



Virtual Memory: Concepts

15-213/15-513/14-513: Introduction to Computer Systems

11th Lecture, Sept 30, 2025

Midterm “Exam”

■ Take home exam (15-213/15-513/14-513) :

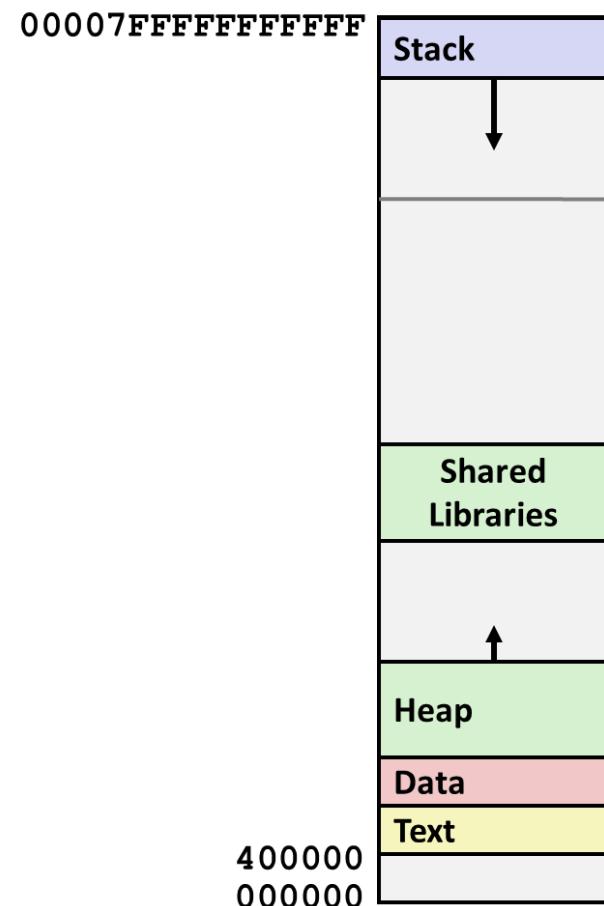
- In-person take home exam released on Wed, Oct 1
- Due on Wed, Oct 8 at 11:59pm
- Submitted to Gradescope
- 5% of course grade for 15-213/14-513 students
- 15% of course grade ONLY for 15-503 students

■ In person exam (15-213/14-513-only):

- In-person individual exam on 10/21 at 12:30pm
- 80 minutes to complete, worth 10% of course grade
- 2 double sided (letter or A4) sheets of notes allowed
- Worth 10% of final grade
- Questions may be similar or identical to take home questions

This Picture is a Lie

- This is RAM, we said...
- But the computer can run more than one program at a time!
- Where are all the other programs?
- Let's *investigate*.



Processes

■ Definition: A *process* is an instance of a running program.

- One of the most profound ideas in computer science
- Not the same as “program” or “processor”

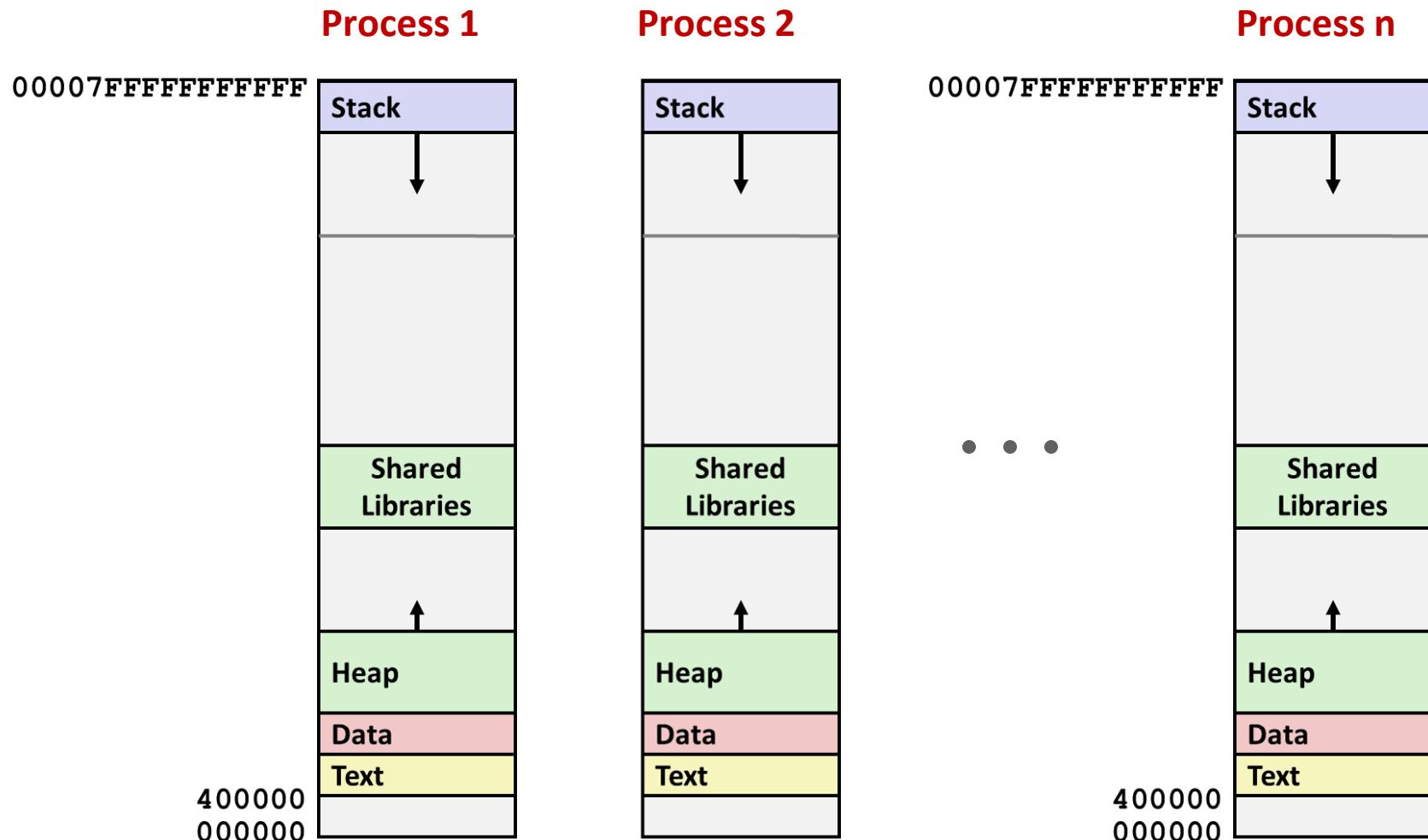
■ Unix: A *parent process* creates a new *child process* by calling `fork`

- Child is (sort of) a copy of the parent
- `fork` returns *twice*—once in each process
 - Different return value in each

■ Parent can wait for child to finish by calling `waitpid`

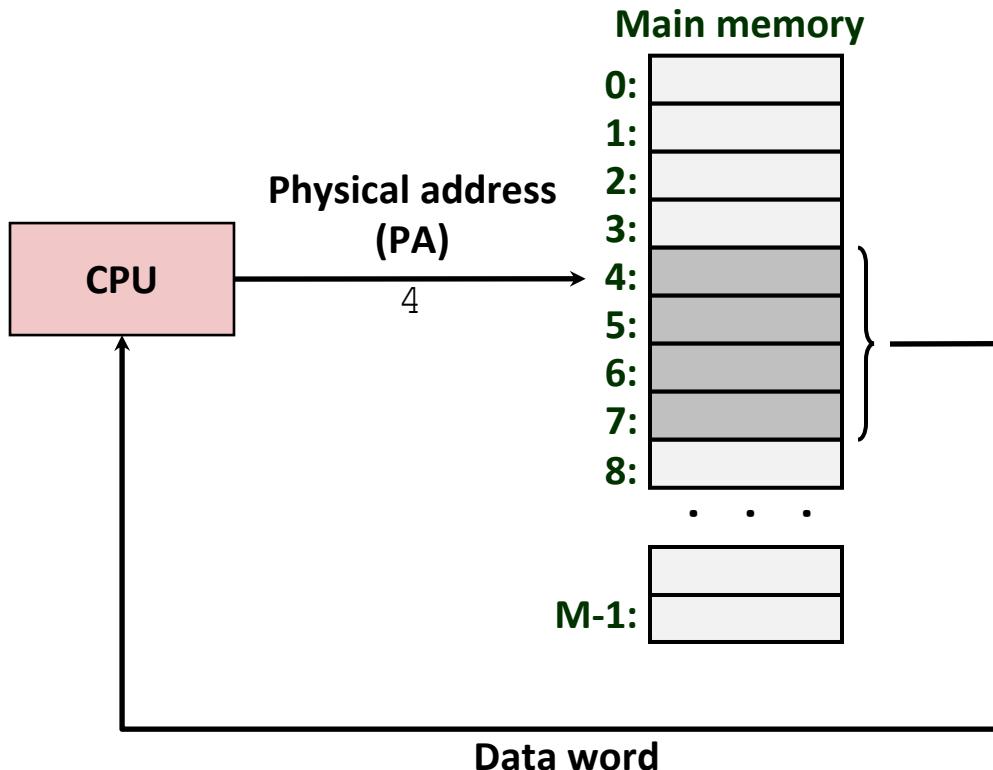
- For now, think of this as “what `main` returns to”

Hmmm, How Does This Work?!



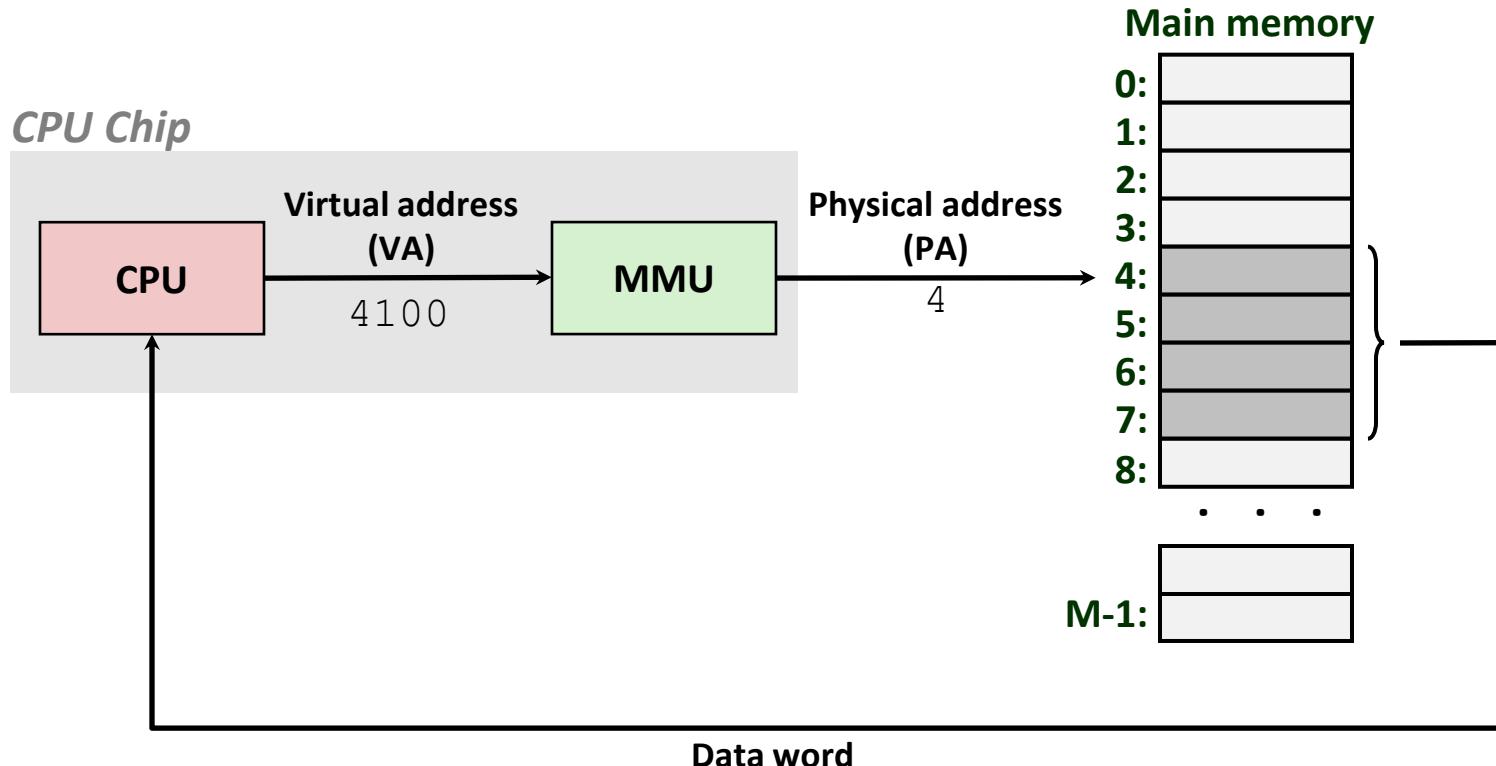
Solution: Virtual Memory (today and next lecture)

