Problem 3 If X is a random variable, so is $|X|$. The converse may be false.

(a) *Proof.*
Suppose $X : \Omega \to \mathbb{R}$, then X is $\mathcal{B}/\mathcal{B}(\mathbb{R})-$measurable.
Let $\pi = |\cdot|$, then $\pi : \mathbb{R} \to \mathbb{R}$ is a continous function.
So $\pi \circ X = |X|$ is also $\mathcal{B}/\mathcal{B}(\mathbb{R})$-measurable.
Thus, $|X|$ is a random variable. $\square$

(b) Counterexample:
Let $A \subset \mathbb{R}$ and $A \notin \mathcal{B}(\mathbb{R})$.
Suppose $X : \mathbb{R} \to \mathbb{R}$.
Define $X = 1_A - 1_{A^c}$ be a simple funtion.
Then X is not $\mathcal{B}(\mathbb{R})/\mathcal{B}(\mathbb{R})$-measurable since
for $1 = [1, 1] \in \mathcal{B}(\mathbb{R})$, $X^{-1}(1) = A \notin \mathcal{B}(\mathbb{R})$.
However, $|X| = 1_{\mathbb{R}}$ is a constant funtion.
Since constant function is a continous function and continuous function is $\mathcal{B}(\mathbb{R})/\mathcal{B}(\mathbb{R})$-measurable,
$|X|$ is $\mathcal{B}(\mathbb{R})/\mathcal{B}(\mathbb{R})$-measurable.