Safety Multiclass Incremental Learning From Biased Source

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***Abstract*—abs**

I. Introduction

Our human beings can learn new things progressively in time. At a very young age, we can distinguish thousands of different objects and this ability can increasingly develop as we grow up. Moreover, we actually don't treat a new concept in isolation, but try to connect the new concept to the knowledge we already learned, which is referred to as transfer learning. For example, we recognize the animal zebra by referring it to the normal horse with distinctive black and white striped coats. Given a task of the target learning problem, transfer learning works on the scenario that knowledge learned from one or several prior (source) tasks can help the target learning task. Based on this, how to utilize the knowledge from multiple sources leads to the research of Multiple Source Transfer Learning (MSTL). Transferring the knowledge from multiple priors can make the learning procedure extremely efficient by mining the recurrent patterns as well as inferring inductively on the target task [1]. Taking advantage of this, the first implementation proposed by [2] using Bayesian approach shows that even with a single example, transfer learning can still get impressive results. Some methods using discriminative approach are proposed in recent years [1] [3] [4]. Previous study shows that the more prior knowledge the system acquired, the easier a new concept can be learned [5].

However, transferring knowledge can consistently boost the learning performance (positive transfer) is based on the fact that the learning procedure can benefit from sufficient related prior knowledge. In some extreme situation, where the source domain and target one are not related, brutal-force transferring the knowledge between them could even degrade the performance of the classifier for target domain. This is often referred to as negative transfer (see Figure 1). How to avoid negative transfer is still an open question in transfer learning [6]. Specifically, how to the measure the transferability of different prior knowledge and obtain a comprehensive and accurate measurement to prevent negative transfer should be studied profoundly. Our work focus on a scenario that transfers the knowledge from multiple sources and we can only access to the model of the source task rather than the data itself. This indicates that the prior knowledge could be incorrect and negative transfer may happen. This scenario can be a very interesting and practical setting in real life, especially for the image recognition task. For certain image database, we can only access to its pre-trained feature representations (like PHOG or color histogram) rather than the images due to the copyright reason or computational prohibition. Meanwhile, the detail of extracting these is not clear as well. Taking PHOG for example, different angle leads to different feature representation and eventually leads to different prior knowledge for classifiers. Without knowing the utility of the prior knowledge, the algorithm should set different weight for different prior knowledge according to their transferability instead of simply ignoring or fully utilizing all of them.

On the other hand, when transferring the knowledge from multiple sources, our human beings are able to distinguish their relationships to the target task by trial-and-error learning and make the decision by balancing the weights between the prior knowledge and empirical knowledge from specific task. For example, a student is asked to pick some pictures of the horse and told what a horse looks like (prior knowledge). After picking some pictures according to the prior knowledge, the student is able to deduce whether the prior knowledge is correct or not (trial-and-error) and pick the correct pictures. If the prior knowledge is incorrect (unrelated prior knowledge), at the beginning, the student would make some mistakes. Then the student will change the strategy to rely more on his/her own knowledge learned from previous pictures over the prior knowledge (increase the weight of empirical knowledge). Inspired by this, we proposed our method that performs with a similar manner. We use Least Square Support Vector Machine (LS-SVM) [7], which is adopted by previous works [1] [3], as our basic model. The decision of each binary LS-SVM is the linear combination of the prior knowledge and empirical knowledge controlled by some transfer parameters. To measure the transferability of each prior knowledge, we estimate our transfer parameters using closed-form leave-one-out (LOO) error. Previous works theoretically suggest that closed-from LOO error can be an efficient way for parameter estimation with a small training set [8] [9]. Then these transfer parameters are optimized by solving a strongly convex problem that can balance the weight between the prior knowledge and empirical knowledge from target task.

In this paper, we also provide the theoretical proof that the transfer parameters estimate by our algorithm can prevent negative transfer. Extensive empirical experiments show that other transfer learning baselines suffer from negative transfer while our method can autonomously ignore the unrelated prior knowledge to prevent negative transfer. Then, we also show

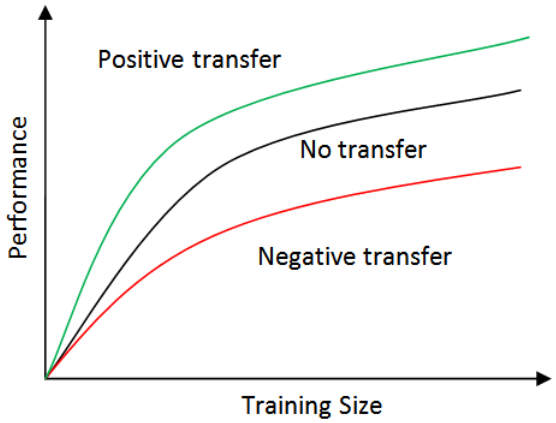

Fig. 1: Positive transfer VS Negative transfer. Relying on unrelated priors could lead to negative transfer.

that when the prior knowledge is highly related to the target task, our method can outperform the transfer learning baselines as well.