A System Using Physical Addressing



- Used in “simple” systems like embedded microcontrollers in devices like cars, elevators, and digital picture frames

A System Using Virtual Addressing

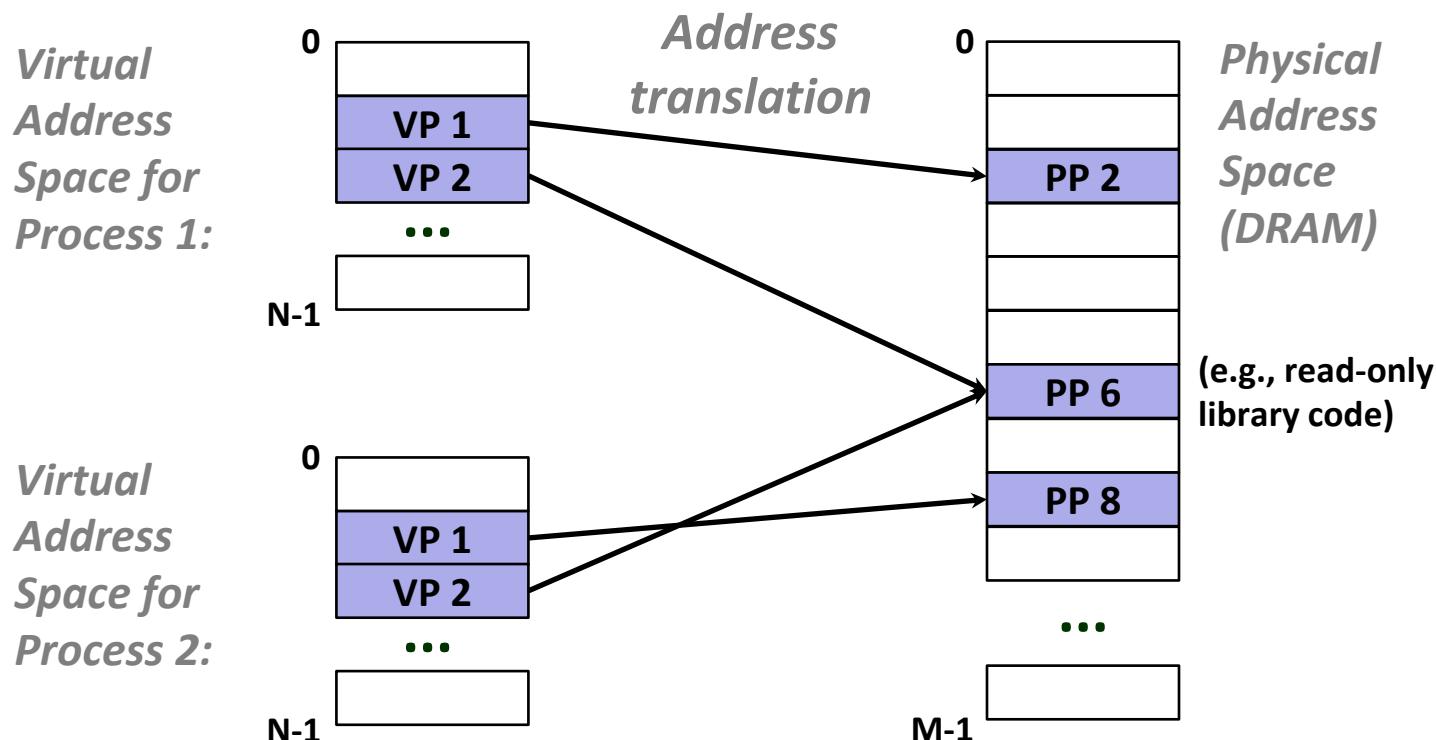


- Used in all modern servers, laptops, and smart phones
- One of the great ideas in computer science

VM as a Tool for Memory Management

■ Key idea: each process has its own virtual address space

- It can view memory as a simple linear array
- Mapping function scatters addresses through physical memory
 - Well-chosen mappings can improve locality



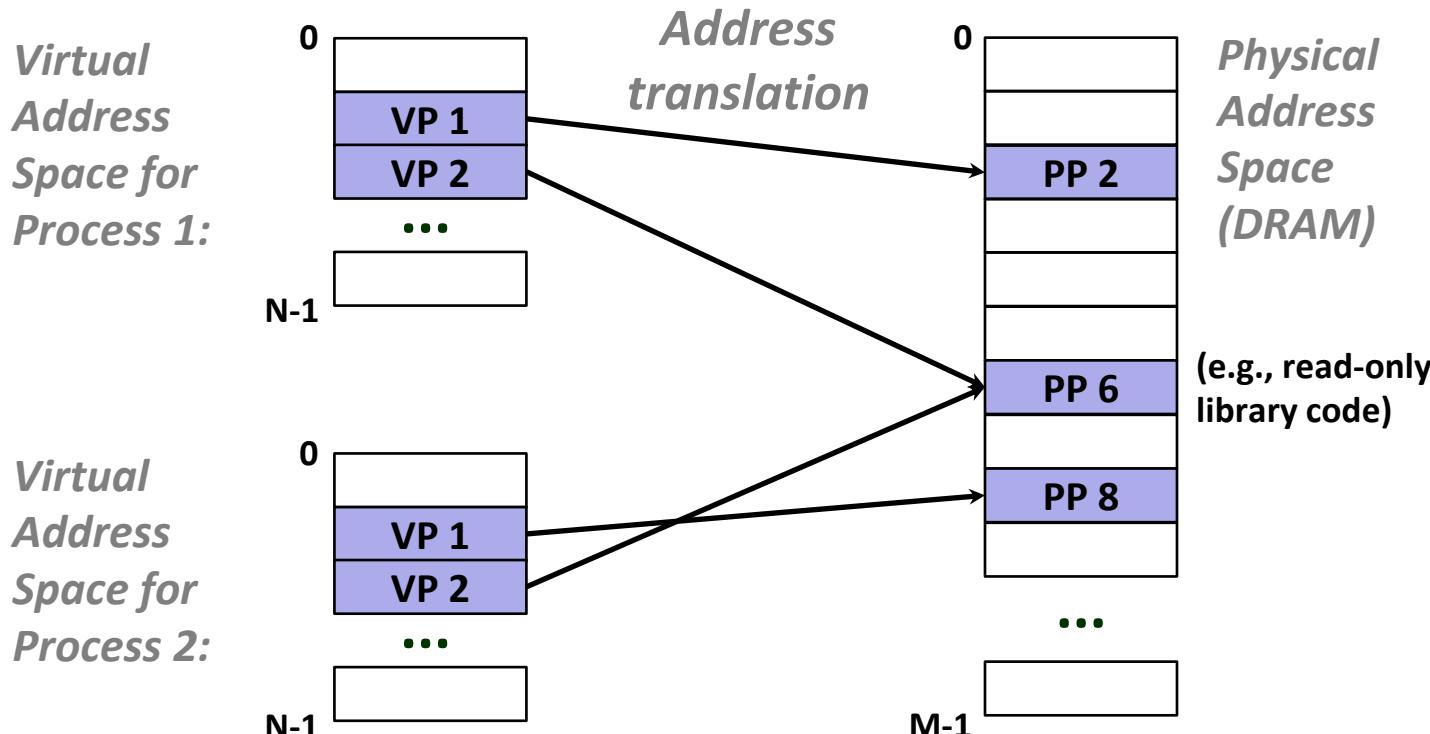
VM as a Tool for Memory Management

■ Simplifying memory allocation

- Each virtual page can be mapped to any physical page
- A virtual page can be stored in different physical pages at different times

■ Sharing code and data among processes

- Map virtual pages to the same physical page (here: PP 6)



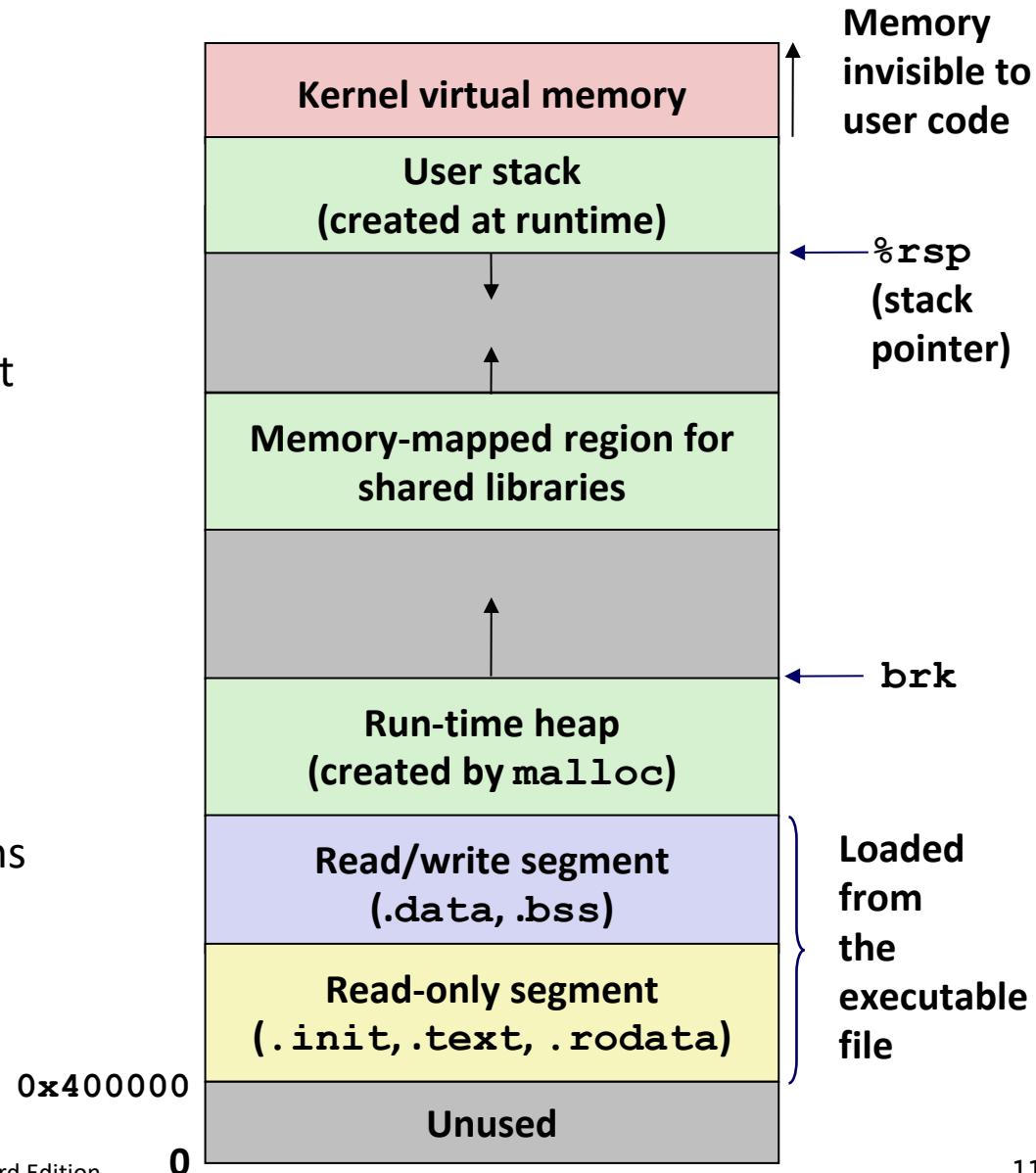
Simplifying Linking and Loading

■ Linking

- Each program has similar virtual address space
- Code, data, and heap always start at the same addresses.

■ Loading

- `execve` allocates virtual pages for `.text` and `.data` sections & creates PTEs marked as invalid
- The `.text` and `.data` sections are copied, page by page, on demand by the virtual memory system



Address Spaces

- **Linear address space:** Ordered set of contiguous non-negative integer addresses:

$\{0, 1, 2, 3 \dots\}$

- **Virtual address space:** Set of $N = 2^n$ virtual addresses

$\{0, 1, 2, 3, \dots, N-1\}$

- **Physical address space:** Set of $M = 2^m$ physical addresses

$\{0, 1, 2, 3, \dots, M-1\}$

Why Virtual Memory (VM)?

