1. Suppose that a one-celled organism can be in one of two states – either A or B. An individual in state A will change to state B at an exponential rate α; an individual in state B divides into two new individuals of type A at an exponential rate β. Define an appropriate CTMC for a population of such organisms and determine the appropriate parameters for this model.

2. Potential customers arrive at a single-server station in accordance with a Poisson process with rate λ. However, if the arrival finds n customers already in the station, then he will enter the system with probability α_n. Assuming an exponential service rate μ, set this up as a birth and death process and determine the birth and death rates.

3. There are N individuals in a population, some of whom have a certain infection that spreads as follows. Contacts between two members of this population occur in accordance with a Poisson process having rate λ. When a contact occurs, it is equally likely to involve any of the $\binom{N}{2}$ pairs of individuals in the population. If a contact involves an infected and a noninfected individual, then with probability p the noninfected individual becomes infected. Once infected, an individual remains infected throughout. Let $X(t)$ denote the number of infected members of the population at time t. Is $\{X(t); t \geq 0\}$ a CTMC?

4. Consider two machines. Machine i operates for an exponential time with rate λ_i and then fails; its repair time is exponential with rate μ_i ,$i = 1, 2$. The machines act independently of each other. Define a four-state continuous-time Markov chain which jointly describes the condition of the two machines. Use the assumed independence to compute the transition probabilities for this chain and then verify that these transition probabilities satisfy the forward and backward equations.

5. The birth and death process with parameters $\lambda_n = 0$ and $\mu_n = \mu, n > 0$ is called a pure death process. Find $p_{ij}(t)$.