
- Tested stability and accuracy of solvers by testing full range of expected inputs
- Implemented full vehicle dynamics and powertrain simulation
- Produced realistic shift curves
- Real time performance achieved, sim can run >>1000 fps

PDR Recap

- Created full system FBD
- Derived governing equations with fewer simplifying assumptions
- Attempted numerical solution of system using gradient descent

Issues after PDR

- Initial derivation of secondary force balance was incorrect
- Gradient descent solution was too unstable

Overall Goals - Recap

Create an accurate simulation of powertrain which is useful to the team.

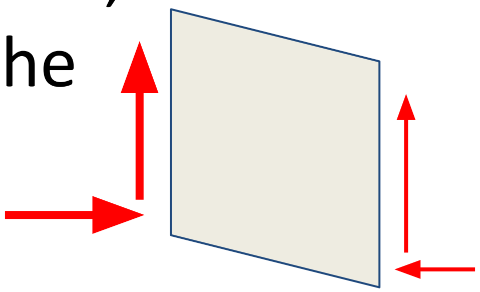
- How accurate?
  - Matches real world acceleration runs such that predicted acceleration times are accurate within +/- 0.01s
    - Chosen b/c in competition, acceleration can be decided by ms
- Useful in what way?
  - Optimize CVT tunes for acceleration, hill climb, sled pull
  - Predict top speed
  - Optimize gear ratio

Assumptions from PDR

- **Forces and moments within the CVT are assumed to be quasi-static**
- Constant temperature (no thermal expansion, constant belt friction coefficient)
- Friction from bearings/rollers is negligible
- Friction resisting belt shifting is negligible (belt “climbs”)
- All unknown friction will be represented as an single constant force resisting shifting, which will be tuned based on collected data

New Assumptions

- **Constant belt length (no belt deformation)**
  - Justification: belt is reinforced with tensile cables, making it stiff
    - However, doing the math, the belt could stretch 6+ cm, so this might not be a reasonable assumption: $\varepsilon_{max}=(1000N/1.5e\text{-}4m^2)/(95e6\ N/m^2) = 0.07$
  - Benefit: this simplification reduces DOF from 3 to 1, making numerical solutions simpler, faster and more stable
- **Friction applied to the belt is split equally between sheaves**
  - Justification: friction applied by the sheaves is proportional to clamping force. Because the belt does not move side to side, the clamping force from each sheave must be equal, so the applied friction force should also be equal.
  - Benefit: makes the system solvable



CVT System Forces

Primary Subsystem
$F_{sp} = k_p(d_{0p} + d_p)$
$F_{bp} = \frac{\alpha(F_f)}{\tan(\phi)\ln(F_f + 1)} - \frac{\alpha}{\tan(\phi)}(T_0 - 1)$
$F_{flyarm} = \frac{0.25 m_{fly}(r_{shoulder} + L_{arm}\sin(\theta_1))\omega_p^2 L_{arm}\cos(\theta_1)\cos(\theta_2)}{L_{arm}\sin(\theta_1 + \theta_2) + r_{roller}\sin(2\theta_2)}$
Secondary Subsystem
$F_{ss} = k_s(d_{0s} + d_s)$
$F_{bs} = \frac{\beta(F_{f,s})}{\tan(\phi)\ln(F_{f,s} + 1)} - \frac{\beta}{\tan(\phi)}(T_0 - 1)$
$\tau_{ss} = \kappa_s(\theta_{s0} + \theta_s)$
$F_{helix} = \frac{\tau_s / 2 + \tau_{ss}}{r_{helix}\tan(\theta_{helix})}$

No belt stretch assumption:

$$L_{b0} = r_p\alpha + r_s\beta + 2\sqrt{L^2 - (r_p - r_s)^2}$$

$$F_{bs} = F_{bp}\frac{\beta}{\alpha}$$

Linear belt force is proportional to wrap angle for equilibrium

$$\boxed{0 = \beta(F_{sp} - 4F_{flyarm}) + \alpha\left(F_{ss} + \frac{\tau_s / 2 + \tau_{ss}}{r_{helix}\tan(\theta_{helix})}\right)}$$

CVT Force Equilibrium

Single variable: d_p (primary sheave displacement)

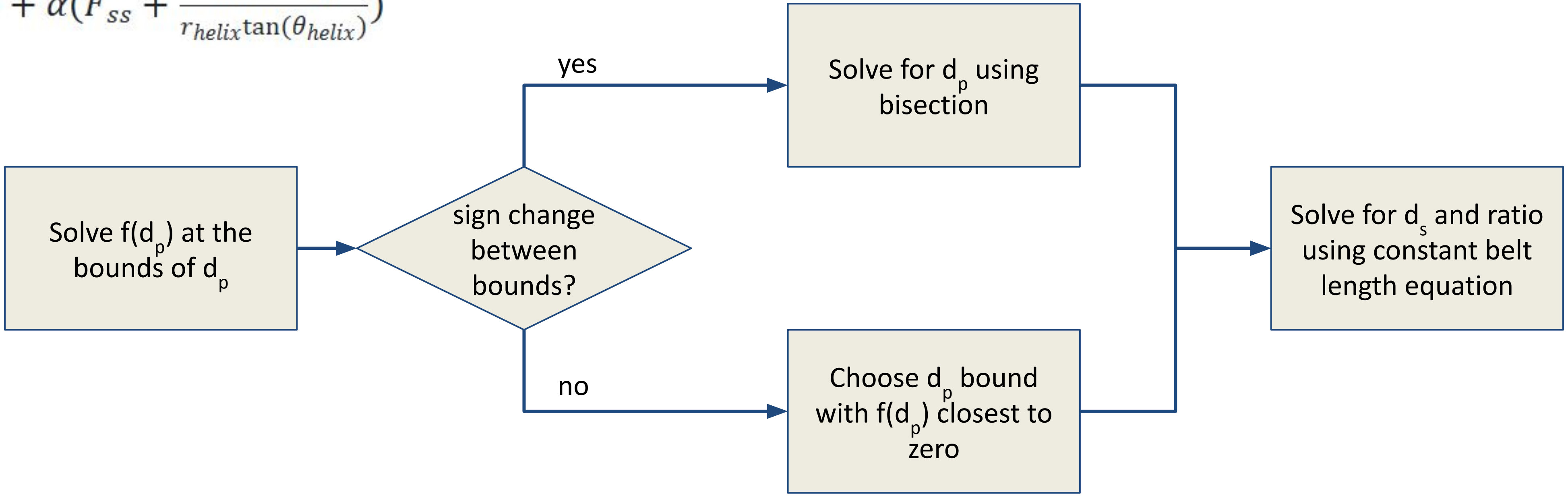
d_s (secondary sheave displacement) is derived using the constant belt length assumption

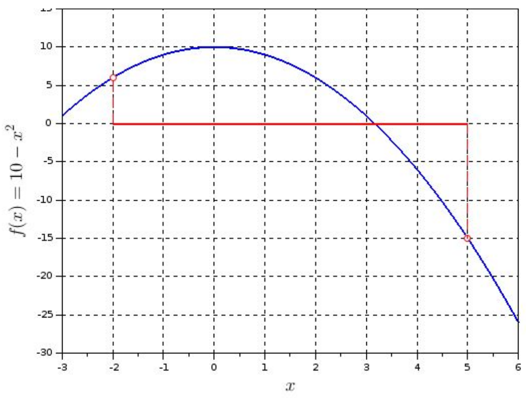
Numerical Solution to CVT System

An equation in the form f(x)=0 is solvable using root-finding.

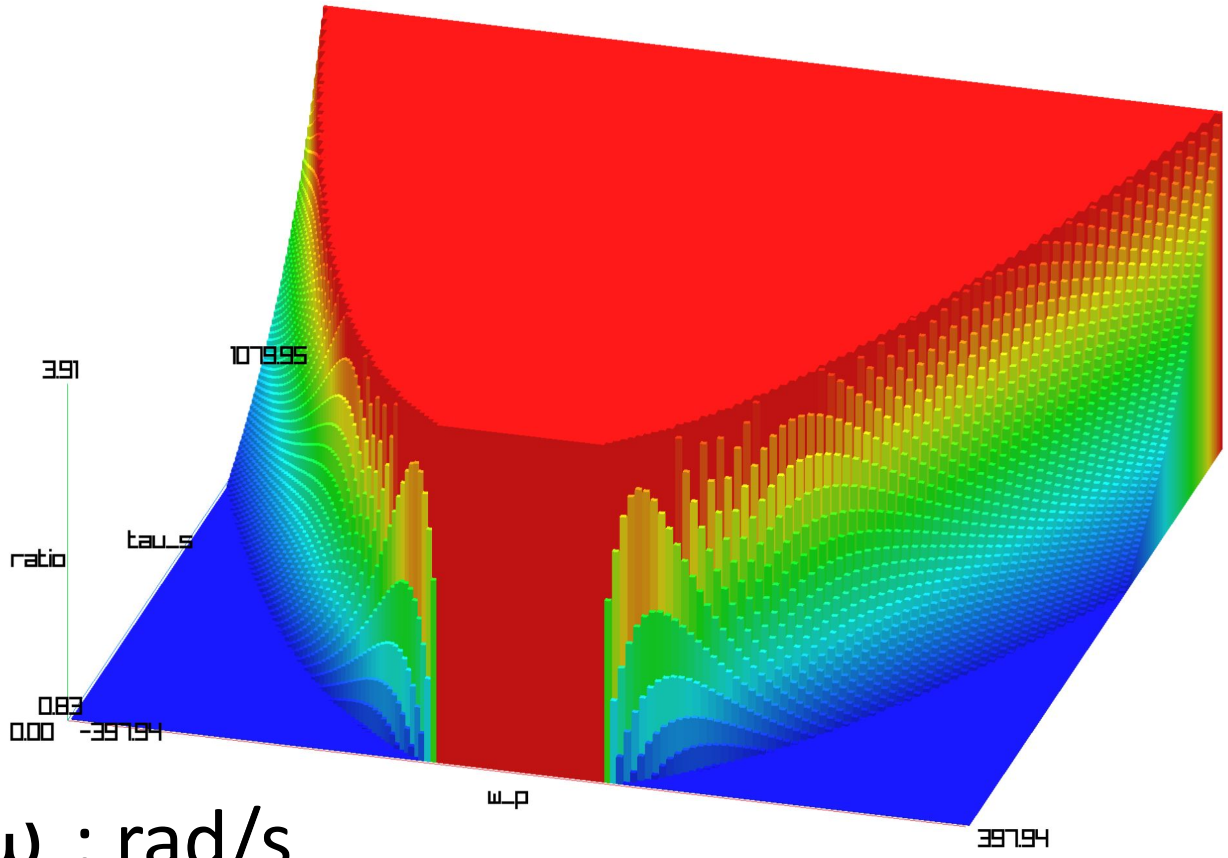
The most reliable root finding method is bisection, which works only when the function has a sign change within known bounds.