■ Uses main memory efficiently

- Use DRAM as a cache for parts of a virtual address space

■ Simplifies memory management

- Each process gets the same uniform linear address space

■ Isolates address spaces

- One process can't interfere with another's memory
- User program cannot access privileged kernel information and code

VM Address Translation

■ Virtual Address Space

- $V = \{0, 1, \dots, N-1\}$

■ Physical Address Space

- $P = \{0, 1, \dots, M-1\}$

■ Address Translation

- $MAP: V \rightarrow P \cup \{\emptyset\}$
- For virtual address a :
 - $MAP(a) = a'$ if data at virtual address a is at physical address a' in P
 - $MAP(a) = \emptyset$ if data at virtual address a is not in physical memory
 - Either invalid or stored on disk

MAP: V \rightarrow P

■ Mapping function from virtual pages to physical pages

- Page is the granularity of mapping set by the ISA

■ Function must be simple and efficient

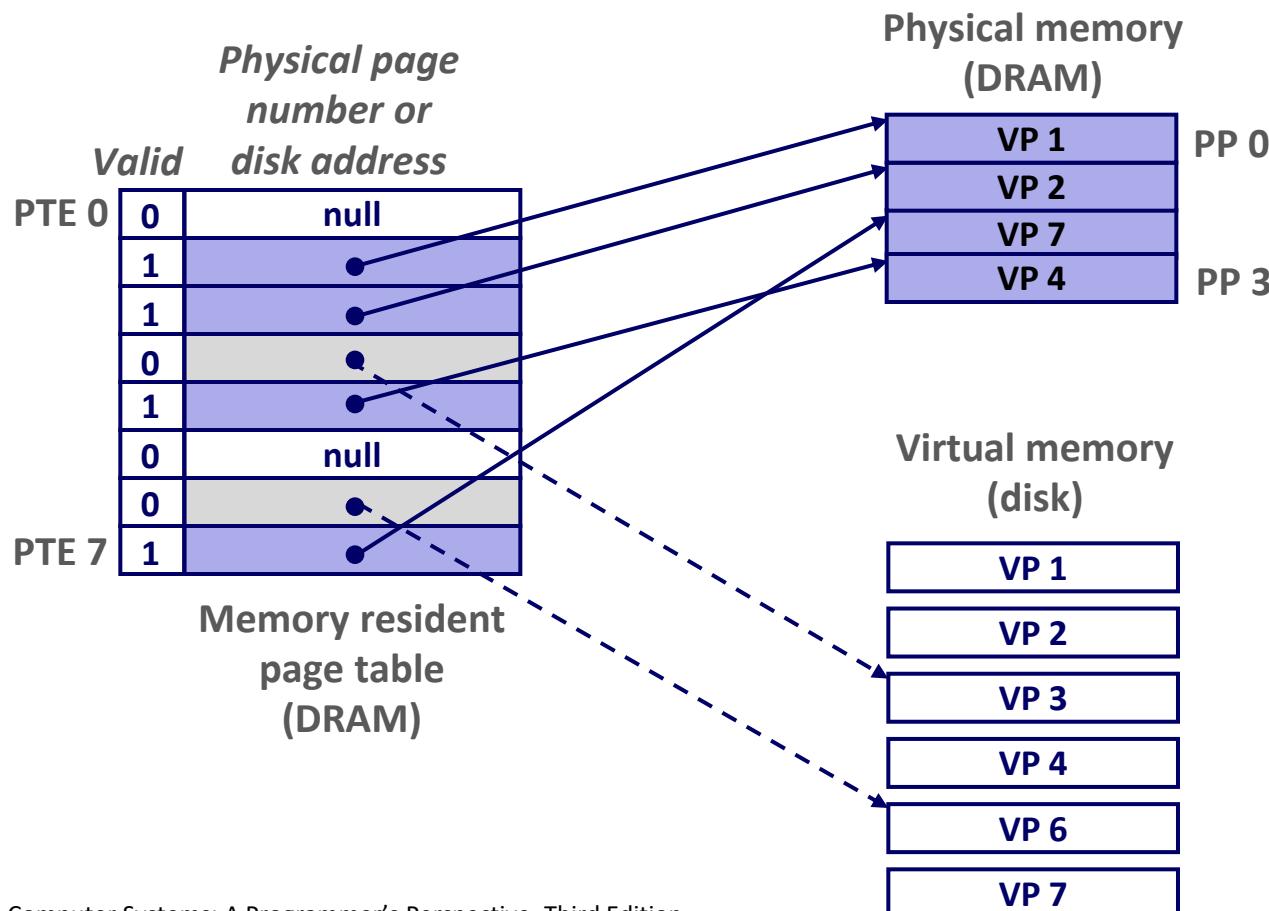
- Implemented in hardware
- Significant design constraints

■ K-nary tree aka Page Table

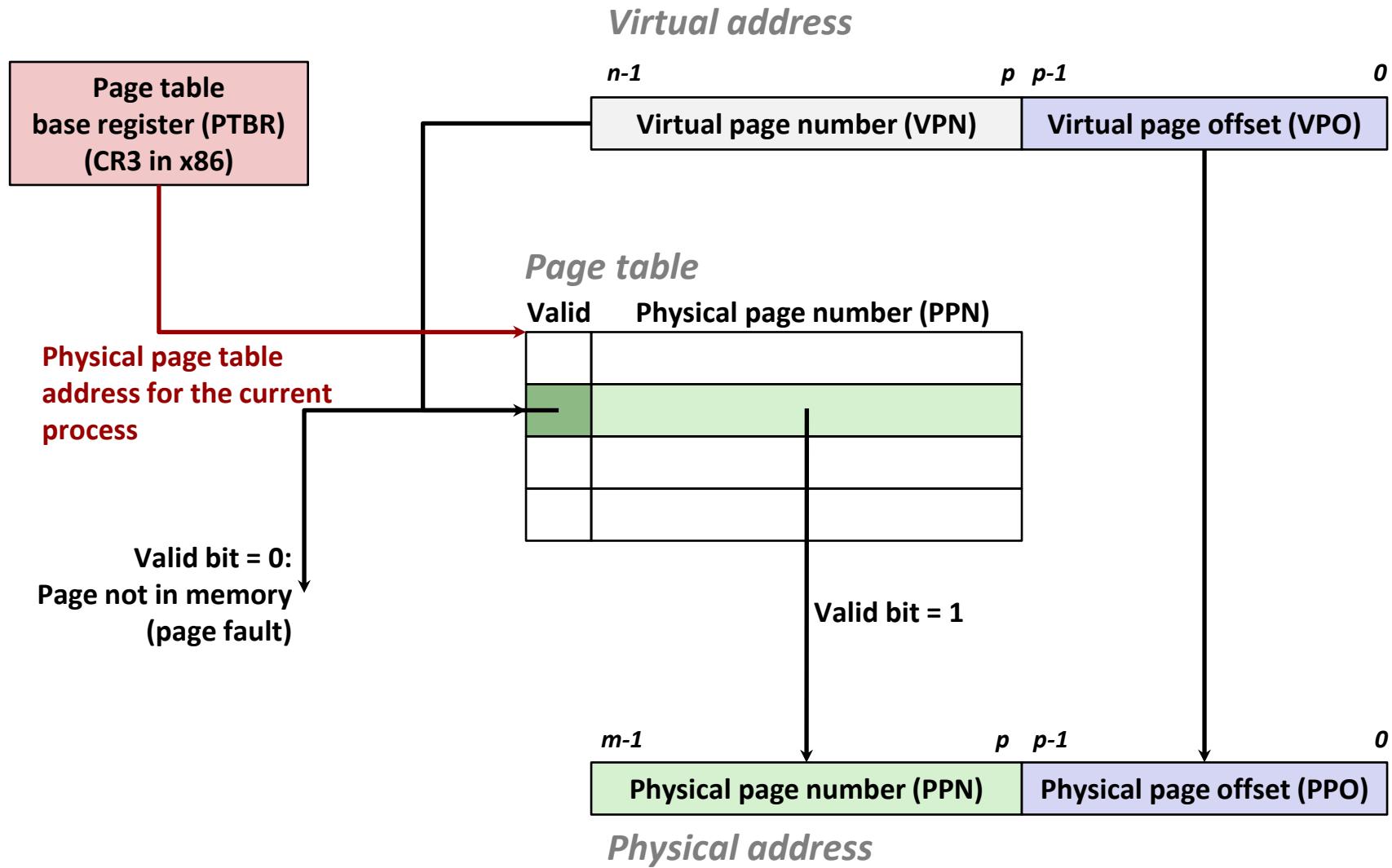
- Each node of the tree is 1 page in size

Enabling Data Structure: Page Table

- A **page table** is an array of page table entries (PTEs) that maps virtual pages to physical pages.
 - Per-process kernel data structure in DRAM

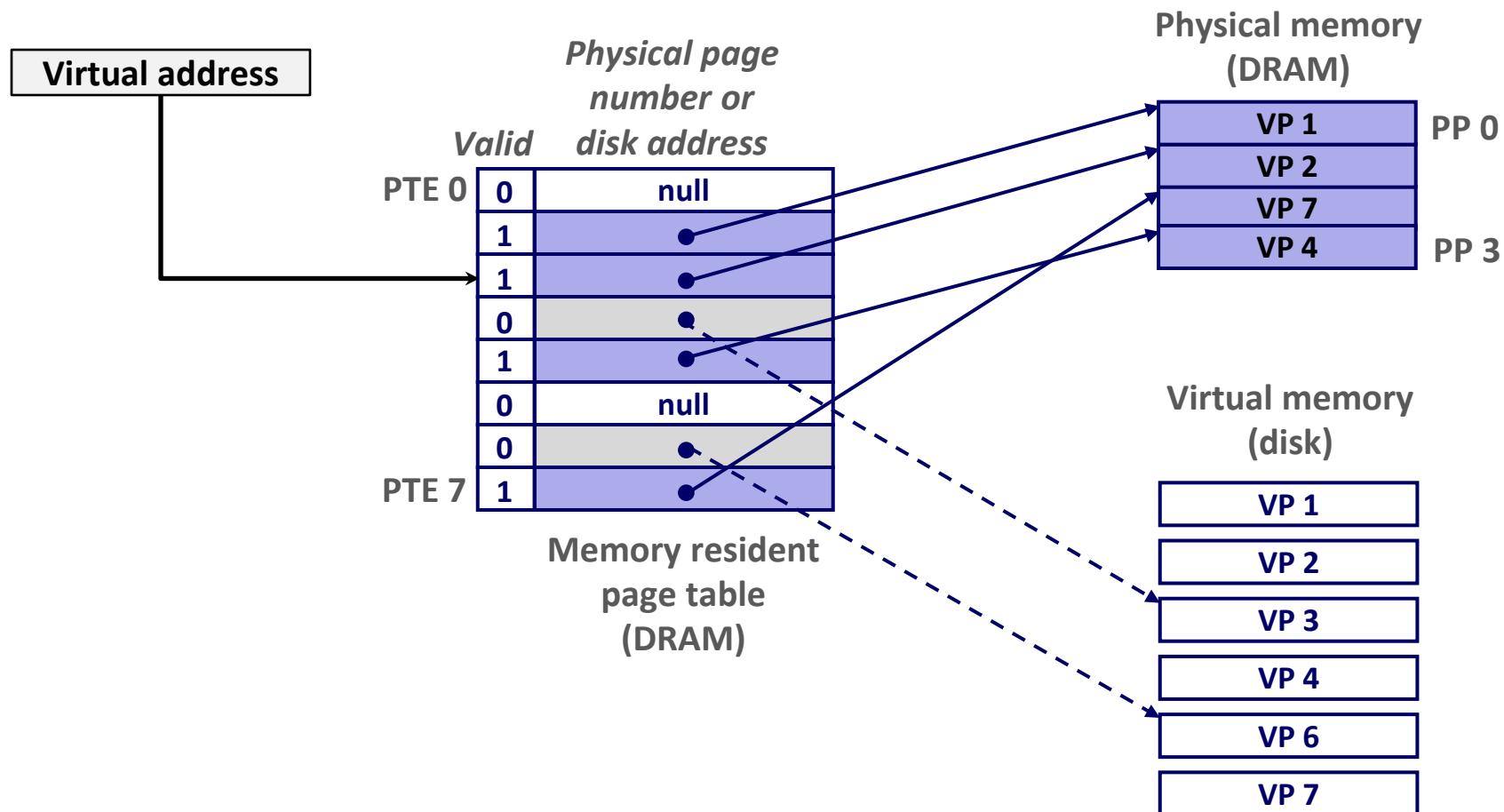


Address Translation With a Page Table

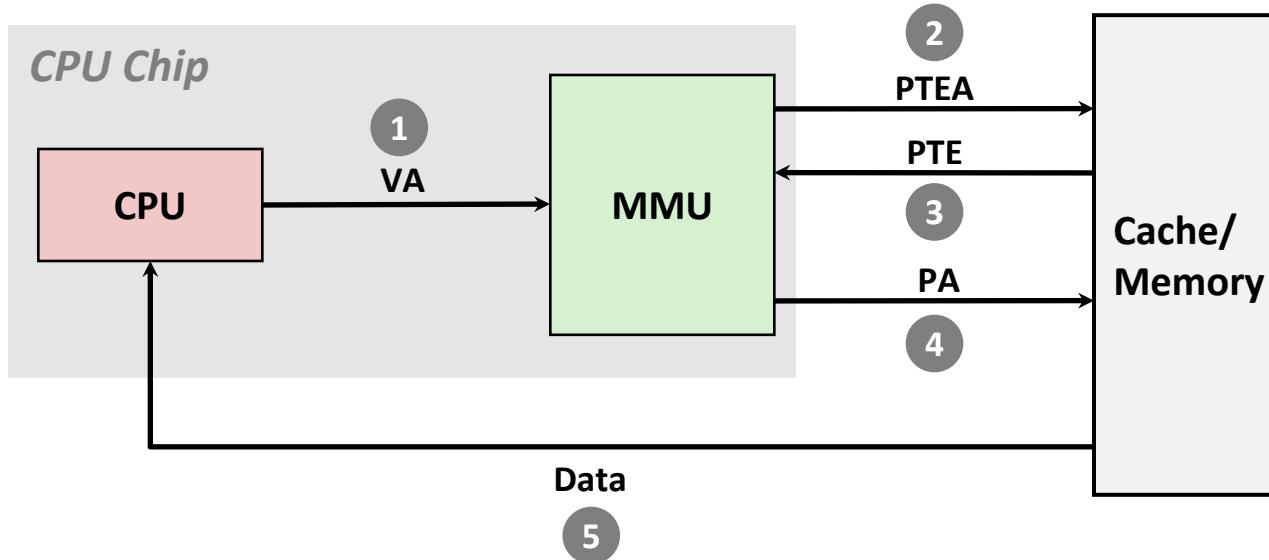


Page Hit

- **Page hit:** reference to VM word that is in physical memory (DRAM cache hit)



