

Homework 3, MATH 9010
Due on Thursday, September 15
Shuai Wei

Problem 2 Suppose

$$f : \mathbb{R}^k \mapsto \mathbb{R} \text{ and } f \in \mathcal{B}(\mathbb{R}^k)/\mathcal{B}(\mathbb{R}).$$

Let X_1, \dots, X_k be random variables on (Ω, \mathcal{B}) . Then

$$f(X_1, \dots, X_k) \in \sigma(X_1, \dots, X_k)$$

.

Proof.

Let $X = (X_1, X_2, \dots, X_k)$ be a random vector, then X is $\mathcal{B}/\mathcal{B}(\mathbb{R}^k)$ measurable.

$$\sigma(X) = [X^{-1}(A), A \in \mathcal{B}(\mathbb{R}^k)].$$

Since f is $\mathcal{B}(\mathbb{R}^k)/\mathcal{B}(\mathbb{R})$ measurable, $f^{-1}(B) \in \mathcal{B}(\mathbb{R}^k)$.

Assume the range of X is \mathcal{C} .

$\forall B \in \mathcal{B}(\mathbb{R})$,

if $f^{-1}(B) \notin \mathcal{C}$, $X^{-1}f^{-1}(B) = \emptyset \in \sigma(X)$,

else, $f^{-1}(B) \in \mathcal{C}$, then $X^{-1}f^{-1}(B) \in \sigma(X)$.

So

$$\forall B \in \mathcal{B}(\mathbb{R}), X^{-1}f^{-1}(B) = (f(X))^{-1}(B) \in \sigma(X).$$

Thus, $f(X) \in \sigma(X)$.

Namely, $f(X_1, \dots, X_k) \in \sigma(X_1, \dots, X_k)$.

□