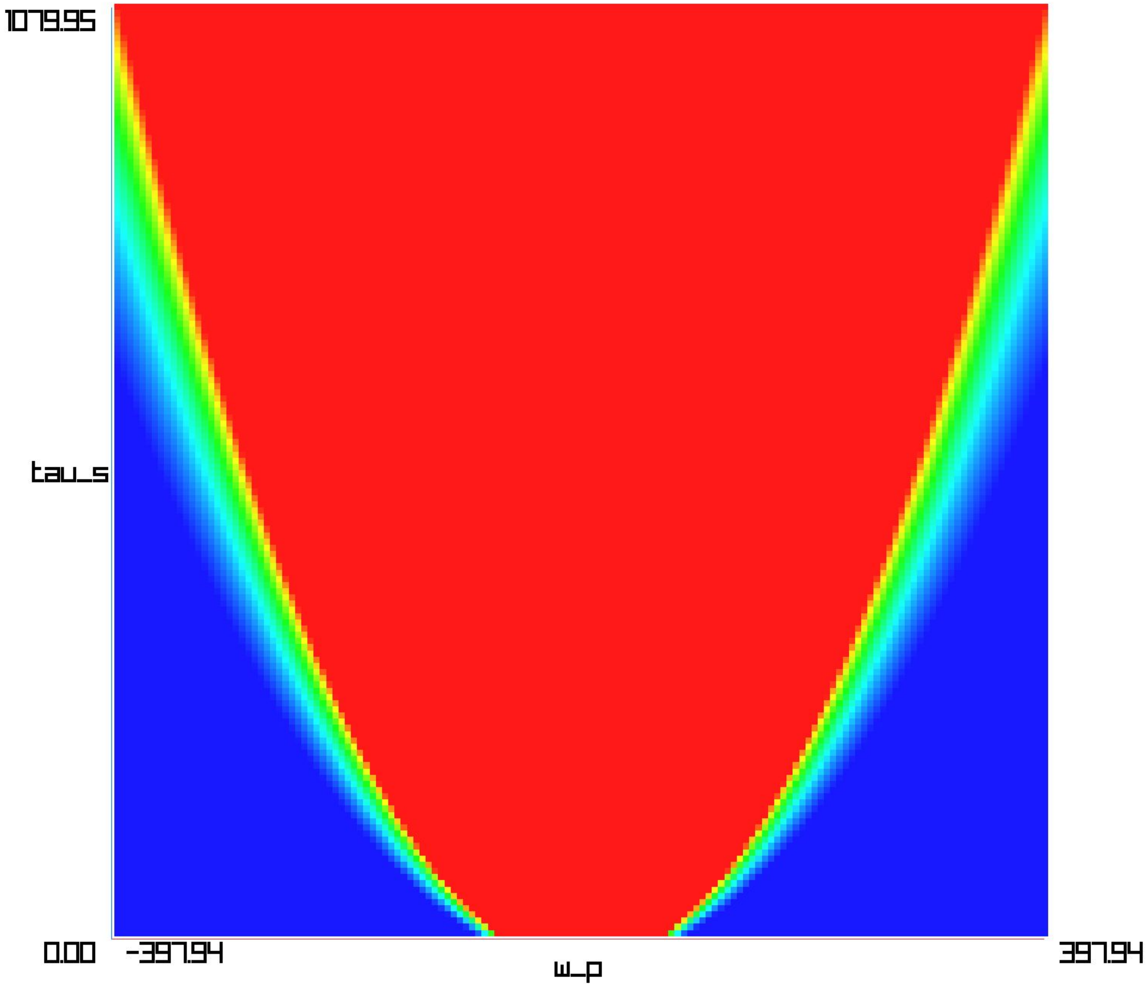
$$f(d_p) = \beta(F_{sp} - 4F_{flyarm}) + \alpha\left(F_{ss} + \frac{\tau_s / 2 + \tau_{ss}}{r_{helix}\tan(\theta_{helix})}\right)$$





CVT Ratio vs. ω_p and τ_s, Flyweights Fixed





ω_p: rad/s

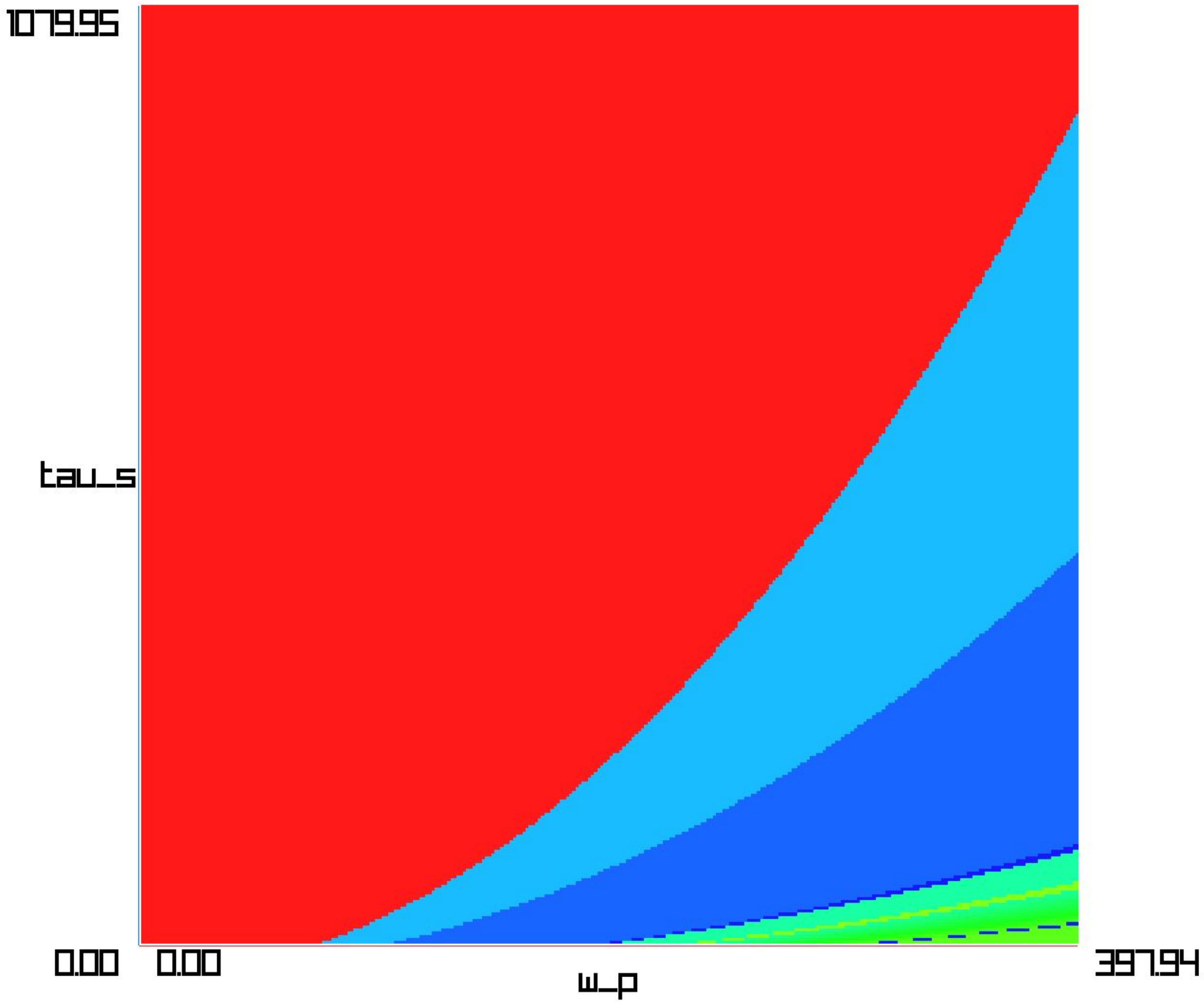
τ_s: N-m

Flyweight force $\propto \omega_p^{\ 2}$. Secondary force $\propto \tau_s$. Result: parabola in XY plane