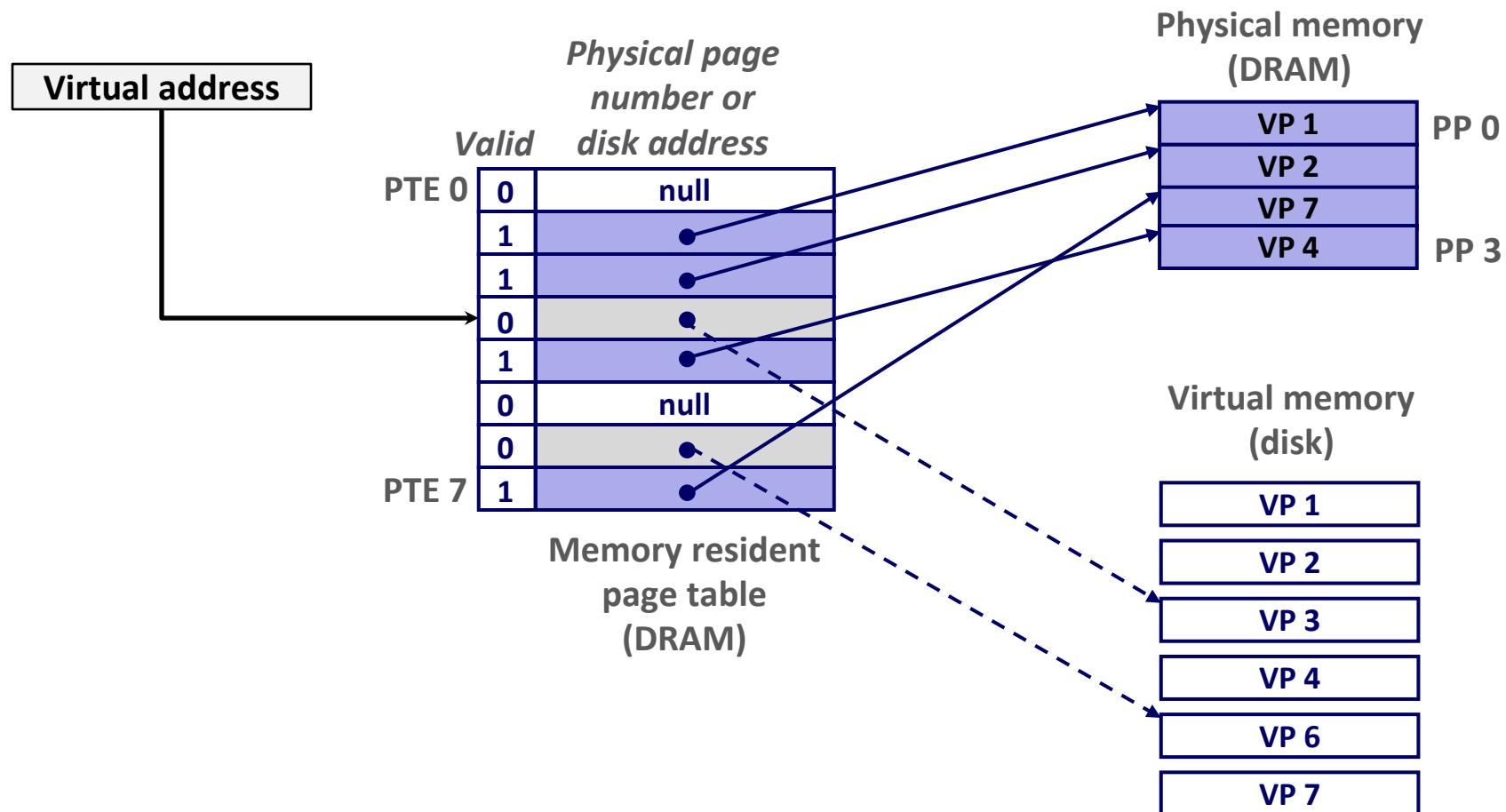
Address Translation: Page Hit



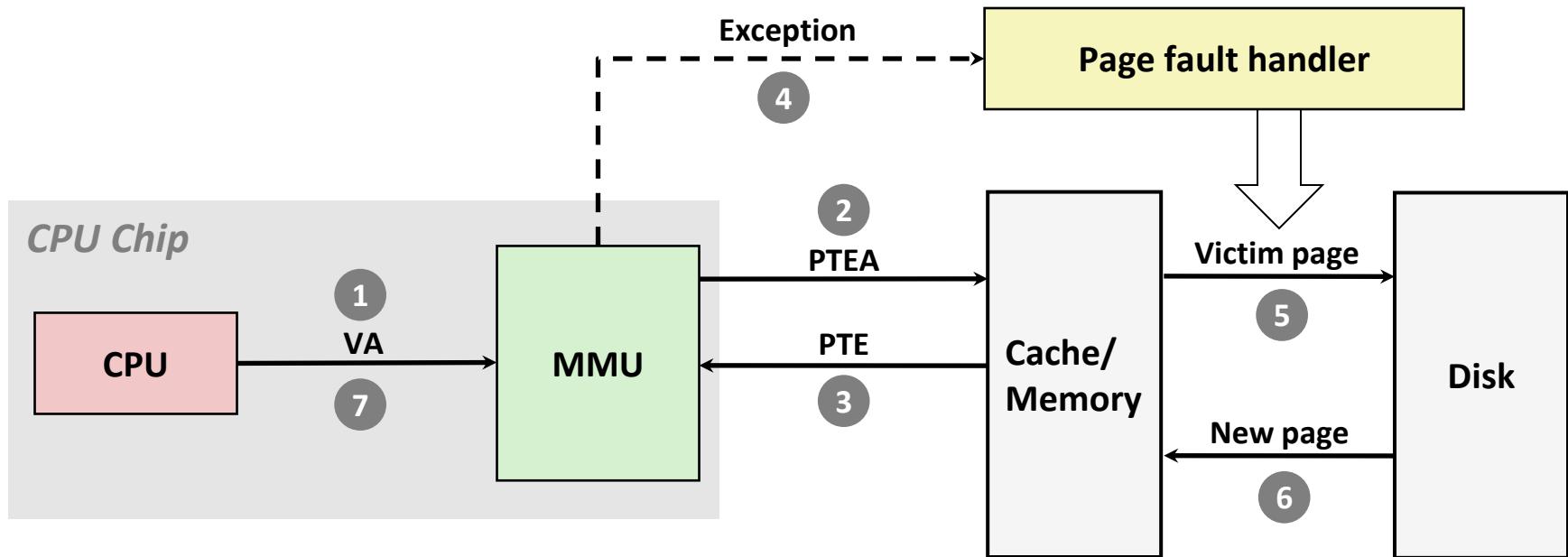
- 1) Processor sends virtual address to MMU
- 2-3) MMU fetches PTE from page table in memory
- 4) MMU sends physical address to cache/memory
- 5) Cache/memory sends data word to processor

Page Fault

- **Page fault:** reference to VM word that is not in physical memory (DRAM cache miss)



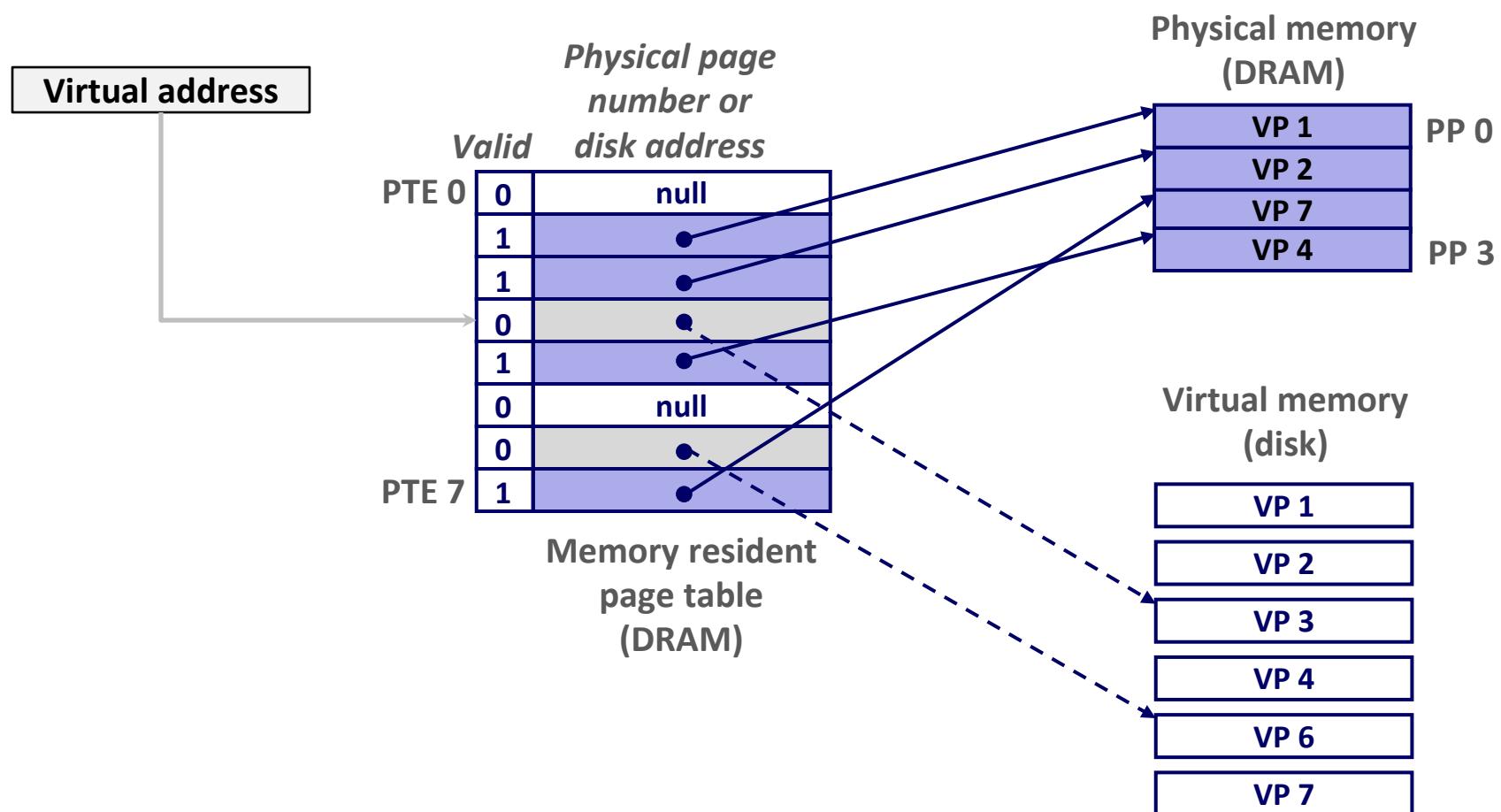
Address Translation: Page Fault



- 1) Processor sends virtual address to MMU
- 2-3) MMU fetches PTE from page table in memory
- 4) Valid bit is zero, so MMU triggers page fault exception
- 5) Handler identifies victim (and, if dirty, pages it out to disk)
- 6) Handler pages in new page and updates PTE in memory
- 7) Handler returns to original process, restarting faulting instruction

