Sparse Bayesian logistic regression

Model formulation

Given design matrix X and response $y \in \{-1, 1\}$, the indicator logistic regression model assumes

$$P(y = 1) = \sigma(X\Gamma\beta),$$

where $\Gamma = diag(\gamma_1, \cdots, \gamma_p), \beta = (\beta_1, \cdots, \beta_p)^T$. Here we introduce the latent indicator variable γ_j who equals to 1 if jth variable x_j is important and 0 if x_j is unimportant.

To complete the Bayesian inference, it assumes $\beta_j \mid \alpha_j \sim N(0, \alpha_j^{-1})$, $\alpha_j \sim Gamma(a_0, b_0)$, $\gamma_j \sim Bernoulli(\rho)$, where a_0, b_0, ρ are hyper-parameters. Generally speaking, a_0, b_0 can be set to $10^{-2}, 10^{-4}$ respectively, in the uninformative fashion. As for ρ, it controls the sparsity of the model and is tuned by BIC. Other parameters are inferred by variational Bayesian.

Example

The following example shows how to apply this software.

1.Consider a dataset with 30 features and 100 samples. Let 6 variables be important and takes value 2,1.5,1,-1,-1.5,2.

```
p = 30;
n = 100;

beta = zeros(p, 1);
beta([1,6,11]) = [-2,-1.5,-1];
beta([16,21,26]) = [1,1.5,2];
inx = beta~=0;

X = mvnrnd(zeros(p,1),eye(p),n);
y = random('bino', 1, logsig(X*beta));
y(y==0) = -1;
```

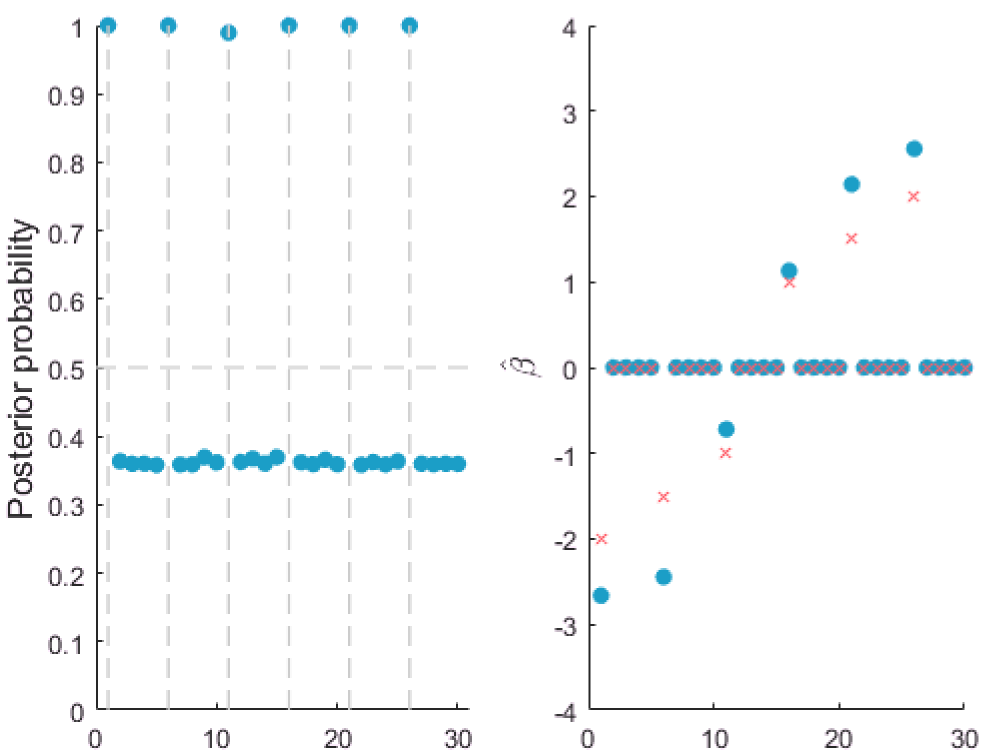
2.Function `vbvs_logit` carries out the varitional Bayes inference and automatically select the optimal ρ according to BIC. It admits parallel computing by by default.

```
% Specific parameters
opt.maxiter = 100;
opt.a0 = 1e-2*ones(p,1);
opt.bo = 1e-4*ones(p,1);
opt.tol = 1e-4;
opt.intercept = false;
% Create parallel pool
% parpool;
model = vbvs_logit(X,y,opt);
model.rhomin
```

```
ans = 0.3669
```

