Homework 2, MATH 9010
Due on Thursday, September 8
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Problem 1 Suppose $\mathcal{C}$ is a collection of subsets of Ω that satisfies the following properties: (i) $\Omega \in \mathcal{C}$; (ii) if $A \in \mathcal{C}$, then $A^c \in \mathcal{C}$; (iii) If $\{A_i\}_{i=1}^{\infty}$ is a collection of disjoint subsets in $\mathcal{C}$, then $\cup_{i=1}^{\infty} A_i \in \mathcal{C}$.

Show that $\mathcal{C}$ is a λ-system.

Proof. We will show $\mathcal{C}$ satisfies the definition of λ-system.

(1) $\Omega \in \mathcal{C}$ by property (i).

(2) If $A, B \in \mathcal{C}$ and $A \subset B$, then $B \setminus A = B \cap A^c = (B^c \sqcup A)^c$.
$B^c \in \mathcal{C}$ by property (ii), and then $(B^c \sqcup A) \subset \mathcal{C}$ by property (iii) since $B^c \cap A = \emptyset$.
So $B \setminus A = (B^c \sqcup A)^c \in \mathcal{C}$.

(3) Let $A_n \in \mathcal{C}, n \geq 1$ and $A_1 \subset A_2 \subset A_3 \subset ....$
Define $B_1 = A_1 \in \mathcal{C}$ and $B_n = A_n \setminus A_{n-1}$ for $n \geq 2$.
Then $B_n \in \mathcal{C}$ by (2) and $B_j \cap B_k = \emptyset$.
So $\bigcup_{n=1}^{\infty} A_n = \bigsqcup_{n=1}^{\infty} B_n \in \mathcal{C}$ by property (iii).

Thus, $\mathcal{C}$ is a λ-system. $\square$