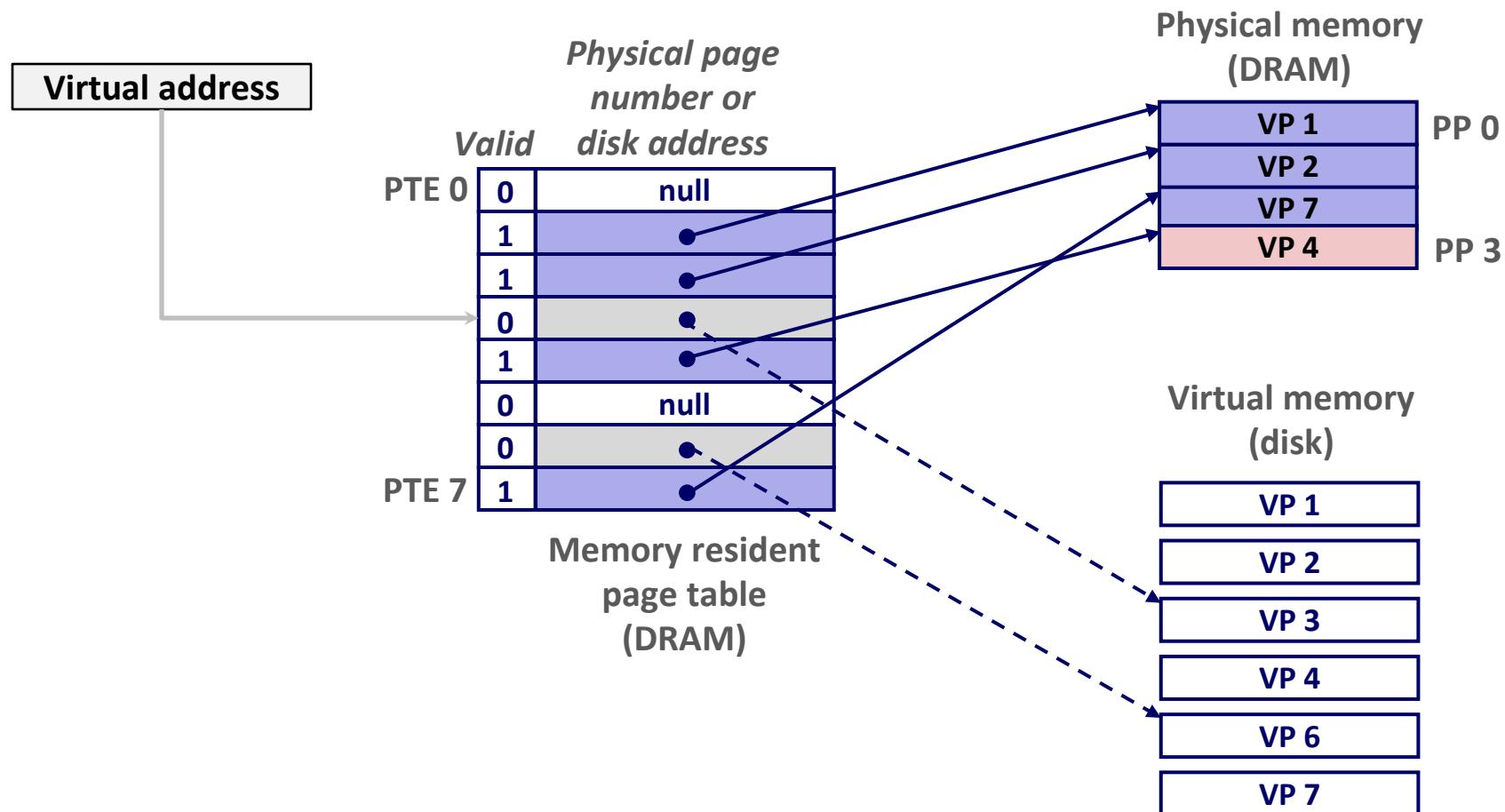
Handling Page Fault

- Page miss causes page fault (an exception)



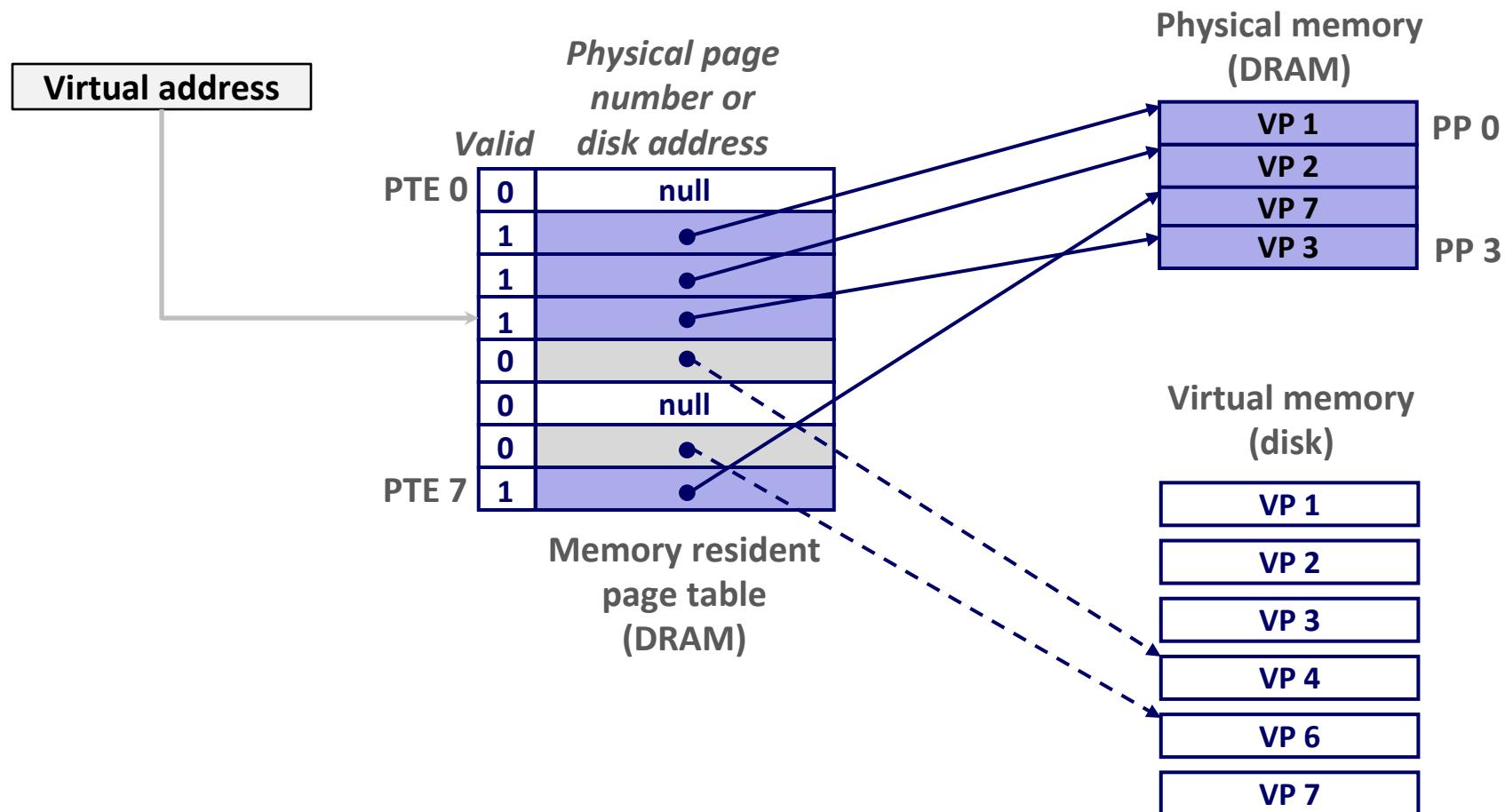
Handling Page Fault

- Page miss causes page fault (an exception)
- Page fault handler selects a victim to be evicted (here VP 4)