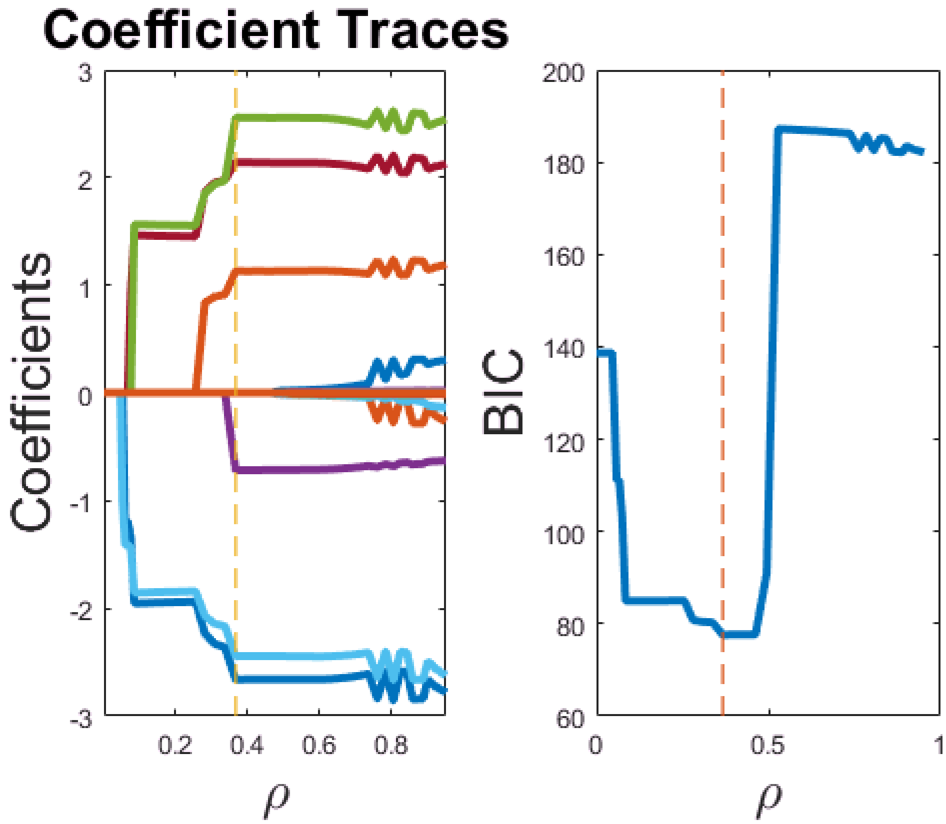
Plot the posterior probability of $\gamma = 1$ and estimates of β.

```
post = model.modelOPT.theta;
beta_hat = model.mu;
red = [1.0000    0.3686    0.4118];
blue = [0.1059    0.6196    0.7804];
cb = [red; blue];
% posterior probability
subplot(1,2,1)
hold on
scatter(1:p, post, 40, cb(((post>0.5)==inx)+1,:), 'filled' )
for i=[1,6,11,16,21,26]; plot([i i], [0 1], '--', 'LineWidth', 1, 'color', 0.8*ones(3,1));end
plot(0:31, 0.5*ones(32), '--', 'LineWidth', 1, 'color', 0.8*ones(3,1)),xlim([0,31])
ylabel('Posterior probability', 'FontSize', 13)
% estimate of beta
subplot(1,2,2)
hold on
scatter(1:p, beta_hat, 40,repmat(cb(2,:),30,1), 'filled')
scatter(1:p, beta, 30,repmat(cb(1,:),30,1),'x')
ylim([-4,4])
ylabel('$\hat{\beta}$', 'FontSize', 13,'interpreter','latex')
```



3.Plot the coefficients trace and BIC as ρ varying. The dashed line corresponds to optimal ρ.

```
figure
subplot(1,2,1),vbvs_plot(model), subplot(1,2,2),vbvs_plot(model,'bic')
```



4. Compare with traditional logistic regression.

```
% Train logistic regression
y(y==-1) = 0;
[B,dev,stats] = mnrfit(X, categorical(y));
% Generate test set
Xtest = mvnrnd(zeros(p,1),eye(p),1000);
ytest = random('bino', 1, logsig(Xtest*beta));
% Make prediction
y_vbvs = vbvs_logit_pred(Xtest, model);
y_logit = mnrval(B,Xtest); y_logit = y_logit(:,2);
display(['Accuracy of sparse model is ',num2str(sum(y_vbvs>0.5 == ytest)/1000)])
```

```
Accuracy of sparse model is 0.862
```

```
display(['Accuracy of generalized linear model is ',num2str(sum(y_logit>0.5 == ytest)/1000)])
```

```
Accuracy of generalized linear model is 0.797
```