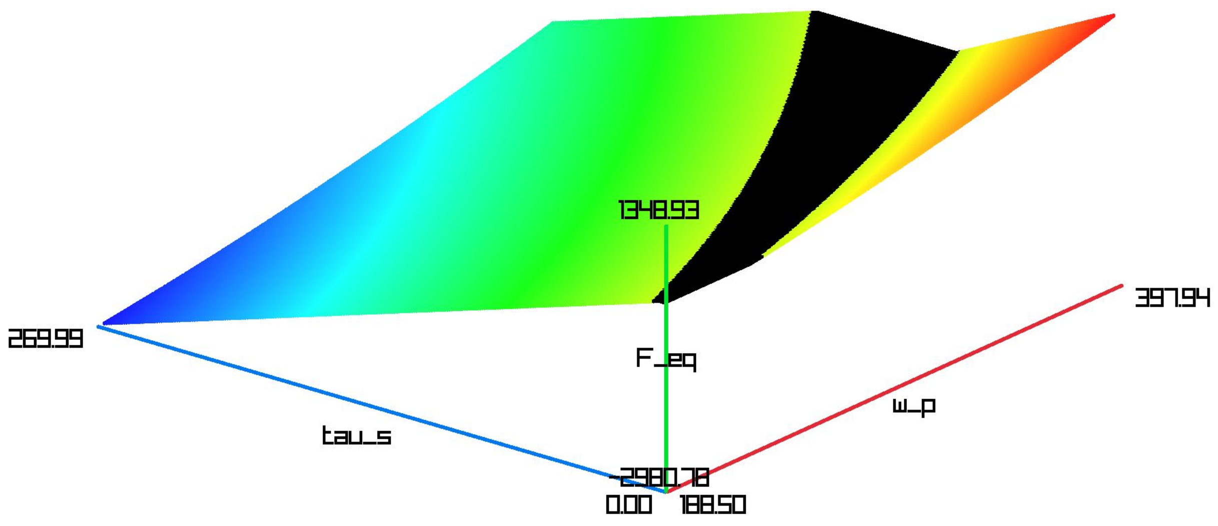
CVT Ratio vs. ω_p and τ_s, Flyweights Free

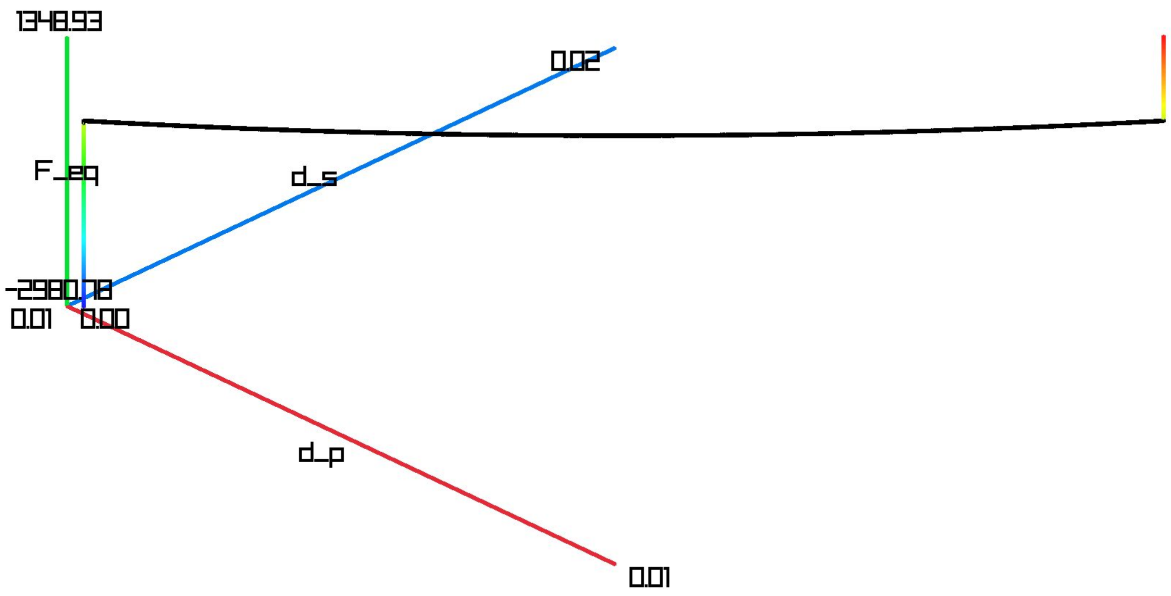
Results in regions of a constant ratio.
Simulation is much less stable,
forces never reach equilibrium.
So, this strategy will not be used.

ω_p: rad/s
τ_s: N-m



Force equilibrium (N)

