Homework 4, MATH 9010
Due on Tuesday, September 27.

Problem 1 Jacod and Protter, page 61 Problem 9.4.

Problem 2 Resnick, page 157 Problem 7.

Problem 3 Resnick, page 157 Problem 10.

Problem 4 Resnick, page 159 Problem 15.

Problem 5 Resnick, page 161 Problem 21.

Problem 6 The Cauchy-Schwarz Inequality is as follows: given two random variables X and Y,

$$\mathbb{E}[|XY|] \leq \sqrt{\mathbb{E}[|X|^2]}\sqrt{\mathbb{E}[|Y|^2]}.$$

Use this inequality to prove the following statement: given a random variable X satisfying $\mathbb{E}[X^2] = 1$, and $\mathbb{E}[|X|] \geq a$ for some $a > 0$, show that for each $\lambda \in [0, 1]$,

$$\mathbb{P}(|X| \geq \lambda a) \geq (1 - \lambda)^2 a^2.$$

Hint: I admit that this is a tricky problem: it helps if you start with the nonnegative number $a(1-\lambda) = a - \lambda a$ and try to get a suitable upper bound on this number that is in terms of quantities associated with X.