The rest of this paper is organized as follow:

II. Related Works

The motivation of transfer knowledge between different domains is to apply the previous information from the source domain to the target one, assuming that there exists certain relationship, explicit or implicit, between the feature space of these two domains [10]. Technically, previous work can be concluded into solving the following three issues: what, how and when to transfer [1].

What to transfer. Previous work tried to answer this question from three different aspects: selecting transferable instances, learning transferable feature representations and transferable model parameters. Instance-based transfer learning assume that part of the instances in the source domain could be re-used to benefit the learning for the target domain. Lim et al. proposed a method of augmenting the training data by borrowing data from other classes for object detection [11]. Learning transferable features means to learn common feature that can alleviate the bias of data distribution in target domain. Recently, Long et al. proposed a method that can learn transferable features with deep neural network and showed some impressive results on the benchmarks [12]. Parameter transfer approach assumes that the parameters of the model for the source task can be transfered to the target task. Yang et al. proposed Adaptive SVMs by transferring parameters by incorporating the auxiliary classifier trained from source domain [13]. On top of Yang's work, Ayatar et al. proposed PMT-SVM that can determine the transfer regularizer according to the target data automatically [14]. Tommasi et al. proposed Multi-KT that can utilize the parameters from multiple source models for the target classes [1]. Kuzborskij et al. proposed a similar method to learn new categories by leveraging over the known source [3].

When and how to transfer. The question *when to transfer* arises when we want to know if the information acquired from previous task is relevant to the new one (i.e. in what situation, knowledge should not be transfered). *How to transfer* the prior knowledge effectively should be carefully designed to prevent inefficient and negative transfer. Some previous work consists in using generative probabilistic method [15] [16] [17]. Bayesian learning methods can predict the target domain by combining the prior source distribution to generate a posterior distribution. Alternatively, some previous max margin methods show that it is possible to learn from a few examples by minimizing the Leave-One-Out (LOO) error for the training model [3] [18]. Previous work shows that there is a closed-form implementation of LOO cross-validation that can generate unbiased model estimation for LS-SVM [9].

Our work correspond to the context above. In this paper, we propose a method based on parameter transfer approach with LS-SVM. We address our work on how to prevent negative transfer when the source data is not accessible. By optimizing the convex objective funtion, our method can autonomously adjust the transfer parameters for different prior knowledge. We theoretically and empirically show that, without any data distrbution assumption, the superior bound of the training loss for our transfer method is the loss of a method learning directly (i.e. without using any prior knowledge). This indecates that when the prior knowledge hurts the transfer procedure, our method can avoid negative transfer. Extensive experiments also show that when the prior knowledge is very related (positive transfer), our method can outperform other methods by relying on the decision of prior knowledge greatly.

III. Problem Statement

Our method works in the following scenario. There is a image dataset (source data) containing N categories and a classifier trained from this dataset to distinguish these N categories. This (source) classifier and the features used to learn it is publicly accessible while the dataset itself is private (unknown distribution). Now we collect our own image dataset (target data) coming from $N+1$ categories. This target dataset consists of N identical categories to the source data and one new category related to the previous N categories. In order to train a new classifier for our new task, we would expect our classifier to get better results with respect to

- Maximize positive transfer. Since we know that these two task share some information, our classifier should transfer useful information as much as possible.
- Minimize negative transfer. The data distribution of the source data is unknown. In the extreme situation, the data distribution of the two task could be totally different. Then these irrelevant prior knowledge should be considered as irrelevant and disposed.

In this paper, we focus our work on transferring the knowledge with LS-SVM as the classifier for multi-class transfer problem. In the following we briefly introduce the mathermatical setting of our problem and show

A. LS-SVM Setting and Definition

Here we introduce the notations used in the rest of the paper. We use any letter with apostrophe to denote the information from the source data, e.g. if $f(x)$ denotes the model for the target task, $f'(x)$ denotes the model for the source one.