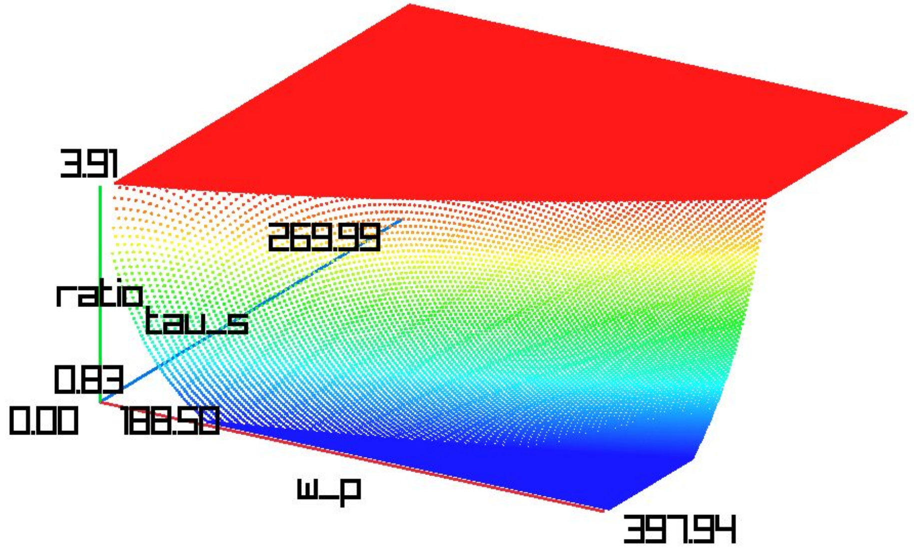




Black region is at equilibrium

Positive F_{eq} = high ratio

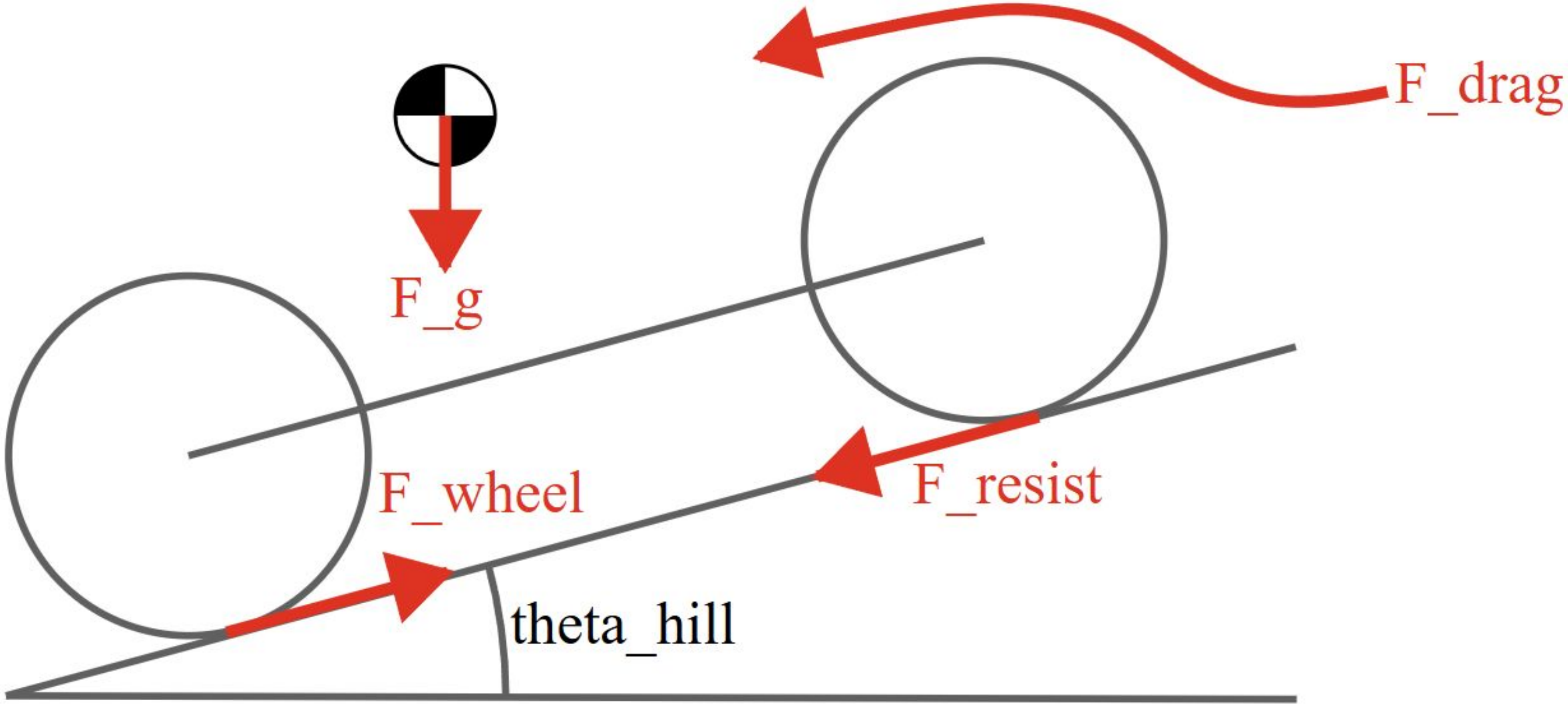
Negative F_{eq} = low ratio



Vehicle FBD

Assumptions

- 1D space, point mass
- No wheel slip



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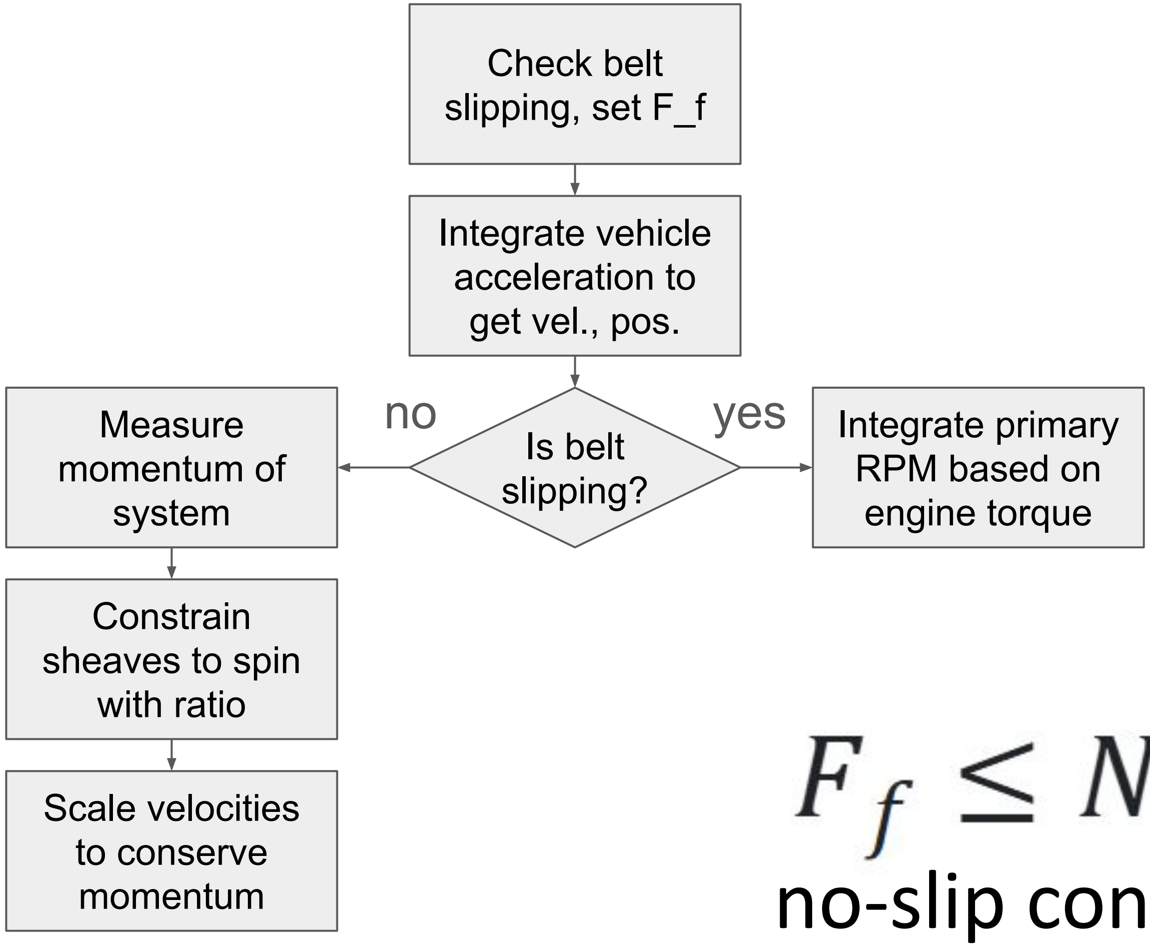
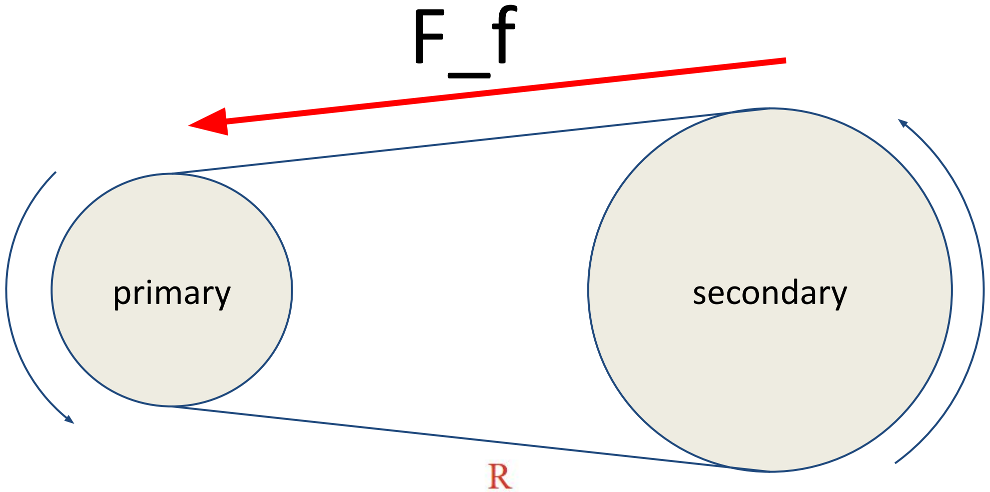
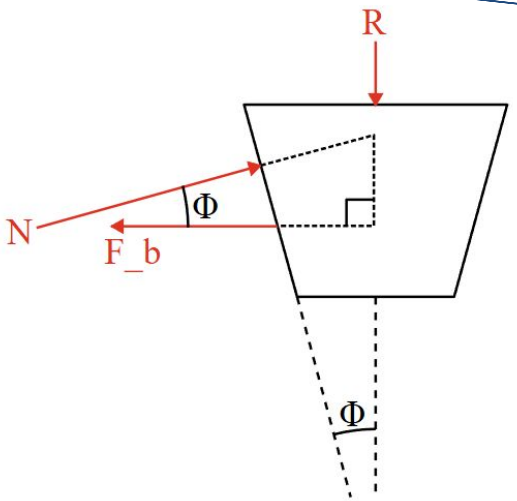
Vehicle Dynamics Implementation



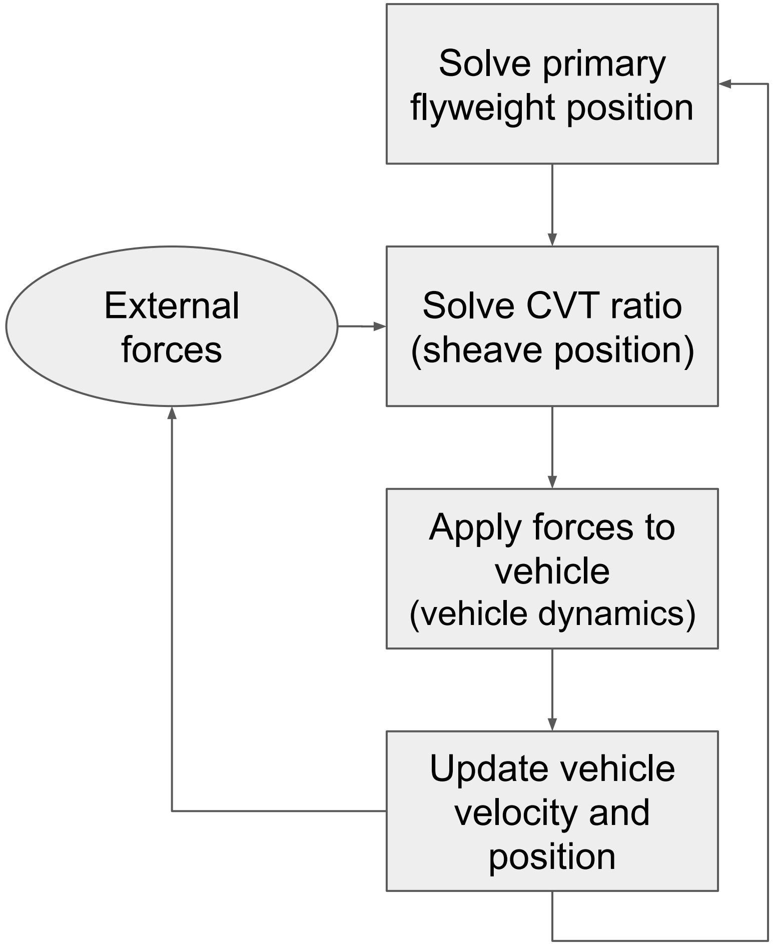


$$F_f \leq N_p \mu_b$$

no-slip condition



Full Vehicle Simulation




```
BajaDynamicsResult trb_sim_step(BajaState &baja, double dt) {
    // 1. Solve flyweight position
    auto S_fly = solve_flyweight_position(baja.theta1, baja.theta2,
    baja.theta1 = S_fly.x(0);
    baja.theta2 = S_fly.x(1);
    // 2. External forces
    baja.tau_s = baja.calc_tau_s();
    // 3. CVT shift ratio
    auto d_p = solve_cvt_shift(baja);
    baja.set_ratio_from_d_p(d_p);
    // 4. Vehicle dynamics
    auto res = solve_dynamics(baja, dt);
    apply_dynamics_result(baja, res);
    return res;
}
```

