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Another Assignment Problem Example

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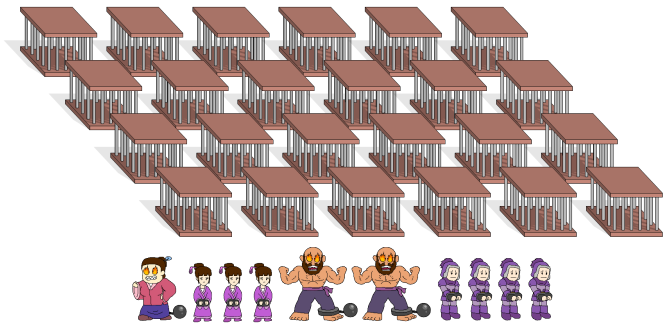
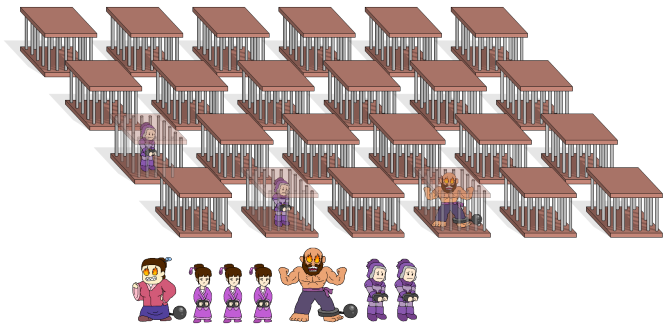
The CellBlock Problem



No Two Prisoners in the Same Cell



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Dangerous Prisoners

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Dangerous Prisoners

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Dangerous Prisoners

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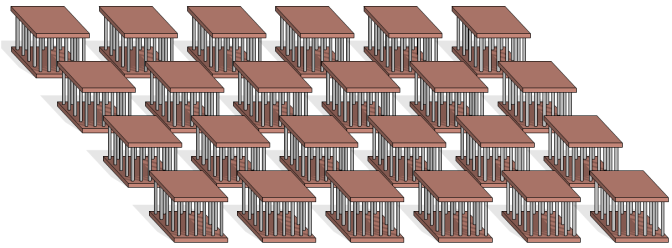
Separation of Males and Females



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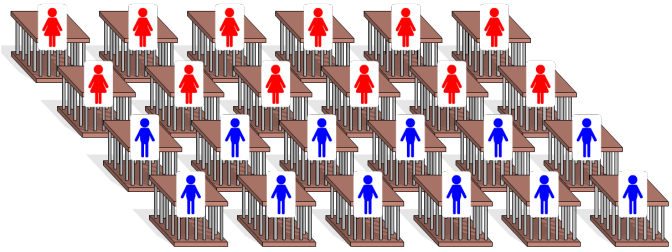
Separation of Males and Females



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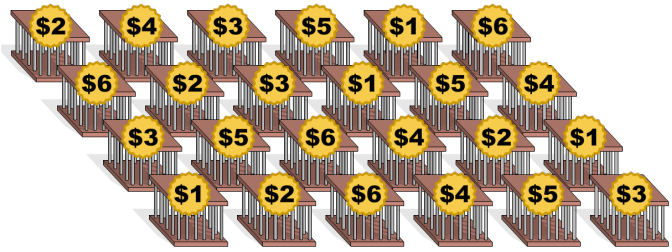
Minimizing the Cost



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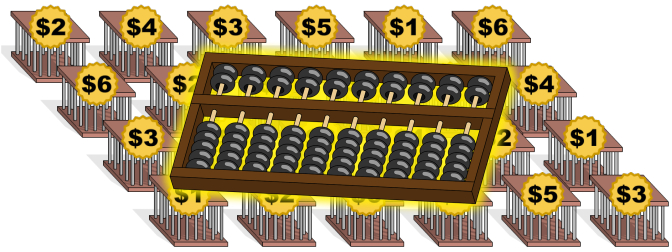
Minimizing the Cost



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CellBlock Data (cellBlock.mzn)

```
enum PRISONER;

int: n;
set of int: ROW = 1..n;
int: m;
set of int: COL = 1..m;
array[ROW,COL] of int: cost;

set of PRISONER: danger;
set of PRISONER: female;
set of PRISONER: male = PRISONER diff female;
```

dependent parameter declarations

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CellBlock Decisions (cellBlock.mzn)

- What are the objects of the domain?
  - DOM = PRISONER
- What are the objects of the codomain?
  - COD = ROW x COL
- Representation: two functions

```
array[PRISONER] of var ROW: r;
array[PRISONER] of var COL: c;
```

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CellBlock Constraints (cellBlock.mzn)

- No two prisoners in the same cell

```
forall(p1, p2 in PRISONER where p1 < p2)
   (abs(r[p1]-r[p2]) + abs(c[p1]-c[p2]) > 0);
```

- Can't we use alldifferent?
- Yes

```
alldifferent([r[p] * m + c[p] | p in PRISONER]);
```

- Mapping each cell block to a unique number
- None adjacent to dangerous prisoners

```
forall(p in PRISONER, d in DANGER where p != d)
   (abs(r[p] - r[d]) + abs(c[p] - c[d]) > 1);
```

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CellBlock Constraints + Objective (cellBlock.mzn)

- Gender constraints

```
forall(p in female)(r[p] <= (n + 1) div 2);
forall(p in male)(r[p] >= n div 2 + 1);
```

- Note the use of `male`
  - clearer than replacing with definition

- Objective function

```
var int: tCost =
   sum(p in PRISONER)(cost[r[p],c[p]]);
solve minimize tCost;
```

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Solving the Model

```
Female: P1 in cell[1,1]
Female: P6 in cell[2,4]
Female: P7 in cell[1,5]
Female: P10 in cell[2,2]

Male: P2 in cell[3,1]
Male: P3 in cell[4,6]
Male: P4 in cell[4,4]
Male: P5 in cell[4,2]
Male: P8 in cell[3,5]
Male: P9 in cell[4,1]

Total Cost: 21
```

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Summary

- Assignment subproblems are common in many applications
- In a next problem for Liu Bei, there is also an assignment subproblem but the function is bijective
  - These are matching problems.

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Image Credits

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