TABLE I: useful notations in this paper

$f'(x)$	binary function for source task
$f(x)$	binary function for target task
$\phi(x)$	function mapping the input sample into a high dimensional feature space.
$K(x,x)$	kernel matrix with $\phi(x_i)\cdot\phi(x_j)$ corresponding to its element (i,j)
X	instance matrix with each row representing one instance
W	(N+1)-column hyperplane matrix for target task. Each column represents one hyperplane of a binary model
W'	hyperplane matrix for the source task
a'	the Lagrangian multiplier matrix for source problem. Each column represents a set of
a	the Lagrangian multiplier matrix for target problem
b', b	the bias vector for source and target task
a_i, w_i	i_{th} column of matrix a and w
d_γ	diagonal matrix with$[\gamma_1, ..., \gamma_N]$ in its main diagonal
β	row vector $[\beta_1, ..., \beta_N]$ to control the prior knowledge for the new category
ε_{ny_i}	loss parameter. $\varepsilon_{ny_i}=1$ if $n=y_i$ and 0 otherwise

Assume that, for our $(N+1)$-category target task, $x\in\mathcal{X}$ and $y\in\mathcal{Y}=\{1,2,...,N+1\}$ are the input vector and output for the learning task respectively. Meanwhile, we have a set of binary linear classifiers $f'_n(x)=\phi(x)w'_n+b'_n$, for $n=1,...,N$ trained from an unknown distribution with One-Versus-All (OVA) strategy. Now we want to learn a set of new classifier $f_n(x)=\phi(x)w_n+b_n, n=1,...,N+1$, so that example x is assigned to the category j if $j\equiv\arg\max_{n=1,...,N+1}\{f_n(x)\}$. In LS-SVM, the solution of the model parameters (w_n,b_n) can be found by solving the following optimization problem:

$$\textbf{min}\quad R(w_n)+\frac{C}{2}\sum_{i}^{l}(Y_{i,n}-\phi(x_i)w_n-b_n)^2$$

Where $R(w_n)$ is the regularization term to guarantee good generalization performance and avoid overfitting. $\mathbf{Y}$ is a encoded label matrix so that $Y_{in}=1$ if $y_i=n$ and -1 otherwise.

In classic LS-SVM setting, the regularization term is set to $\frac{1}{2}\|w_n\|^2$ and the optimal $w_n=\phi(X)^T\alpha_n$ while the parameters $(\aleph_n,b_n)$ can be found by solving

$$\begin{bmatrix} K(X,X)+\frac{1}{C}\mathrm{I} & \mathbf{1} \\ \mathbf{1^T} & 0 \end{bmatrix}\begin{pmatrix}\alpha_n\\ b_n\end{pmatrix}=\begin{pmatrix}Y_n\\0\end{pmatrix} \tag{1}$$

Here I is the identity matrix and $\mathbf{1}$ is a column vector with all its elements equal to 1.

Now our task can be divided into two separate part: learning the N overlapped categories and the new category. We know that the source and target share N categories. From previous work [13], the regularization term can be written as $\frac{1}{2}\|w_n-\gamma_n w'_n\|^2$. Here, γ_n is the regularization parameter controlling the amount of transfer. For the task for new category, we can use multi-source kernel learning strategy in [1]

So the multi-class transfer problem can be solved by optimizing the following objective function:

$$\begin{aligned}\textbf{min}\quad & \frac{1}{2}\sum_{n=1}^{N}\|w_n-\gamma_n w'_n\|^2+\frac{1}{2}\left\|w_{N+1}-\sum_{k=1}^{N}w'_k\beta_k\right\|^2\\ & \frac{C}{2}\sum_{n=1}^{N+1}\sum_{i=1}^{l}e_{i,n}^2\\ \textbf{s.t.}\quad & e_{i,n}=Y_{i,n}-\phi(x_i)w_n-b_n\end{aligned} \tag{2}$$

The closed-form of the optimal solution to Eq. (2) is:

$$\begin{aligned} w_n&=\gamma_n w'_n+\sum_{i}^{l}\alpha_{in}\phi(x_i) \qquad n=1,...,N\\ w_{N+1}&=\sum_{k}^{N}\beta_k w'_k+\sum_{i}^{l}\alpha_{i(N+1)}\phi(x_i)\end{aligned}$$

Here α_{ij} is the element (i,j) in $\boldsymbol{\alpha}$. The intuitive interpretation of the results above is that the hyperplane of the target problem is the linear combination of the prior knowledge (first part of the right side) and empirical knowledge from target task (second part of the right side).

