MATH 8170,
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HW 2
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1. Let $X_A(t)$ denote the number of organisms in state A at time t.
Let $X_B(t)$ denote the number of organisms in state B.
Then $X = \{X_A, X_B\}$ is a CTMC with sojourn time rate $\upsilon_{ij} = \alpha i + \beta j$ for state (i, j).
The state space is

$$E = \{(i, j)\}_{i,j \geq 0, i+j \geq 1},$$

where $i, j \geq 0$ and $i + j \geq 1$.
The sojourn time T_{ij} for the state (i, j) is exponentially distributed with rate $\alpha i + \beta j$.
The transition probability is :

$$P_{(i,j)(i+2,j-1)} = \frac{\beta j}{\alpha i + \beta j};$$

$$P_{(i,j)(i-1,j+1)} = \frac{\alpha i}{\alpha i + \beta j};$$

2. Let $X(t)$ denote the number of customers in the system at time t.
The time until the next entering/birth $\sim exp(\lambda \alpha_i)$.
The time until the next service ending/death $Y \sim exp(\mu)$.
The birth rate $\lambda \alpha_i$, the death rate is μ.
Then $X(t)$ is a CTMC with sojourn time rate $\upsilon_i = \mu + \lambda \alpha_i$ for state i.
Since it can only change its state by increasing by one or decreasing by one, it is a birth and death procee.
The state space is

$$E = \{0, 1, 2, 3, ...\}$$

The transition probability is :

$$P_{ij} = \left\{ \begin{array}{ll} P(X < Y), & j = i+1, i \geq 1 \\ P(Y < X), & j = i-1, i \geq 1 \end{array} \right. = \left\{ \begin{array}{ll} \frac{\lambda \alpha_i}{\mu + \lambda \alpha_i}, & j = i+1, i \geq 1 \\ \frac{\mu}{\mu + \lambda \alpha_i}, & j = i-1, i \geq 1 \end{array} \right.$$

When $i = 0$, $P_{01} = 1$.