Cincinnati Public School Bus Routing and Bell Time Optimization

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Routing: Bi-objective Routing Decomposition (BiRD)

Decomposing the routing problem:

1. single-school
   (a) stop assignment
   (b) single-school routing
2. multi-school
   (a) scenario selection
   (b) bus scheduling

Stop Assignment

Set/Parameters

Identifier	Index/Set Domain	Comment
schools	S	set of schools
students	P	set of students
P_s	$s \in S$	set of students attending school s
Locations	C	set of all locations can serve as bus stops
C_p	$C_p \subseteq C$	Allowed bus stops for student $p \in P_s$
$d_{p,c}$		walking distance for student p to a stop $c \in C_p$

For student p with special needs may require a "door-to-door" pickup: the set C_p is a singleton c, where $d_{p,c} = 0$.

The objective is to minimize the number of stops for each schools and the total walking distance. This is similar to a facility location problem.

Variable

z_c: binary whether stop c is selected for school S.

$y_{p,c}$: binary whether student p is assigned to stop c.

Objective and Constraints

For each school S, minimize the total number of stops and walking distance:

$$min \sum_{c \in C} z_c + \beta \sum_{p \in P_s} \sum_{c \in C_p} d_{p,c} y_{p,c}$$

subject to:

1. student p is assigned to c only if stop c is selected for school s.

$$y_{p,c} \leq z_c, \forall p \in P_s, c \in C_p$$

2. Each student is assigned to 1 stop

$$\sum_{c \in C_p} y_{p,c} = 1, \forall p \in P_s$$

Where β is the trade-off of the total number of stops and walking distance.

Single school routing

After the stop assignment, C_s is the set of bus stops which serve school S. $n_{c,S}$ is number of students from school S at a stop c.

Bus

Identifier	index/set	comment
Bus Type	b/B	
Capacity Q_b	b	
Wheelchair spots W_b	b	fixed number of wheelchair
Bus depot Y		**stored during the night or day???**

Trip

1. Time

Identifier	comment
$t_{c,S}^{pick}$	length of time to pick all p for S at c; function of $n_{c,S}$

Identifier	comment
t_S^{drop}	length of time to drop all p at school S; fixed
$t_{l_1,l_2,\tau}^{drive}$	driving time from l_1 to $l_2 \in S \cup C \cup Y$ at time τ
τ_S	all buses must arrive at school S at τ_S
$t_S^{drop} + \tau_S$	bus free to leave at this time
T^{max}	maximum time that students allowed to spend on the bus

$t_{l_1,l_2,\tau}^{drive}$ are deterministic and known, and usual got from API such as Google Map. Due to reality, such as accident, and to mitigate the impact of the traffic, the drop-off times t_S^{drop} is artificially increase to say 10-15 minutes even it only takes 3 minutes.

2. Trip/Route

A trip/route is a ordered sequence of stops visited or served, by a bus. For school S, a feasible trip/route:

(a) no student spends more than T^{Max} between pickup and drop-off.
(b) there exists a type of bus with capacity and carry all students assigned to the stops severed by the trip.

Given a feasible route R:

- $B_R \subseteq B$: the set of types of buses that have the necessary capacity to serve the trip.
- T_R: service time of the trip, the time between arrival at the stop to the final school
- T_S: A set of feasible trips, each stops $c \in C_s$ is served by a nonempty set of feasible trips $T_c \subset T_S$.

  For example: School S has 4 different sets of feasible trips, stop c4 can be served by two differents trips 3 and 5, so $T_c = 3, 5$.

  The heuristic returns a set of trips T what covers each stop **exactly once**. Then we run it N times to build a set of feasible trips where each stop is covered by serveral trips. The heuristic returns N different solutions and each stop will be covered by N different feasible trips.

Trips in T_S indexed by $1, ..., m_S$.

For given set of trips $T \subseteq T_S$: $I(T)$ is the subset of $\{1, ..., m_S\}$ corresponding to trips in T.

For example: School S has 4 different set of feasible trips, with 25 different trips in total. Stop c4 can be served by two differents trip 3 and 5, so $I(T_c) = I(T_4) = 3, 5 \subset 1, ...25$

find best set of routes

1. Variable

   r_i: binary whether trip i is selected $\forall i \in \{1, ..., m_S\}$

2. Objective

$$min \quad \lambda \sum_{i=1}^{m_S} r_i + \sum_{i=1}^{m_S} r_i \Theta_i$$

3. Constraints

   every stop must be served by at least one trip.

$$\sum_{i \in I(T_c)} r_i \geq 1, \forall c \in C_S$$

   where, for each trip i in T_s, $\Theta_i = \sum_{p:c_p \in C_i} \theta_p^{(i)}$, where C_i reprents the list of stops served by the $i-th$ trip i,

   $\theta_p^{(i)}$ corresponds to tiem spent by student p on the bus druing trip i.

   λ controls the importance of the number of trips relative to the total time students on the bus.

   In the optimal solution above, some stops may be served by more than one trip. Set of trips $T_c^{opt} \subset T_S$ serving each such stop c. For each $i \in I(T_c^{opt})$, we compute the improvement δ_i to the objective that would be obtained by removing stop c from trip i.

Scenario Selection

Node

Type	Notation	comment
depot	y	
trip	$\rho_{S,h,R}$	for each school S, and routing scenario $1 \leq h \leq h_S$ and each trip $R \in \mathcal{T}_S^h$
available node	$a_{S,h}$	for each school S, and routing scenario $1 \leq h \leq h_S$

The set of arcs includes an arc from depot to trip; trip to corresponding available node; from available node back to the depot.

Also, an arc from each available node to each trip node where trip $R' \in \mathcal{T}_{S'}^{h'}$ is compatible with the a bus staring from school S. $\tau_S + t_S^{drop-off} + t_{S,c_{R'}^{start},\tau_S}^{drive} + \mathcal{T}_R \leq \tau_{S'}$

For a note $i \in \mathcal{N}, let\, \mathcal{I}(i) \subseteq \mathcal{N}$ be the in-neighborhood of the node i, and

$\mathcal{O}(i) \subseteq \mathcal{N}$

Variable

integer flow variables f_i^j for each arc(i,j).

Objective

Minimize the total number of buses corresponds to minimizing the total flow out of the yard node y.

$$minimize \sum_{s \in S} \sum_{h=1}^{h_S} \sum_{R \in \mathcal{T}_S^h} f_y^{\rho_{S,h,R}}$$

Constraints

1. flows along all arcs must be integral.
2. $$\mathcal{Z}_{S,h} \in \{0,1\}, \forall S \in S, 1 \leq h \leq h_S$$
3. $$\sum_{h=1}^{h_S} Z_{S,h} = 1$$

The binary variable $Z_{S,h}$ is 1 if $\mathcal{T}_S^h$ is the selected set of trips for school S. 3. exactly one set of trips / one scenario is selected for each school.