Let ψ denotes the first term of left-hand side in Eq. (1) and let:

$$\begin{aligned}\psi\begin{bmatrix}\boldsymbol{\alpha}'\\ \boldsymbol{b}'\end{bmatrix}&=\begin{bmatrix}Y\\0\end{bmatrix}\\ \psi\begin{bmatrix}\boldsymbol{\alpha}''\\ \boldsymbol{b}''\end{bmatrix}&=\begin{bmatrix}X(W')^T\\0\end{bmatrix}\end{aligned} \tag{3}$$

We have:

$$\boldsymbol{\alpha}=\boldsymbol{\alpha}'-\begin{bmatrix}\boldsymbol{\alpha}''d_r & \boldsymbol{\alpha}''\boldsymbol{\beta^T}\end{bmatrix} \tag{4}$$

From Eq. (4) we can see that, the solution of Eq. (1) is completed once $\boldsymbol{\gamma}=[\gamma_1,...,\gamma_N]$ and $\boldsymbol{\beta}$ are set.

B. Optimize γ and β

introduce LOO error estimation. In this part, we introduce our method to estimate proper $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ that can prevent negative transfer. From above, we can see that the hyperplane for the target problem is determined by $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$. Negative transfer happens when the model aggressively leverage over irrelevant prior knowledge, i.e. set a large value to $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$. However, aggressive leverage over informative priors can improve the performance of the transfer model greatly. Inspired by some previous works [1] [3], we proposed our method that can minimize the affect of negative transfer from unrelated priors.

As we mentioned above, another important advantage of LS-SVM over the other model is that we can get unbiased LOO error in closed form [9]. The unbiased LOO estimation for sample x_i can be written as:

$$\hat{Y}_{i,n}=Y_{i,n}-\frac{\alpha_{in}}{\psi^{-1}_{ii}}\ \text{for}\ n=1,...,N+1 \tag{5}$$

Here ψ^{-1} is the inverse of matrix ψ and ψ^{-1}_{ii} is its ith diagonal element.

Let us call ξ_i the empirical error of our multi-class prediction for example x_i, and ξ_i can be defined as [19]:

$$\xi_i(\gamma,\beta)=\max_{n\in\{1,...,N+1\}}\left[1-\varepsilon_{ny_i}+\hat{Y}_{in}(\gamma,\beta)-\hat{Y}_{iy_i}(\gamma,\beta)\right] \tag{6}$$

Where $\varepsilon_{ny_i} = 1$ if $n = y_i$ and 0 otherwise. $\xi_i(\gamma, \beta) > 0$ if example x_i is misclassified. The intuition behind this loss function is to enforce the distance between the true class and other classes to be at least 1.

Then we define our objective function as:

$$\begin{aligned} \textbf{min} \quad & \frac{\lambda_1}{2}\sum_{n=1}^{N}\|\gamma_n\|^2 + \frac{\lambda_2}{2}\sum_{n=1}^{N}\|\beta_n\|^2 + \sum_{i=1}^{l}\xi_i \\ \textbf{s.t.} \quad & 1 - \varepsilon_{ny_i} + \hat{Y}_{in}(\gamma,\beta) - \hat{Y}_{iy_i}(\gamma,\beta) \le \xi_i; \\ & \lambda_1, \lambda_2 \ge 0 \end{aligned} \tag{7}$$

Here λ_1 and λ_2 are two regularization parameters to prevent overfitting. From the objective function above we can see that, for certain λ_1 and λ_2, when the prior knowledge is unrelated and negative transfer happens, increasing $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ leads to larger punishment from both regularization and empirical error from target task. Decreasing the affect of prior knowledge reduces the loss of the objective function and eventually prevents negative transfer. Moreover, we also prove that this objective function can avoid negative transfer (for more details, see Theorem 1). On the other hand, if the prior knowledge is related, even though, increasing $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ leads to larger punishment, it also leads to smaller empirical error on the target problem. So the algorithm compromises between the prior and empirical knowledge. Besides, there are some other properties that make our method efficient.(see Section III-C)