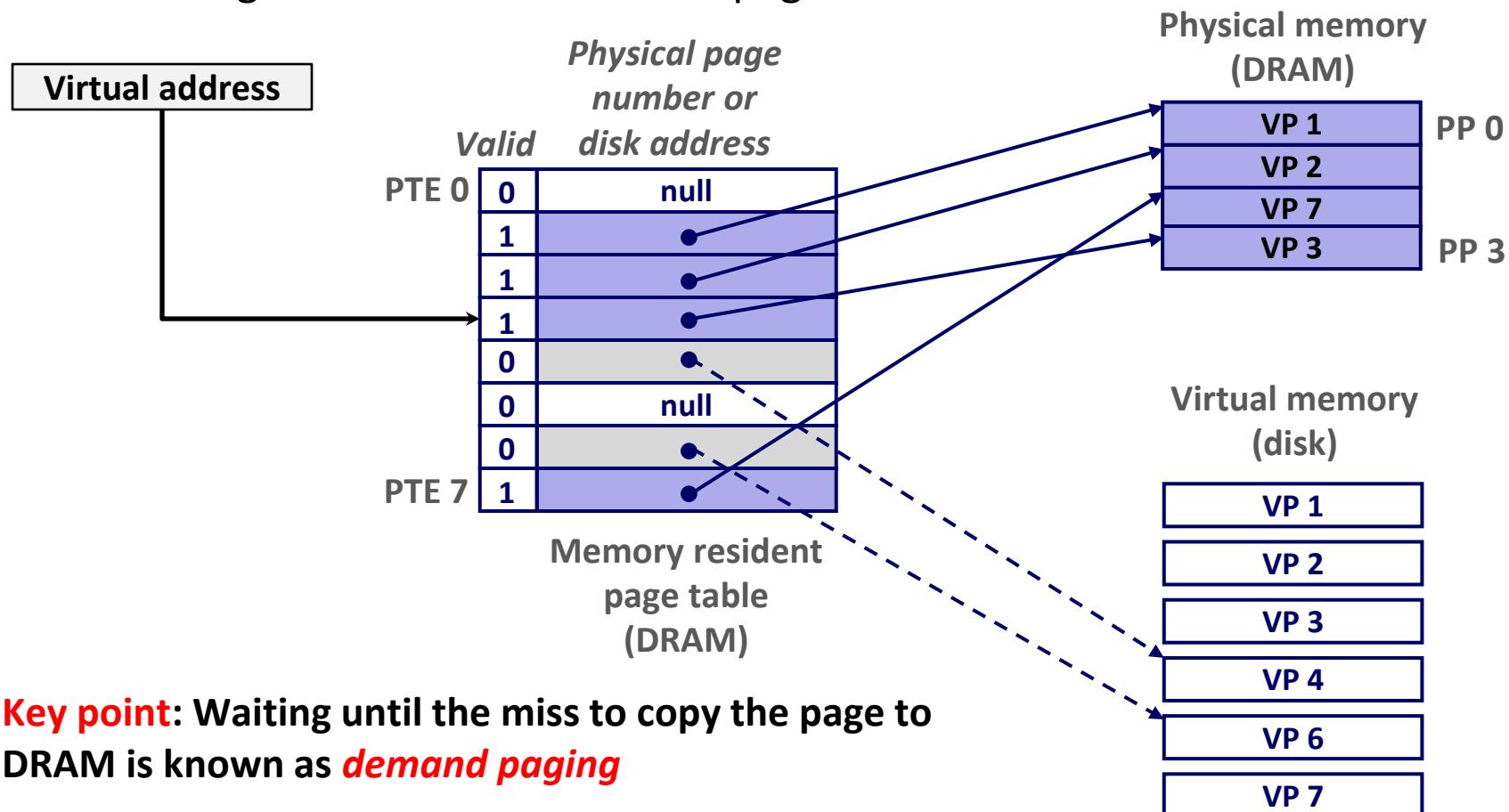
Handling Page Fault

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Handling Page Fault

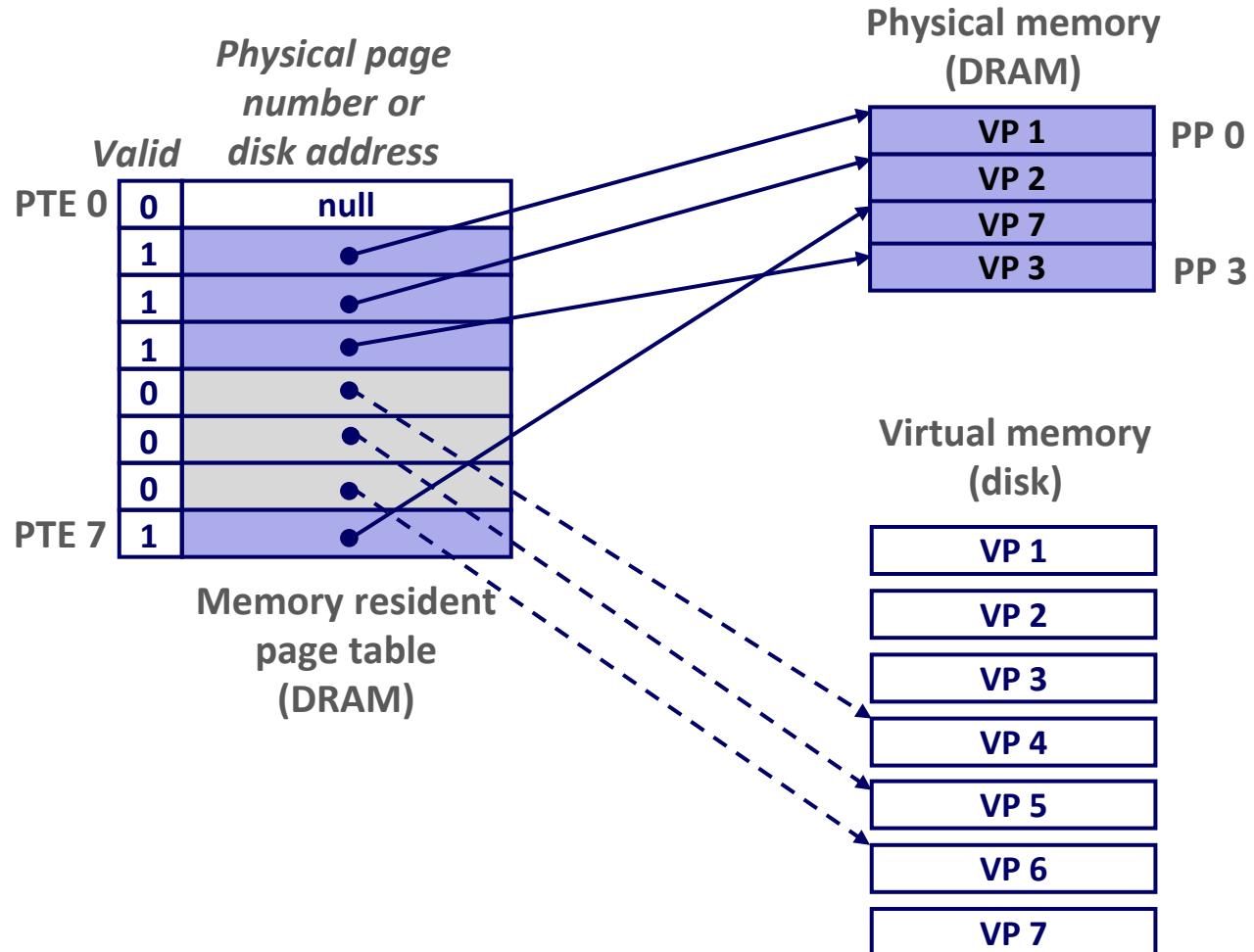
- Page miss causes page fault (an exception)
- Page fault handler selects a victim to be evicted (here VP 4)
- Offending instruction is restarted: page hit!



Key point: Waiting until the miss to copy the page to DRAM is known as *demand paging*

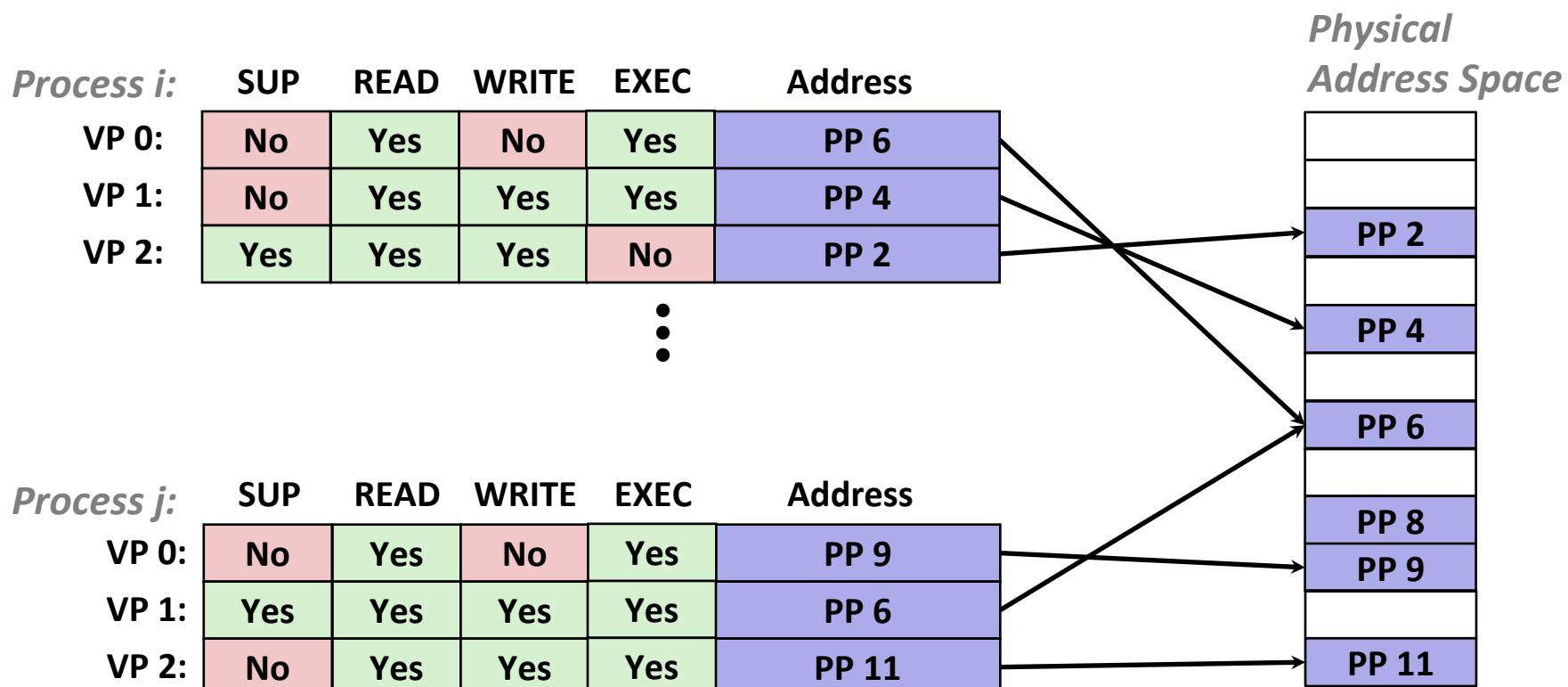
Allocating Pages

Allocating a new page (VP 5) of virtual memory.



VM as a Tool for Memory Protection

- Extend PTEs with permission bits
- MMU checks these bits on each access



VM is also Caching

■ Programs allocate virtual address ranges

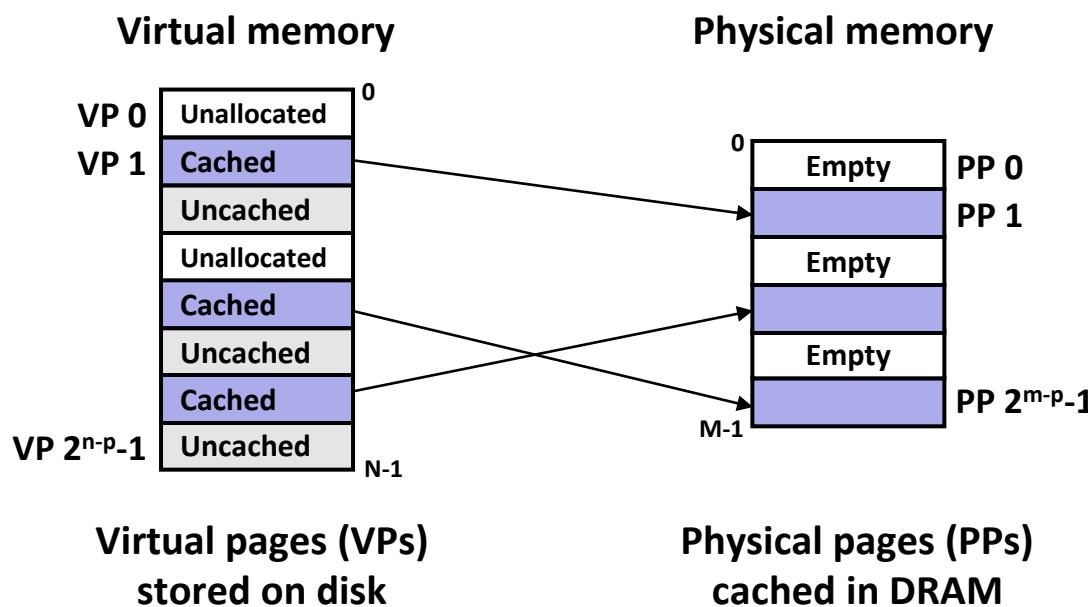
- Implicitly via binaries / libraries
- Explicitly through heap / stack

■ The operating system decides which virtual pages should be **resident** (i.e., in physical memory)

- OS manages the placement / replacement policies between DRAM and disk

VM as a Tool for Caching

- Conceptually, *virtual memory* is an array of N contiguous bytes stored on disk.
- The contents of the array on disk are cached in *physical memory (DRAM cache)*
 - These cache blocks are called *pages* (size is $P = 2^p$ bytes)



DRAM Cache Organization

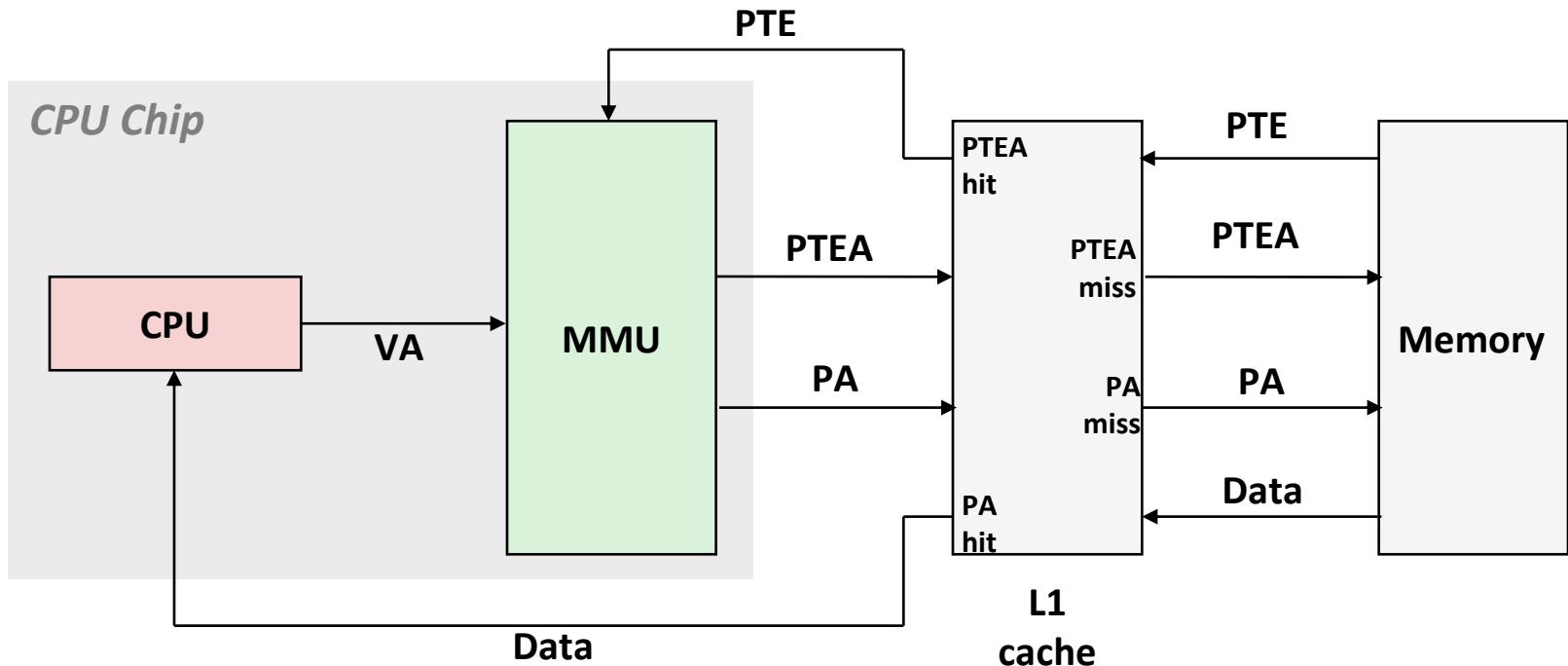
■ DRAM cache organization driven by the enormous miss penalty

- DRAM is about **10x** slower than SRAM
- Disk is about **10,000x** slower than DRAM

■ Consequences

- Large page (block) size: typically 4 KB, sometimes 4 MB
- Fully associative
 - Any VP can be placed in any PP
 - Requires a “large” mapping function – different from cache memories
- Highly sophisticated, expensive replacement algorithms
 - Too complicated and open-ended to be implemented in hardware
- Write-back rather than write-through

Integrating VM and Cache



VA: virtual address, PA: physical address, PTE: page table entry, PTEA = PTE address

Locality to the Rescue Again!