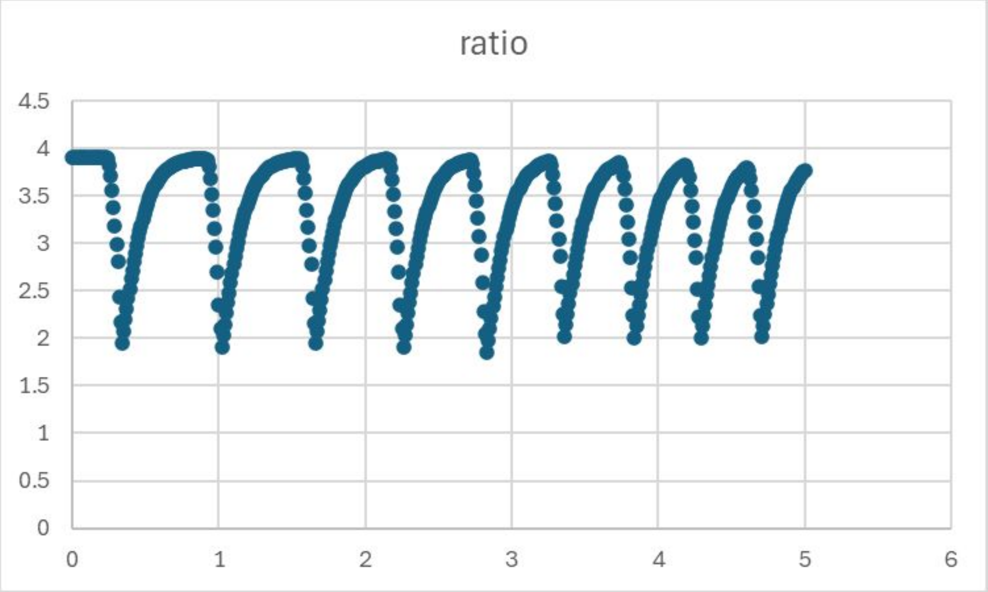
Simulation Results: Bad Belt Slipping

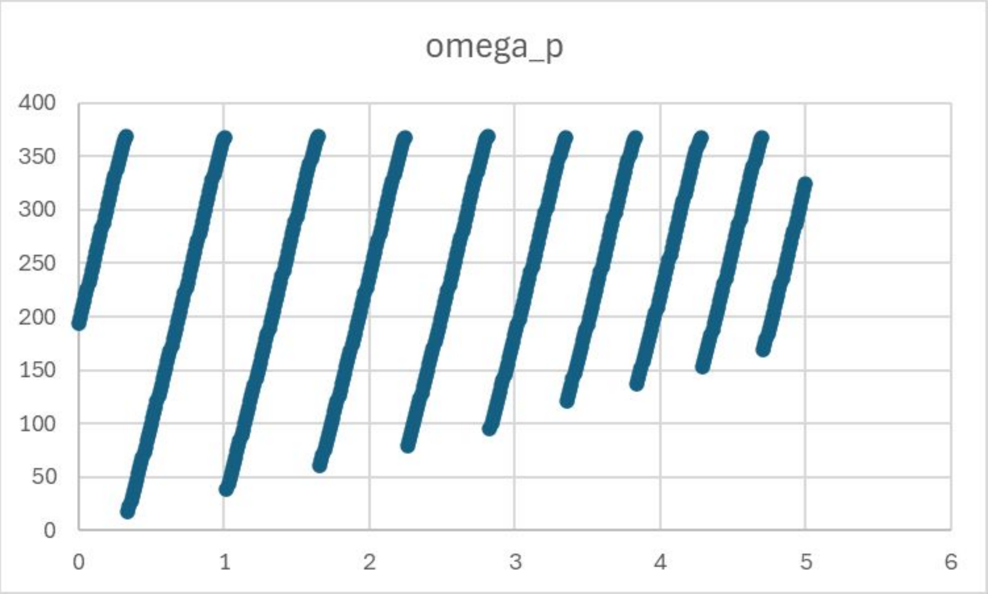
Ran with belt friction μ = 0.6

The ratio is going crazy (technical term)

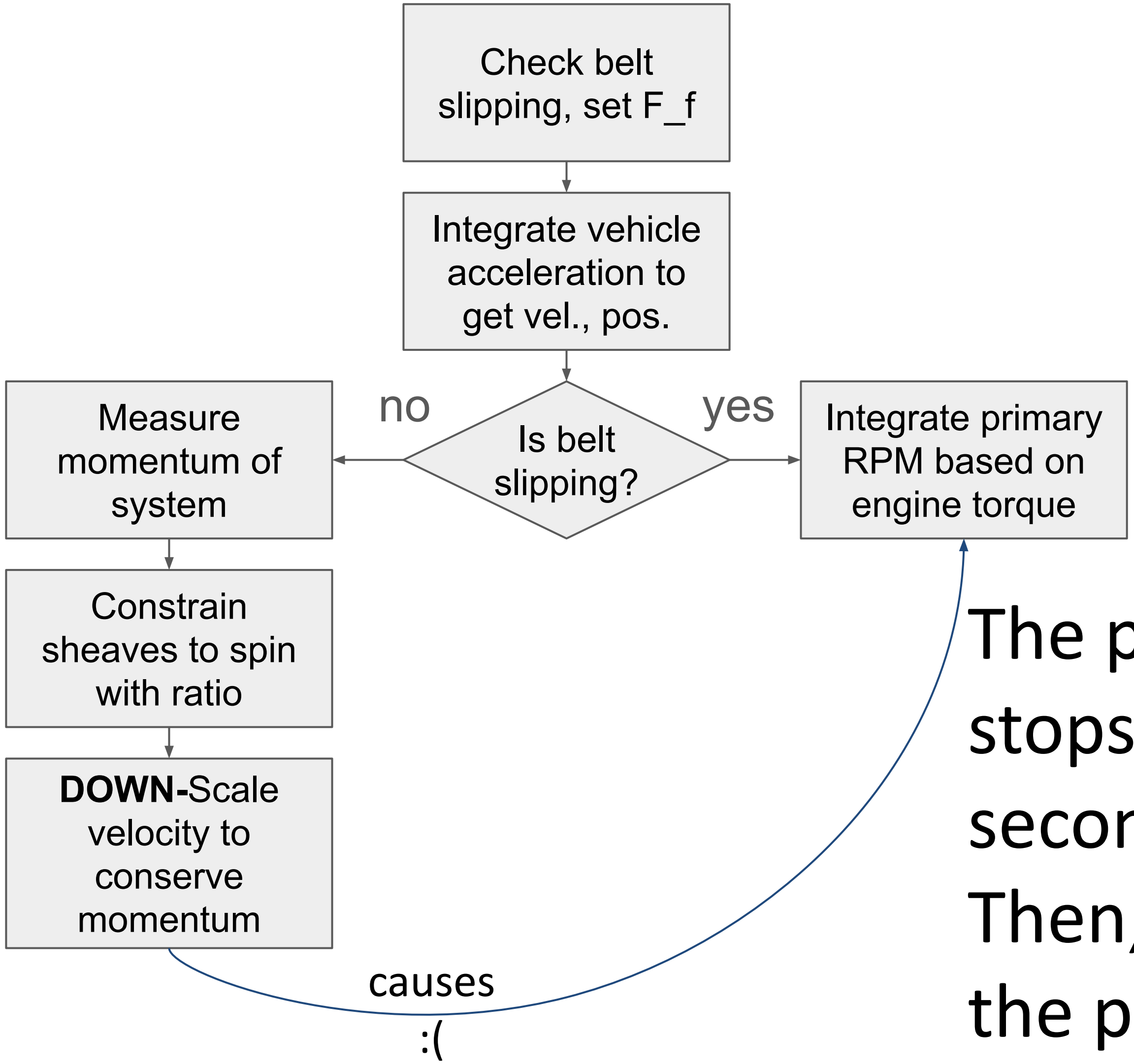
The engine speed looks like it's from a fast and furious movie where the car has 10 speeds