By adding a dual set of variables, one for each constraint, we get the Lagrangian of the optimization problem:

$$\begin{aligned} \textbf{max} \quad & L(\gamma,\beta,\xi,\eta) = \\ & \frac{\lambda_1}{2}\sum_{n=1}^{N}\|\gamma_n\|^2 + \frac{\lambda_2}{2}\sum_{n=1}^{N}\|\beta_n\|^2 + \sum_{i=1}^{l}\xi_i \\ & + \sum_{i,n}\eta_{i,n}\left[1 - \varepsilon_{ny_i} + \hat{Y}_{in}(\gamma,\beta) - \hat{Y}_{iy_i}(\gamma,\beta) - \xi_i\right] \\ \textbf{s.t.} \quad & \forall i,n \quad \eta_{i,n} \ge 0 \end{aligned} \tag{8}$$

The problem of Eq. (8) is a non-differentiable strongly convex problem. The sub-gradient of it can be written as:

$$\Delta_\gamma = \begin{cases} \mathbf{0} & y_i = n \\ \left[0,..,\frac{\alpha''_{in}}{\psi_{ii}^{-1}},..,-\frac{\alpha''_{iy_i}}{\psi_{ii}^{-1}},..,0\right] & y_i, n = 1,...N \\ \left[0,..,\frac{\alpha''_{in}}{\psi_{ii}^{-1}},..,0\right] & y_i = N+1; n = 1,...N \\ \left[0,..,-\frac{\alpha''_{iy_i}}{\psi_{ii}^{-1}},..,0\right] & \text{otherwise} \end{cases}$$

$$\Delta_\beta = \begin{cases} -\sum \alpha''_{ik}\beta_k & y_i = N+1; n = 1,...N \\ \sum \alpha''_{ik}\beta_k & y_i = 1,..N; n = N+1 \\ \mathbf{0} & \text{otherwise} \end{cases}$$

To obtain the optimal values for the problem above, we introduce our method using sub-gradient descent [20] and summarize it in Alg. 1.

C. Analysis

In this part, we mainly discuss our method in two aspects: convergence analysis and mathematical proof of preventing negative transfer.

Algorithm 1 γ optimization

Input: $\psi, \alpha', \alpha'', T, \psi,$
Output: $\gamma = \{\gamma^1, ..., \gamma^n\}, \beta$

$\beta^0 \leftarrow 0, \gamma^0 \leftarrow 1$
for $t = 1$ to T **do**
  $\hat{Y} \leftarrow Y - (\psi \circ I)^{-1}\left(\alpha' - \left[\begin{array}{cc} \alpha'' d_\gamma & \alpha'' \beta^T \end{array}\right]\right)$
  $\Delta_\gamma = 0, \Delta_\beta = 0$
  for $i = 1$ to l **do**
    $\Delta_\gamma \leftarrow \Delta_\gamma + \lambda_1\gamma$
    $\Delta_\beta \leftarrow \Delta_\beta + \lambda_2\beta$
    for $r = 1$ to $N+1$ **do**
      $l_{ir} = 1 - \varepsilon_{y_i r} + \hat{Y}_{ir} - \hat{Y}_{iy_i}$
      if $l_{ir} > 0$ **then**
        if $y_i, r \in \{1, ..., N\}$ **then**
          $\Delta_\gamma^{y_i} \leftarrow \Delta_\gamma^{y_i} - \frac{\alpha''_{iy_i}}{\psi_{ii}^{-1}}$
          $\Delta_\gamma^{r} \leftarrow \Delta_\gamma^{r} + \frac{\alpha''_{ir}}{\psi_{ii}^{-1}}$
        else if $y_i = N+1$ **then**
          $\Delta_\beta \leftarrow \Delta_\beta - \frac{\alpha''_i}{\psi_{ii}^{-1}}$
          $\Delta_\gamma^{r} \leftarrow \Delta_\gamma^{r} + \frac{\alpha''_{ir}}{\psi_{ii}^{-1}}$
        else
          $\Delta_\gamma^{y_i} \leftarrow \Delta_\gamma^{y_i} - \frac{\alpha''_{iy_i}}{\psi_{ii}^{-1}}$
          $\Delta_\beta \leftarrow \Delta_\beta + \frac{\alpha''_i}{\psi_{ii}^{-1}}$
        end if
      end if
    end for
  end for
  $\beta^t \leftarrow \beta^{(t-1)} - \frac{\Delta_\beta}{l \times t}$
  $\gamma^t \leftarrow \gamma^{(t-1)} - \frac{\Delta_\gamma}{l \times t}$
end for