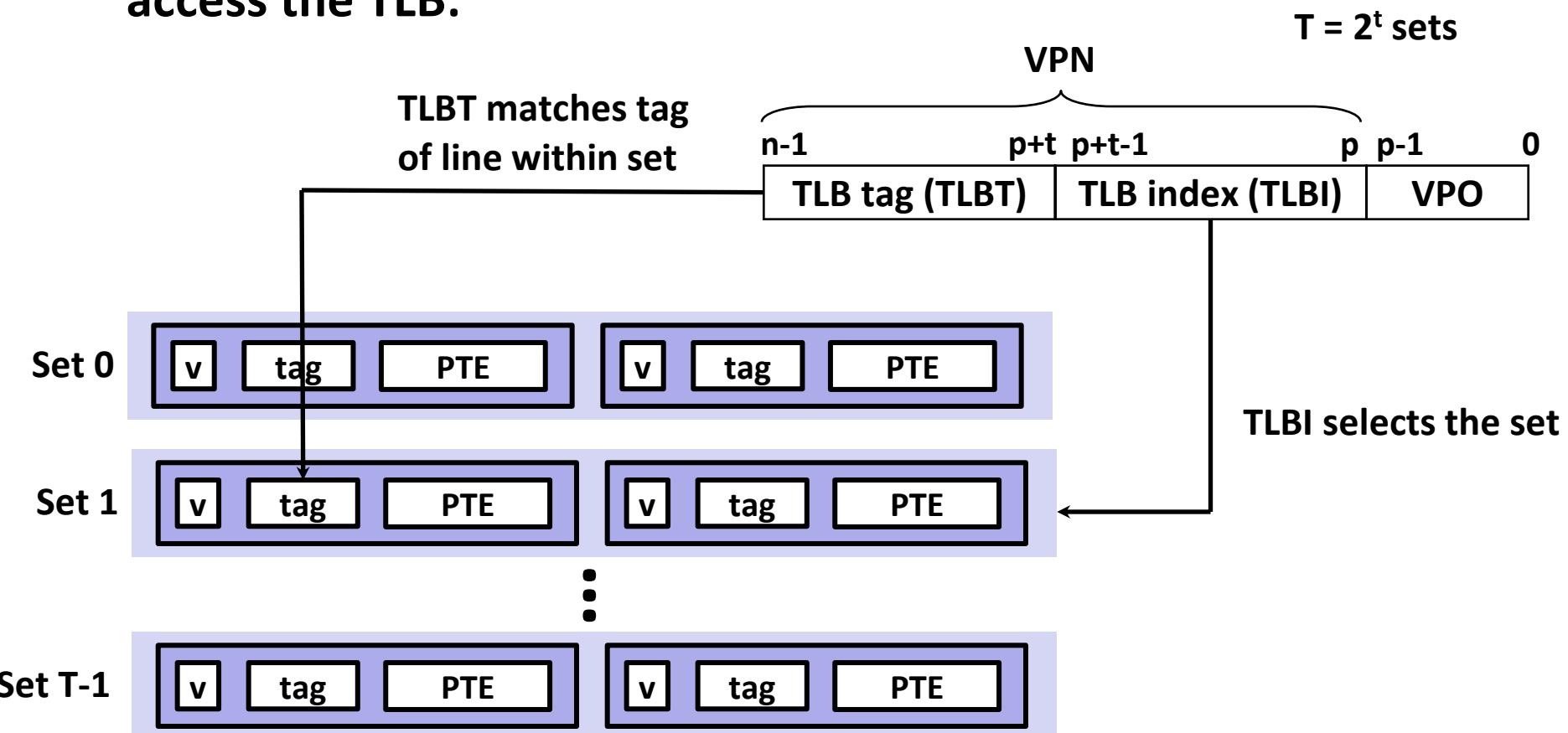
- Virtual memory seems terribly inefficient, but it works because of locality.
- At any point in time, programs tend to access a set of active virtual pages called the *working set*
 - Programs with better temporal locality will have smaller working sets
- If (**working set size < main memory size**)
 - Good performance for one process after compulsory misses
- If (**SUM(working set sizes) > main memory size**)
 - *Thrashing*: Performance meltdown where pages are swapped (copied) in and out continuously

Speeding up Translation with a TLB

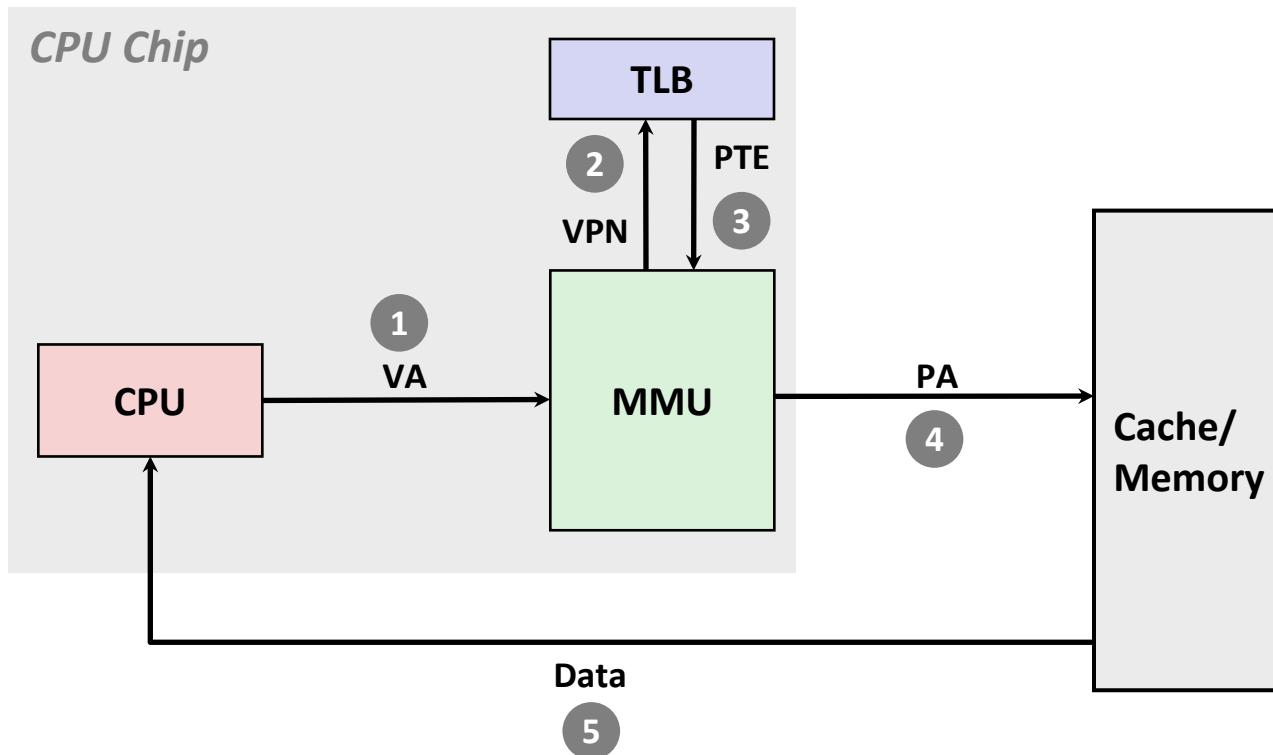
- **Page table entries (PTEs) are cached in L1 like any other memory word**
 - PTEs may be evicted by other data references
 - PTE hit still requires a small L1 delay
- **Solution: *Translation Lookaside Buffer* (TLB)**
 - Small set-associative hardware cache in MMU
 - Maps virtual page numbers to physical page numbers
 - Contains complete page table entries for small number of pages

Accessing the TLB

- MMU uses the VPN portion of the virtual address to access the TLB:

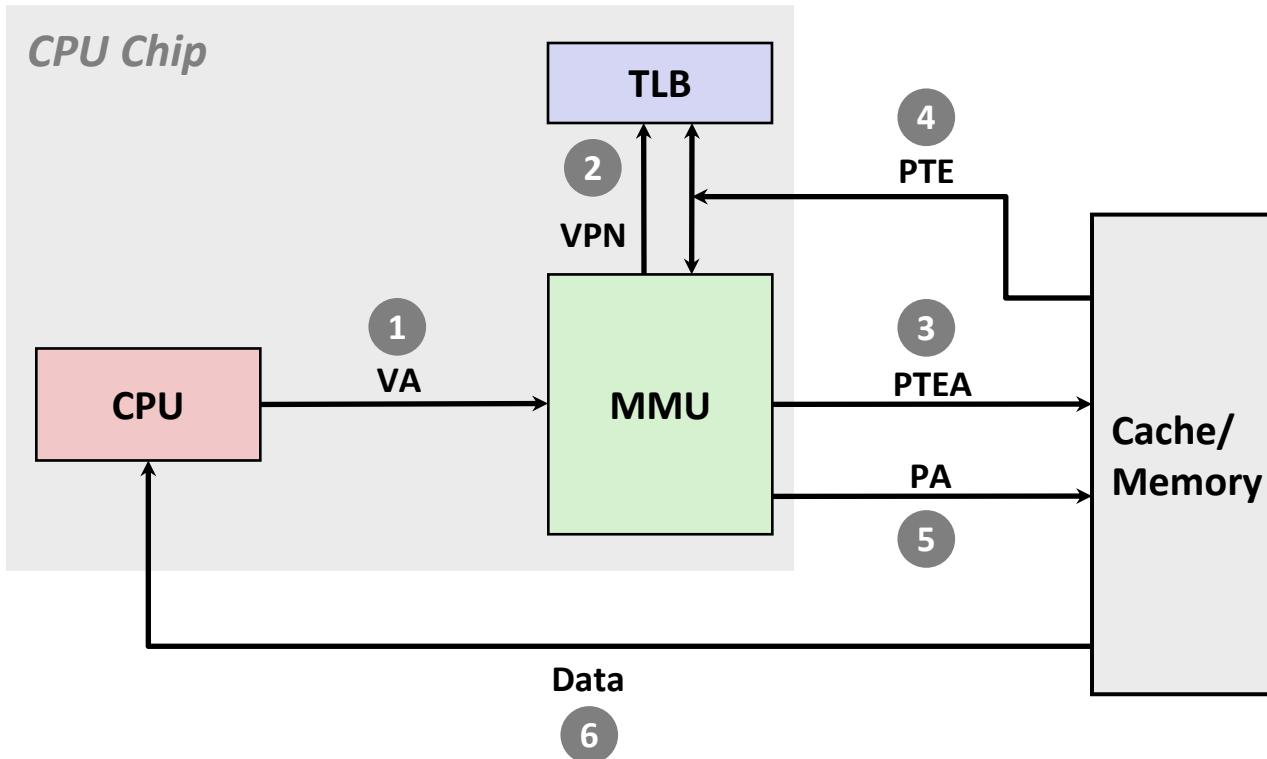


TLB Hit



A TLB hit eliminates a memory access

TLB Miss



A TLB miss incurs an additional memory access (the PTE)
Fortunately, TLB misses are rare. Why?

Summary of Address Translation Symbols

■ Basic Parameters

- $N = 2^n$: Number of addresses in virtual address space
- $M = 2^m$: Number of addresses in physical address space
- $P = 2^p$: Page size (bytes)

■ Components of the virtual address (VA)

- **TLBI**: TLB index
- **TLBT**: TLB tag
- **VPO**: Virtual page offset
- **VPN**: Virtual page number

■ Components of the physical address (PA)