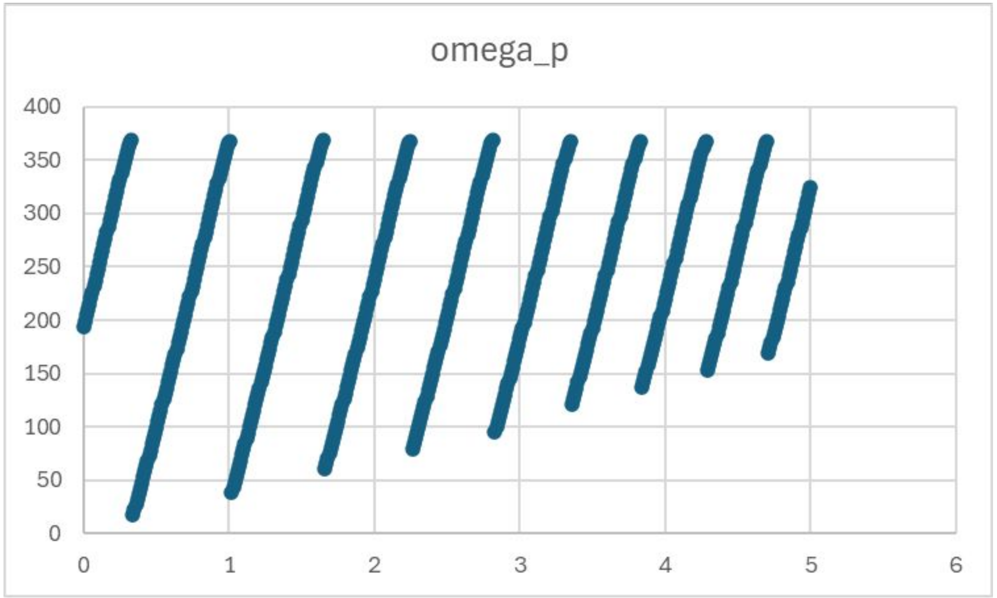
Something has gone horribly wrong…





Why the Bad Belt Slipping?





The primary slows down when the belt stops slipping so it can match the secondary's speed.
Then, it slips in the next frame because the primary isn't spinning fast enough!

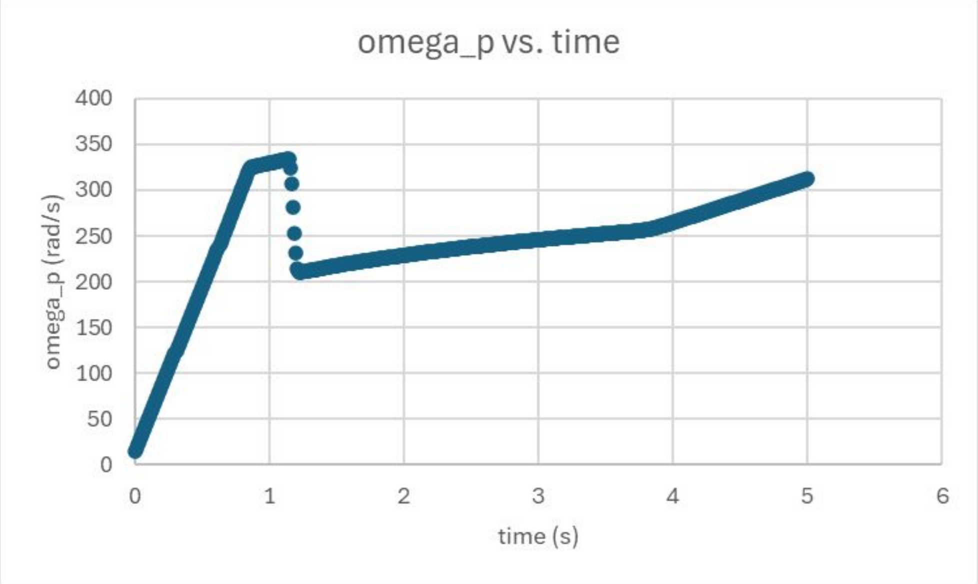
Better Simulation Results

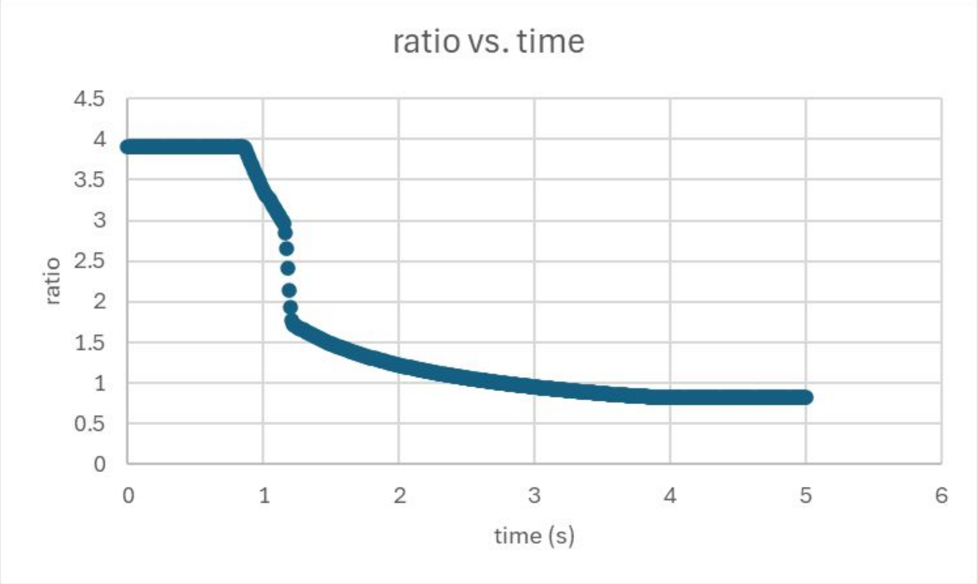
This shape looks reasonable!

How? The belt slipping condition was ignored by making μ ***extremely*** large.

And, ratio change is reduced by a "speed factor" with decay: exp(-S*dt)

In real life, slipping doesn't usually occur once the shift has started, and shifting isn't instant.



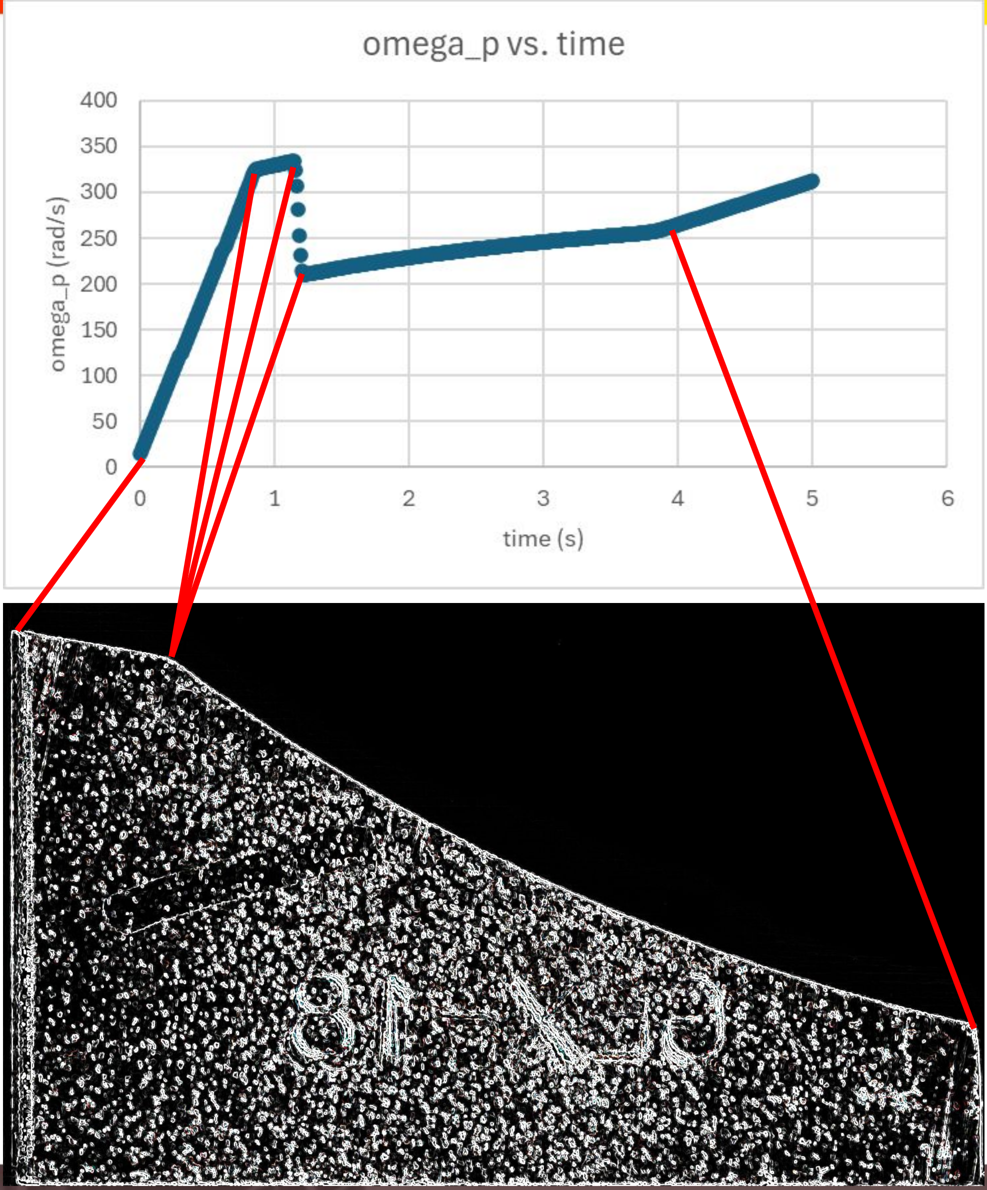


Better Simulation Results - Ramp Effect

Ramp shape is crucial to shift behavior:

The corner causes the drop in speed

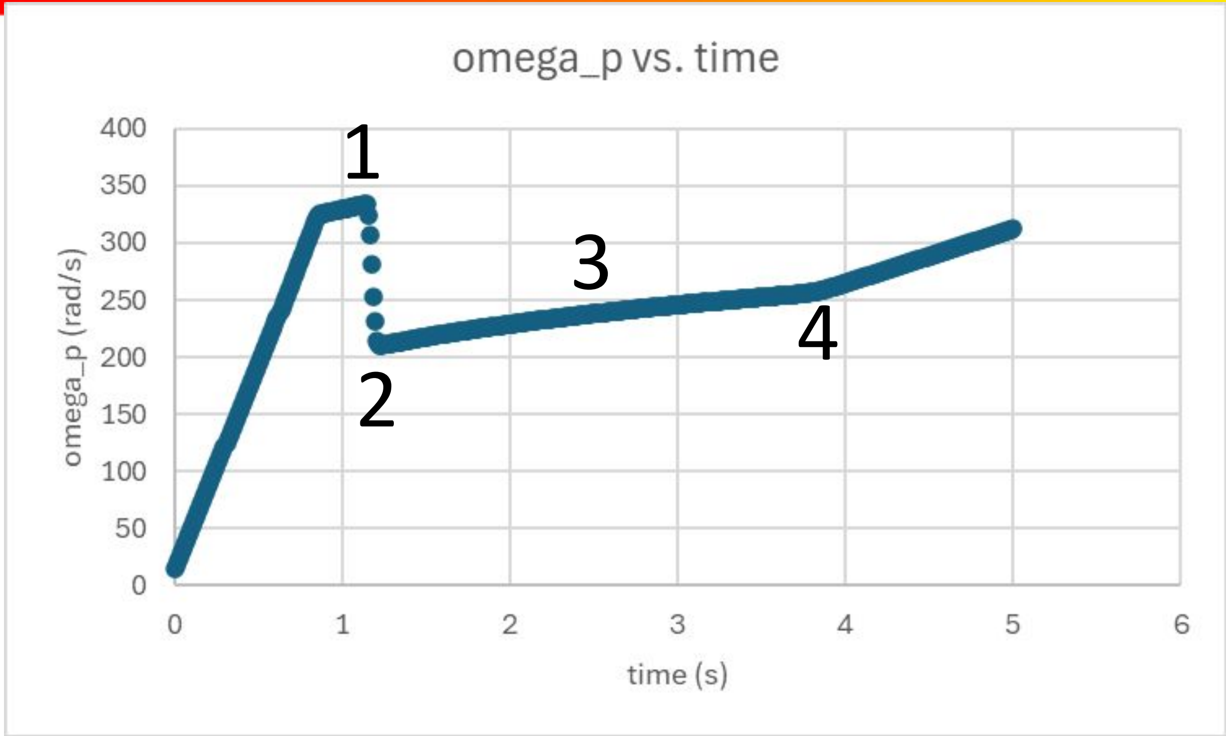
- Steeper slope ⟶ more clamp force
- More clamp force ⟶ upshift
- Upshift ⟶ more resistive torque experienced by engine ⟶ decrease in engine speed



Better Simulation Results - Curve Analysis

Features of this shift curve:

1. Peak is 3150 RPM at 1.1 s
2. Sudden drop in RPM
3. “Flat” section hovers ~2200 RPM
4. Max ratio reached at 3.9 s



But is it accurate?

Nope, but surprisingly close.

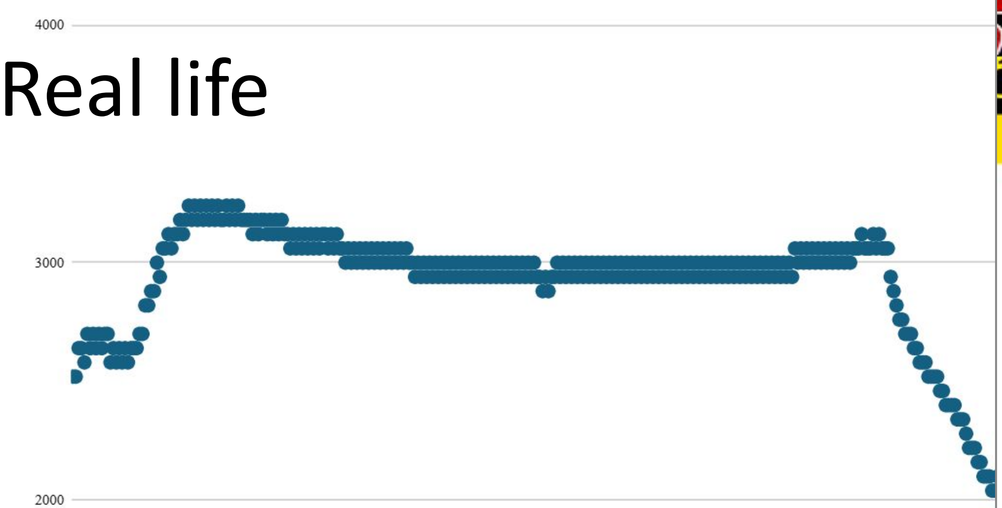
Same cvt tune was used:

- Peak RPM: 3240 vs. 3150
- **“Flat” RPM: 3000 vs. 2200**
- Drop in RPM smooth vs. instant
- Flat vs. sloped shift

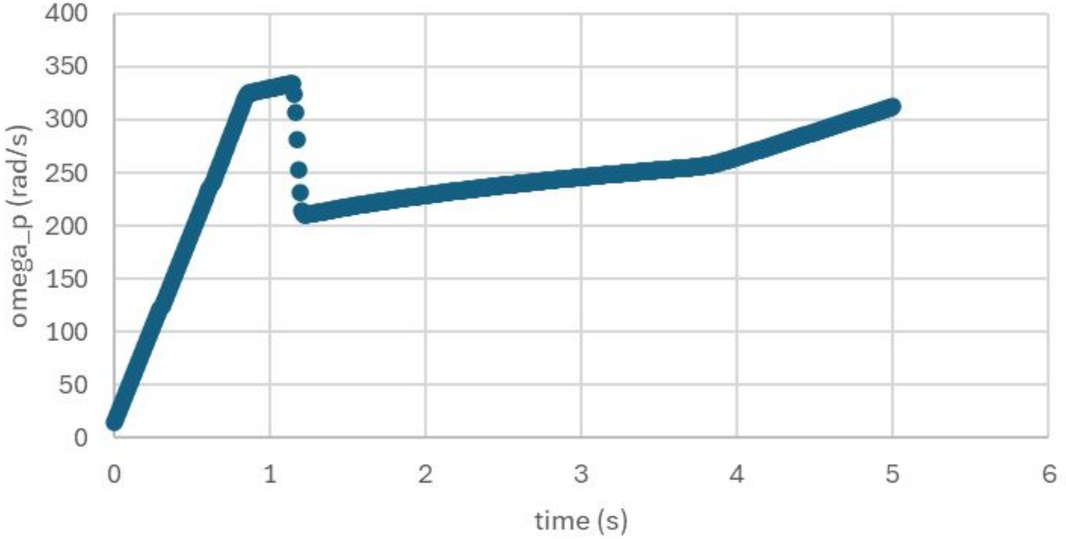
And considering time:

- Real life max shift at 4.3 s
- Sim max shift at 3.9 s

Real life



Simulation



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What could be the cause?