Convergence analysis The primal problem (7) becomes the strongly convex problem by adding the L2 regularization terms. Optimizing the strongly convex problem can lead to the following error bound:

Let $\mu_1, ..., \mu_t$ be a sequence corresponding to $\mu_t = (\sqrt{\lambda_1}\gamma^t, \sqrt{\lambda_2}\beta^t)$. Problem (7) can be rewritten as:

$$J(\mu) = \frac{1}{2}\|\mu\|^2 + \sum_{i=1}^{l}\xi_i(\mu)$$

Let Δ_t be the sub-gradient for $J(\mu_t)$ and $\mu^* = (\sqrt{\lambda_1}\gamma^*, \sqrt{\lambda_2}\beta^*)$ be the optimal solution for it. Assume that $\|\Delta_t\| \le G$. According to Lemma 1 in [21], we have:

$$J(\mu_t) - J(\mu^*) \le \frac{G^2}{2t}(1 + \ln(t)) \tag{9}$$

This means our method converges at the rate of $O(\frac{\log(t)}{t})$.

Superior bound analysis

Theorem 1: Assume that $\bar{\xi}_i$ is the multi-class loss of example x_i when $\gamma = \beta = \mathbf{0}$. Let γ^*, β^* be the optimal solution for Eq. (8) and ξ_i^* be the multi-class loss with respective to example x_i. Then for every example $x_i \in \mathcal{X}$, we have:

$$\sum_i \xi_i \le \sum_i \bar{\xi}_i$$

Proof: For simplification, let $\delta_i = 1$ if $i = N+1$ and 0 otherwise, and $\theta_{ij} = \alpha''_{ij}(1-\delta_j)/\psi_{ii}^{-1}$. Eq. (6) can be written as:

$$\begin{aligned}\xi_i(\gamma,\beta) = &\max_n \left\{ \varepsilon_{ny_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{in})}{\psi_{ii}^{-1}} + \theta_{in}\gamma_n \right. \\ &\left. - \theta_{iy_i}\gamma_{y_i} + (\delta_n - \delta_{y_i}) \sum_k \frac{\alpha''_{ik}\beta_k}{\psi_{ii}^{-1}} \right\}\end{aligned} \tag{10}$$

When $\gamma = \beta = \mathbf{0}$, from Eq. (10) we can get:

$$\bar{\xi}_i = \max_n \left[\varepsilon_{ny_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{in})}{\psi_{ii}^{-1}} \right]$$

To obtain the optimal value of γ and β, we have to seek the saddle point of the Lagrangian problem in (8) by finding the minimum for the prime variables $\{\gamma, \beta, \xi\}$ and the maximum for the dual variables η. To find the minimum of the primal problem, we require:

$$\frac{\partial L}{\partial \xi_i} = 1 - \sum_n \eta_{in} = 0 \rightarrow \sum_n \eta_{in} = 1$$

Similarly, for γ and β, we require:

$$\begin{aligned}\frac{\partial L}{\partial \gamma_n} &= \lambda_1\gamma_n + \sum_i \eta_{in}\theta_{in} - \sum_{i,n=y_i} \left(\sum_q \eta_{iq} \right) \theta_{in}\gamma_n \\ &=_1 \lambda_1\gamma_n + \sum_i \eta_{in}\theta_{in} - \sum_i \varepsilon_{ny_i}\theta_{in} = 0 \\ &\Rightarrow \gamma_n^* = \frac{1}{\lambda_1} \sum_i (\varepsilon_{ny_i} - \eta_{in})\theta_{in}\end{aligned} \tag{11}$$

In $=_1$ we use the facts that $\sum_n \eta_{in} = 1$ and use ε_{ny_i} to replace it.

$$\begin{aligned}\frac{\partial L}{\partial \beta_n} &= \lambda_2\beta_n + \left[\sum_{i,n} \frac{\eta_{in}\alpha''_{in}}{\psi_{ii}^{-1}} (\delta_n - \delta_{y_i}) \right] = 0 \\ &\Rightarrow \beta_n^* = \frac{1}{\lambda_2} \sum_{i,n} \frac{\eta_{in}\alpha''_{in}}{\psi_{ii}^{-1}} (\delta_{y_i} - \delta_n)\end{aligned} \tag{12}$$

