The 3-extra connectivity of Augmented cubes

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Abstract

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1 Introduction

Given a graph G and a non-negative integer g, if there is a set of vertices in the graph G such that the graph G is disconnected after the vertex set is deleted and the number of vertices of each component is greater than g, then we call it a vertex cut. The minimum cardinality of all vertex cuts is referred to as the *g-extra connectivity* of graph G, denoted by $\kappa_g(G)$. g-extra connectivity is a generalization of the connectivity.

2 H-structure connectivity and H-substructure connectivity

Theorem 1. *$\kappa(AQ_1) = 1$, $\kappa(AQ_2) = 3$, $\kappa(AQ_3) = 4$, and for $n \geq 4$, $\kappa(AQ_n) = 2n - 1$.*

$\kappa_2(G) = 3$

Lemma 1. *Let x, y be two nodes in AQ_n, where $n \geq 3$. Then $N_{AQ_n}(\{x, y\}) \geq 4n - 8$ holds.*

Lemma 2. *Let x, y, z be three nodes in AQ_n, where $n \geq 3$. Then $N_{AQ_n}(\{x, y, z\}) \geq 6n - 17$ holds.*

Lemma 3. *$\kappa_1(AQ_n) = 4n - 8$ for $n \geq 6$.*

Lemma 4. *$\kappa_2(AQ_n) < 6n - 17$ for $n \geq 6$.*

Proof. □

Lemma 5. *$\kappa_2(AQ_n) = 6n - 17$ for $n \geq 6$.*

Proof. Let S be an arbitrary set of vertices in AQ_n such that $|S| \leq 6n-18$ and there are no isolated vertices and K_2s in $AQ_n - S$. We will prove that $AQ_n - S$ is connected. Note that $AQ_n = L \oplus R$ where $L \cong AQ^0_{n-1}$ and $R \cong AQ^1_{n-1}$. For convenience, let $S_L = S \cap L$ and $S_R = S \cap R$. Without loss of generality, we may suppose that $|S_L| \geq |S_R|$. Then $|S_R| \leq 3n-9$.

Then we have the following two cases:

Case 1: S_R is connected.

We need to prove that any vertex in $L - S_L$ is connected via a path to a vertex in $R - S_R$. Let u be any vertex in $L - S_L$, if its two neighbours u_n and $\overline{u}_n$ don't belong to $R - S_R$. Then we are done. Otherwise, we have $\{u_n, \overline{u}_n\} \subset R - S_R$.

Since $AQ_n - S$ does not contain isolated vertices, then there exists a neighbour v of u, if $\{v_n, \overline{v}_n\} \not\subset R - S_R$, we are done. Otherwise, we have $\{v_n, \overline{v}_n\} \subset R - S_R$.

Since $AQ_n - S$ does not contain K_2s, then there exists a neighbour w of u and v, if $\{w_n, \overline{w}_n\} \not\subset R - S_R$, we are done. Otherwise, we have $\{w_n, \overline{w}_n\} \subset R - S_R$. If $\{u, v, w\}$ is an isolated K_3, then by Lemma 2, we have $N_{AQ_n}(\{u, v, w\}) = 6n - 17$, a contradiction. So we assume $\{u, v, w\}$ is not an isolated K_3. Since $N_{AQ_n}(\{u, v, w\}) = 6n - 17$ and $|S_R| \leq 3n - 9$, and for any vertex m in L, m can connect L via m_n and $\overline{m}_n$. Thus, for any vertex u in $L - S_L$, we can get a path $\{u, y, z\}$ where $y \in N_L(u, v, w)$ and $z \in \{y_n, \overline{y}_n\}$ and $z \in R - S_R$;

Case 2: S_R is disconnected.

And there is an isolated vertex x in $R - S_R$ since $|S_R| \leq 3n - 9$ and $\kappa(AQ_{n-1}) = 2n - 3$ and $\kappa_1(AQ_{n-1}) = 4n - 12$.

Since $AQ_n - S$ does not contain isolated vertices and K_2s, then there exists a $K_2 = \{v, w\}$ that $\{v, w\} \subset L - S_L$ and u connects $\{v, w\}$. Since u is an isolated vertex in $R - S_R$, then $S_L <= 6n-18-(2n-3) = 4n-15$. And by Lemma 1, $N_{AQ^0_n}(v, w) >= 4n-12$, there there is a vertex $y \notin L - S_L$ and $y \in N_{L-S_L}\{v, w\}$. Then by Lemma 2, we have $N_{L-S_L}\{v, w, y\} >= 6n-23 > 4n-15$ for $n \geq 6$. Then according to the definition of AQ_n and similar to Case 1,

□

3 Conclusions

conclusion

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References

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