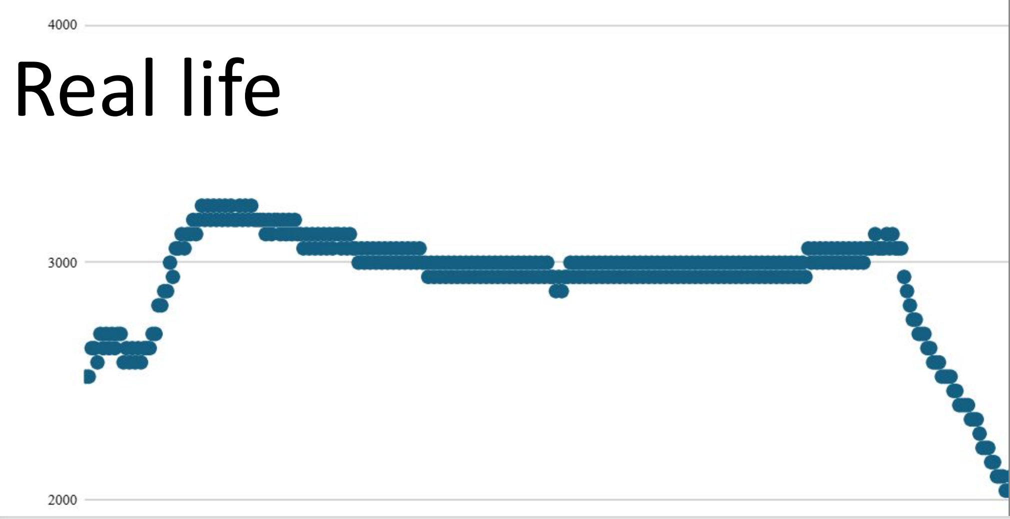
"Flat" section RPM is too low:

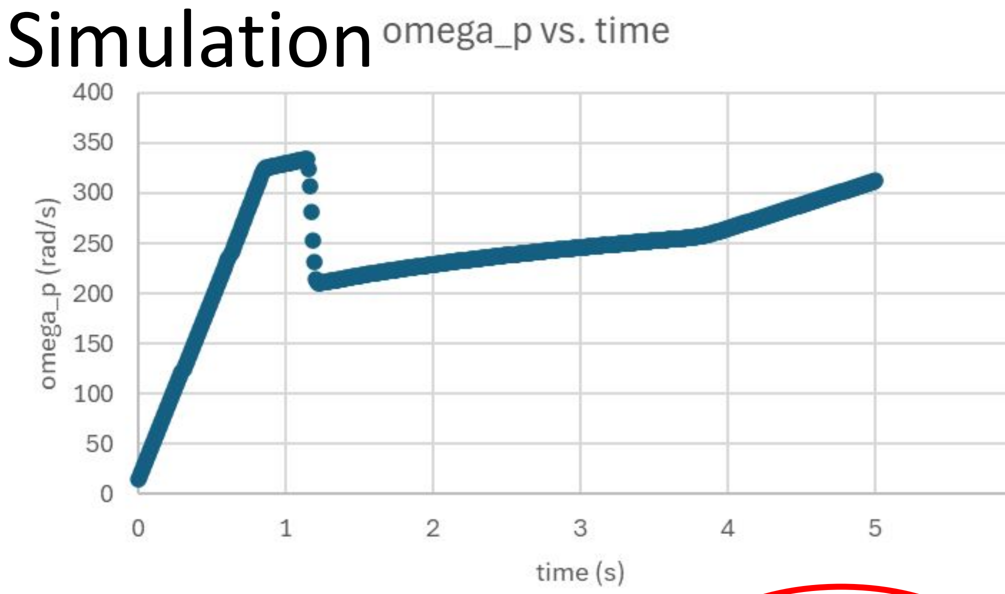
- **If resistive torque is higher, more engine RPM needed for equilibrium**

Max shift time is too early:

- **If resistive torque is higher, then vehicle will shift more slowly**

Seems like resistive torque is too low. (tau_s)





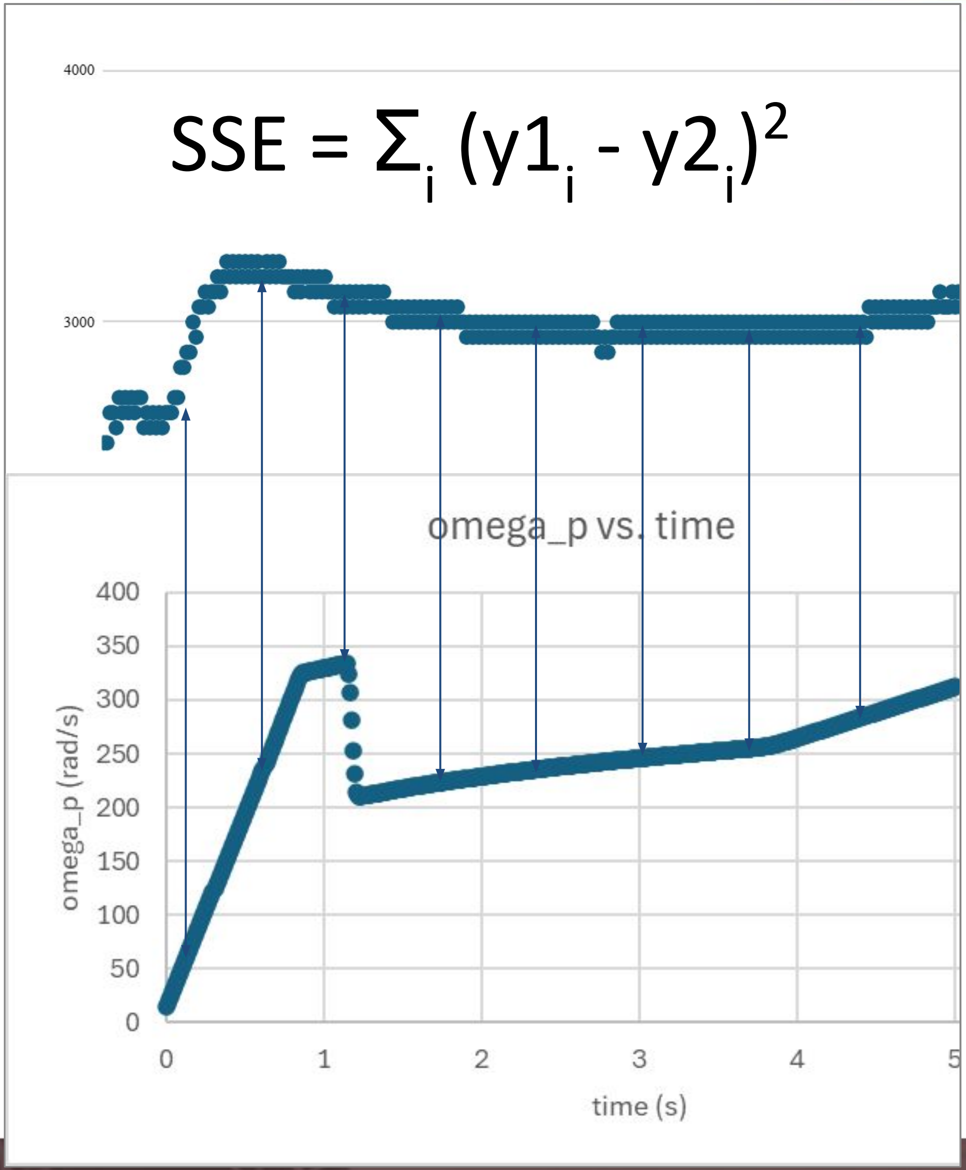
$$0 = \beta(F_{sp} - 4F_{flyarm}) + \alpha\left(F_{ss} + \frac{\tau_s / 2 + \tau_{ss}}{r_{helix}\tan(\theta_{helix})}\right)$$

Improving Accuracy

Quantify comparison and optimize:

1. Measure accuracy - SSE
   - For shift ratio, engine speed, vehicle speed
2. Modify free variables to decrease SSE
   - Rolling resistance force
   - Shift speed factor
3. Minimize SSE with optimization

$$SSE = \Sigma_i (y1_i - y2_i)^2$$



Limits of Accuracy

- Noise prevents perfect replication of sensor data
- Optimizing SSE will include measurement error from sensors
- Certain conditions of testing are unknown
  - When exactly did the driver hit the throttle?
  - How heavy was the car and driver on that day?
  - Was the belt slipping when the driver hit the throttle?
- Gradient descent may not converge to global min

Most importantly:

- Simplifying assumptions introduce modeling error

Precision

Good news! The sim is very precise!

Belt length constraint error: < 1e-8 m (limits of floating point)

Flyweight angle error: < 1e-3 rad (~1/20 deg)

CVT equilibrium error: < 0.002 N

Dynamics integration error:

- Vehicle position & velocity: 4th order accurate, error $\propto 1/dt^4$
- Engine speed: 1st order accurate, error $\propto 1/dt$

Todo

- Validation with sensor data!
- Represent comp events in simulation
- Optimize CVT tune for accel and hill climb
- Documentation for future teams
- Optimize gear ratio (if time permits)

Conclusion

“Create an accurate simulation of powertrain which is useful to the team.”

- Expected results: **a vehicle simulation which is as accurate as our data allows**
- Have you met all of your objectives? **Not yet, but some are met**
- Risk assessment of current design? **Inaccuracy** → **suboptimal designs**
- Greatest takeaways from this project?
  - **Complex systems are understandable by applying first principles and breaking down into smaller parts.**
  - **Numerical solvers can’t solve all of your problems, but most of them.**

Future Work

If you had to do it all over again what would you do differently?

- **Attempt a Lagrangian mechanics solution. This could simplify derivation since Lagrange is energy-based.**
- **Include driving mechanics such as braking, steering, tilting, and sliding**

What projects/research/testing would enhance this simulation?

- **More controlled testing of the car with less noisy, more precise data**
  - For example, having sensors to test how much throttle the user is giving, using interrupts to measure engine RPM rather than sampling
- **CVT dyno testing for data on steady state CVT behavior**
- **Engine dyno testing for more precise engine torque curve**
  - Measuring how engine responds to a non-wide-open throttle
- **Measuring inertia of physical components rather than using best guess**

See the Github for more derivation details!

https://github.com/shua5115/Terps-Racing-Baja-Simulator

See “CVT derivation.md” for derivation of governing equations

Any questions?