As the strong duality holds,the primal and dual objectives coincide. Plug Eq (11) and (12) into Eq. (8), we have:

$$\sum_{i,n} \eta_{in} \left[1 - \varepsilon_{ny_i} + \hat{Y}_{in}(\gamma^*, \beta^*) - \hat{Y}_{iy_i}(\gamma^*, \beta^*) - \xi_i^* \right] = 0$$

Expand the equation above, we have:

$$\begin{aligned}&\sum_{i,n} \eta_{in} \left[\varepsilon_{n,y_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{in})}{\psi_{ii}^{-1}} - \xi_i \right] \\ &= \lambda_1 \sum_r \|\gamma_r^*\|^2 + \lambda_2 \sum_r \|\beta_r^*\|^2 \geq 0\end{aligned}$$

Rearranging the above, we obtain:

$$\begin{aligned}&\sum_{i,n} \eta_{in} \left[\varepsilon_{n,y_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{in})}{\psi_{ii}^{-1}} \right] \\ &\geq \sum_{i,n} \eta_{in}\xi_i = \sum_i \xi_i\end{aligned} \tag{13}$$

The left-hand side of Inequation (13) can be bounded by:

$$\begin{aligned}&\sum_{i,n} \eta_{in} \left[\varepsilon_{ny_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{in})}{\psi_{ii}^{-1}} \right] \\ &\leq \sum_i \left(\sum_n \eta_{in} \max_r \left\{ \varepsilon_{ry_i} - 1 + \frac{(\alpha'_{iy_i} - \alpha'_{ir})}{\psi_{ii}^{-1}} \right\} \right) \\ &= \sum_i \left(\sum_n \eta_{in}\bar{\xi}_i \right) = \sum_i \bar{\xi}_i\end{aligned} \tag{14}$$

■

When setting $\gamma = \beta = \mathbf{0}$, we don't utilize any knowledge from previous task (see Eq. (2)). From Theorem 1 we can conclude our method can always outperform the method learning directly.

discuss λ

IV. Experiment

In this section, we show empirical results of our algorithm on different transferring situations on two datasets: AwA[1] [22] and Caltech-256[2] [23]. We design the following 3 sets of experiments: learning from informative prior, irrelevant prior and mixed prior, to show the effectiveness of our algorithm.

A. Dataset & Settings

Caltch-256 contains 30607 images from 256 categories. We select the following 10 categories: *bat, bear, dolphin, giraffe, gorilla, horse, leopard, raccoon, skunk, zebra* as our dataset.

AwA dataset consists of 50 animal categories. Its source images is not publicly accessible and we can only access the six pre-extracted feature representations for each image. This property makes it natural as the unknown distribution source dataset to train the prior knowledge. We choose the identical 10 categories as those in Caltech-256 as the source dataset.

B. Baselines

We compare our algorithm with two kinds of baselines. The first one is methods without leveraging any prior knowledge (no transfer baselines). The second consists of some methods with transfer techniques. Here are the no transfer baselines.

0+T(arget): LS-SVM trained only on target data. This baseline can be the indicator as the best performance in the bad oracle experiment.

S(ource)+T(arget): This baseline is only used in good oracle experiment. We combined the source and target data, assuming that we have fully access to all data, to train the LS-SVM. The result of this baseline might be considered as the best performance achieved in the experiment as well as an important reference for assessing the models with transfer learning methods.

S(ource)+1: This method only train a new binary LS-SVM for the new category. For the rest of the classes, we use the

[1]The features of AwA dataset is available from http://attributes.kyb.tuebingen.mpg.de/

[2]Images for Caltech-256 is available from http://www.vision.caltech.edu/Image_Datasets/Caltech256/

predictions of the classifiers trained from source data directly. This is arguably the easiest way for transfer learning. In some of our experiments, it is a good indicator when negative transfer happens.

We select the following 3 methods as our transfer baselines. The general property of these 3 methods is that they all try to leverage multiple prior knowledge to benefit the transfer procedure.

MKTL [4]: This method uses the output of prior models as extra feature inputs, and automatically determine from which prior models to transfer and how much to transfer.

Multi-KT [1]: This method has similar idea with MKTL. It uses LOO error to determine how much to transfer from prior models and convert it into solving the convex optimization problem.