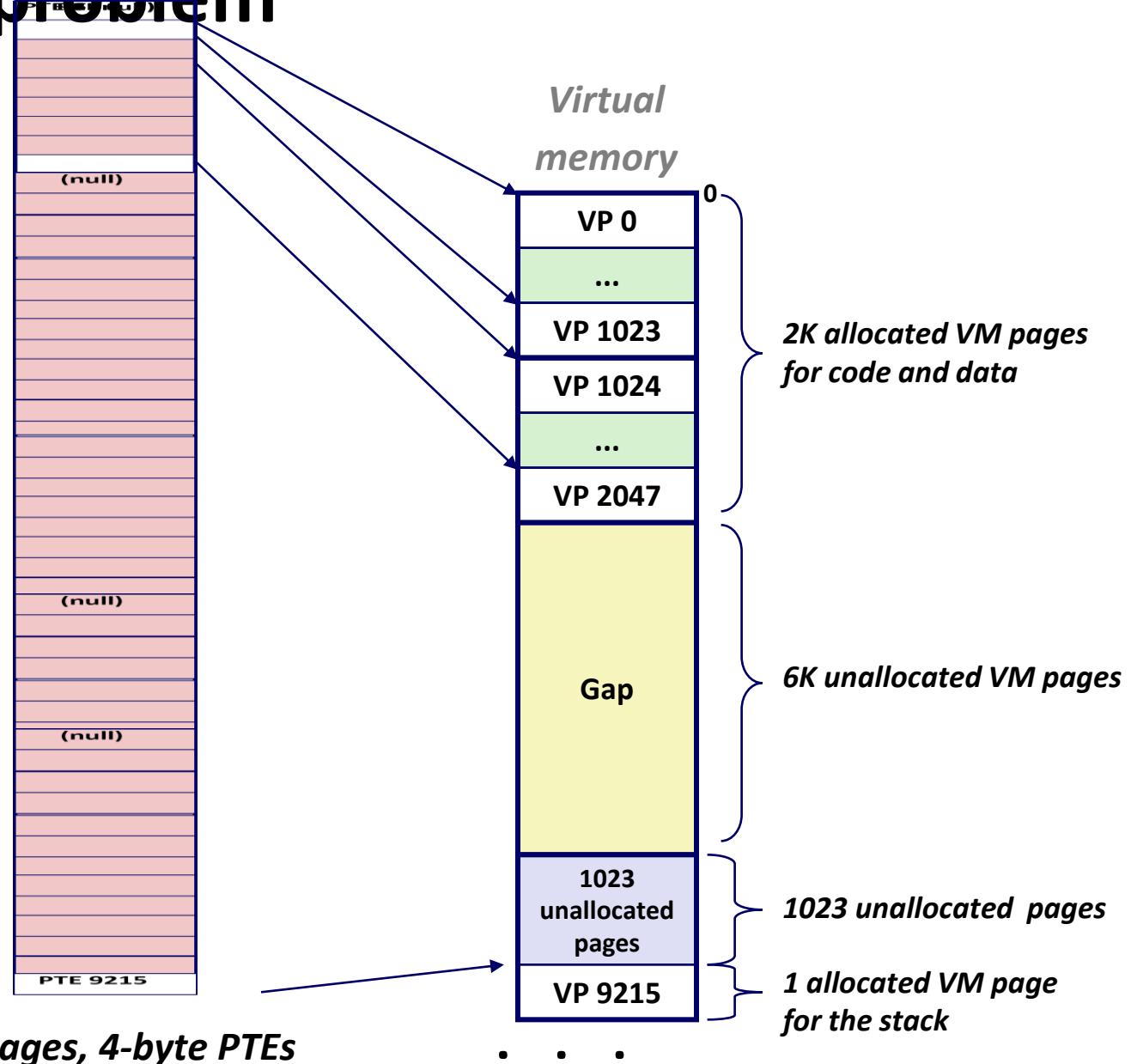
- **PPO**: Physical page offset (same as VPO)
- **PPN**: Physical page number

Quiz

<https://canvas.cmu.edu/courses/49105/quizzes/150041/>

We have a problem

2^{20} Entries of
4 bytes each



32 bit addresses, 4KB pages, 4-byte PTEs

Multi-Level Page Tables

■ Suppose:

- 4KB (2^{12}) page size, 48-bit address space, 8-byte PTE

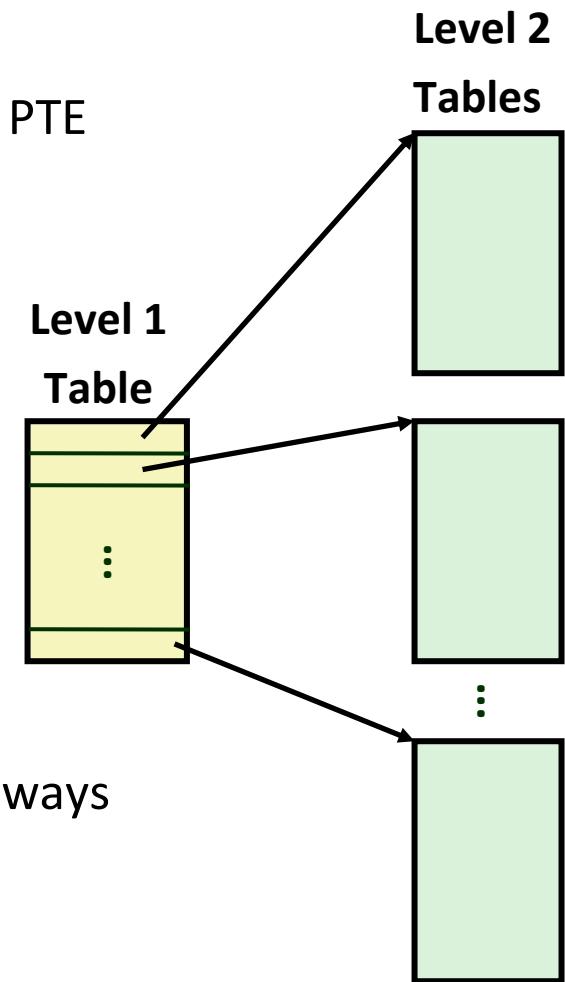
■ Problem:

- Would need a 512 GB page table!
 - $2^{48} * 2^{-12} * 2^3 = 2^{39}$ bytes

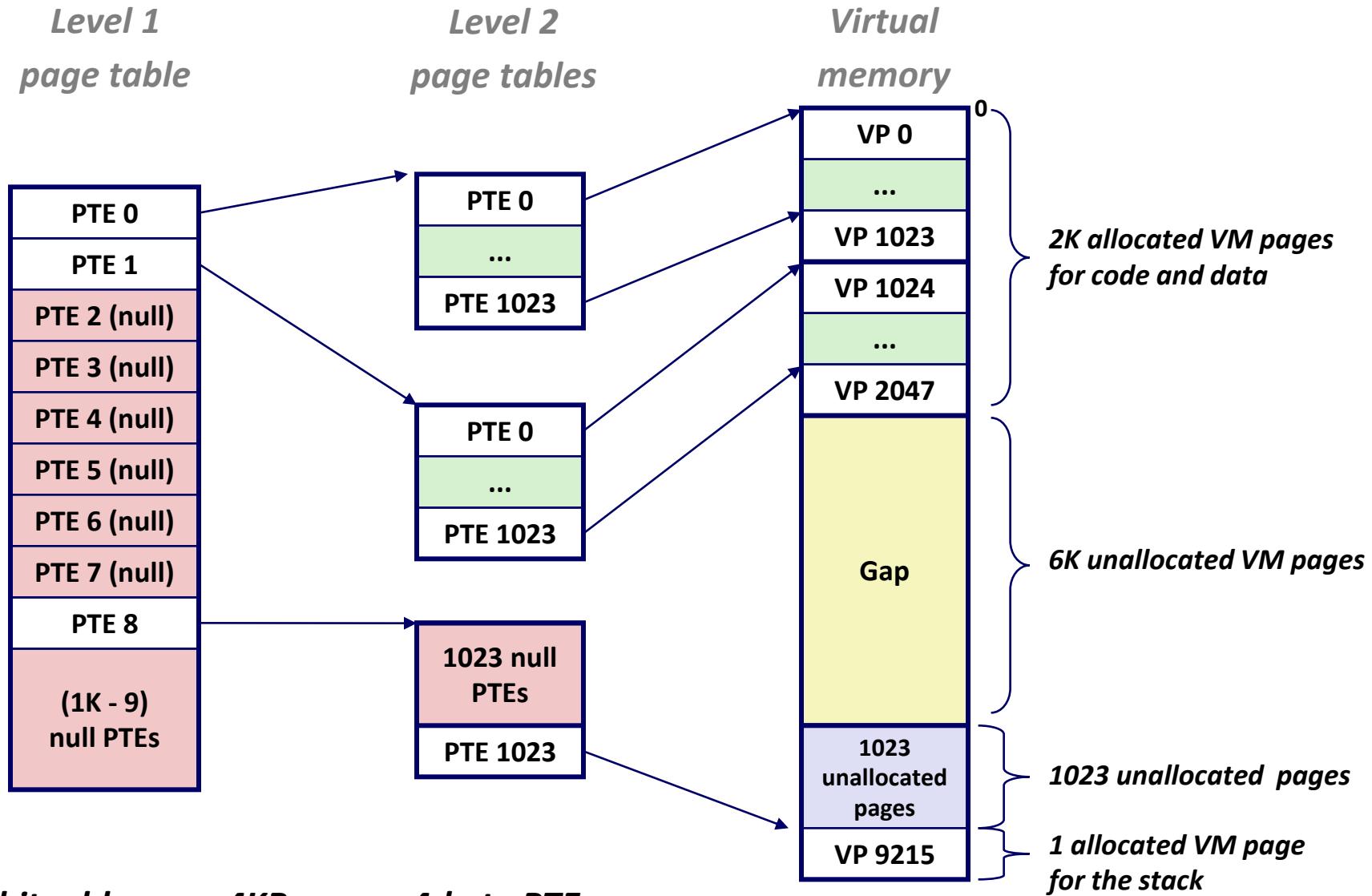
■ Common solution: Multi-level page table

■ Example: 2-level page table

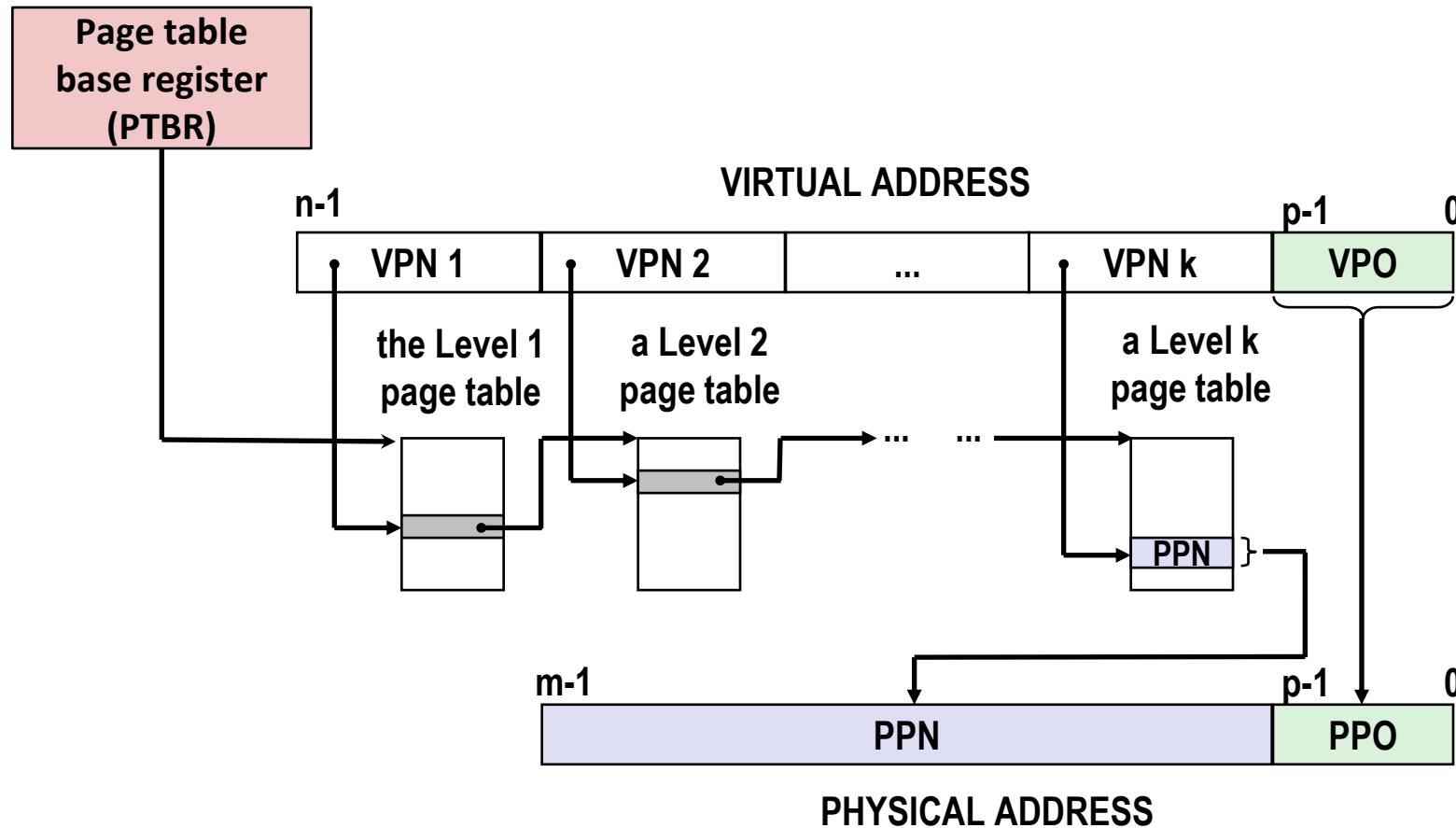
- Level 1 table: each PTE points to a page table (always memory resident)
- Level 2 table: each PTE points to a page (paged in and out like any other data)



A Two-Level Page Table Hierarchy



Translating with a k-level Page Table



TLBs and k-level Page Tables

■ TLBs cache the complete virtual to physical mapping

- Regardless of the levels of page tables,
the TLB stores the VPN \rightarrow PPN

Summary

■ Programmer's view of virtual memory

- Each process has its own private linear address space
- Cannot be corrupted by other processes

■ System view of virtual memory

- Uses memory efficiently by caching virtual memory pages
 - Efficient only because of locality
- Simplifies memory management and programming
- Simplifies protection by providing a convenient interpositioning point to check permissions