MULTIpLE [3]: The basic setting of this method is similar like ours. It is designed to balance the performance between learning the new category and preserving the model from prior knowledge.

C. transfer from good oracle

TABLE II: Overall Caltech to Caltech

	5	10	15	20
No transfer	27.33	31.53	35.73	38.47
Source+1	43.33 ± 5.96	43.87 ± 6.14	44.33 ± 6.00	44.57 ± 5.99
MKTL	38.89 ± 8.54	43.27 ± 7.66	45.72 ± 6.41	47.44 ± 6.96
MULTIKT	37.96 ± 5.99	42.89 ± 5.77	45.96 ± 6.25	47.32 ± 5.73
MULTIpLE	42.63 ± 6.35	45.63 ± 5.99	47.81 ± 5.92	48.73 ± 6.01
Gama	**43.53 ± 5.53**	**46.45 ± 5.31**	**48.25 ± 5.74**	**49.15 ± 5.90**
Batch	43.77 ± 5.99	44.73 ± 6.01	46.67 ± 5.71	48.00 ± 5.35

TABLE III: Overall AwA to AwA

	5	10	15	20
No transfer	23.52	26.79	29.60	31.50
Source+1	39.00 ± 2.13	39.34 ± 1.95	39.62 ± 1.94	39.74 ± 2.02
MKTL	31.46 ± 2.51	34.76 ± 2.37	37.41 ± 2.10	38.81 ± 2.03
MULTIKT	29.86 ± 1.52	32.86 ± 1.31	35.22 ± 1.30	36.33 ± 1.26
MULTIpLE	37.80 ± 2.11	38.81 ± 2.06	39.80 ± 1.96	40.47 ± 1.96
Gama	**37.83 ± 2.38**	**39.31 ± 2.26**	**40.37 ± 2.18**	**41.09 ± 2.05**
Batch	39.62 ± 1.98	40.18 ± 2.05	40.67 ± 2.03	41.44 ± 1.93

D. from bad oracle

E. mixed

V. Conclusion

Acknowledgment

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TABLE IV: Overall AwA to Caltech

	5	10	15	20	25	30
No transfer	**30.99 ± 2.61**	33.97 ± 2.79	35.95 ± 1.94	37.78 ± 1.81	38.27 ± 1.94	39.39 ± 1.61
Source+1	17.89 ± 3.44	18.69 ± 2.66	18.79 ± 2.32	19.69 ± 2.16	19.39 ± 2.70	20.20 ± 1.94
MKTL	25.19 ± 4.14	30.14 ± 4.18	32.53 ± 4.00	34.30 ± 3.39	35.83 ± 3.19	36.66 ± 3.22
MULTIKT	27.60 ± 1.90	32.19 ± 2.88	34.51 ± 2.52	36.78 ± 1.68	37.79 ± 2.00	39.27 ± 2.00
MULTIpLE	29.79 ± 2.41	33.45 ± 2.20	35.49 ± 1.79	36.77 ± 1.48	37.43 ± 1.61	38.62 ± 1.61
Gama	30.93 ± 2.55	**34.13 ± 2.94**	**36.09 ± 2.09**	**38.01 ± 1.76**	**38.46 ± 1.95**	**39.59 ± 1.65**

TABLE V: AwA leave class 6 as new category. 2345 as bad class

	5	10	15	20	25	30
no transfer	23.45 ± 1.55	26.97 ± 2.08	29.64 ± 2.40	31.62 ± 2.38	32.59 ± 2.64	34.44 ± 2.40
source+1	17.35 ± 1.05	18.43 ± 1.57	17.66 ± 1.15	18.18 ± 1.47	18.59 ± 1.65	19.16 ± 1.44
MKTL	21.73 ± 2.45	25.19 ± 2.83	28.45 ± 2.13	31.46 ± 2.38	31.74 ± 4.62	32.44 ± 3.17
MultiKT	21.78 ± 2.08	25.60 ± 2.34	28.84 ± 2.68	31.29 ± 2.26	32.13 ± 2.56	33.78 ± 2.23
MULTIpLE	22.13 ± 1.83	26.19 ± 2.04	29.29 ± 2.39	31.52 ± 2.36	32.86 ± 2.37	34.37 ± 2.30
Gama	25.14 ± 1.31	27.89 ± 1.91	30.31 ± 2.47	32.14 ± 2.52	33.08 ± 2.70	34.78 ± 2.46