

# Analysis and Design of Algorithms

## Assignment 2

### Google Page Ranking Algorithm

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#### Introduction

Within past few years, Google has become the far more utilized search engine worldwide. A major factor was the superior quality of search results which is based on the Google's page ranking algorithm.

Page Rank algorithms is the link analysis based algorithms named after Larry Page & Sergey Brin and used by the Google Internet search engine, that assigns a numerical weighting to the web pages. The name “Page Rank” is a trademark of Google. However the patent is assigned to Stanford University. Page Ranking is an algorithm which gives a numerical value to each page on the web. It does not rank websites as a whole rather it ranks individual pages. The importance or the ranking of a page improves when a page is found with a link to this page. Hence with every page with one outbound link to this page piling on, the rank of this page improves. It improves further if a page with higher rank has a link to this page.

Using this Page Ranking algorithm, Google is providing its one of the best search results showing higher rank pages containing the keyword searched as the top choices followed by the lower rank pages.

#### Page Ranking

Page Ranking Algorithm is based on the **Random Surfer Model** which is based on one random surfer who is surfing the web by simply clicking on the links with no regard towards content. The surfer visits a page with a certain probability which derives from the page's page rank. The Probability that the surfer clicks on one link is solely given by the number of links on that page. Random Surfer could click on one of the links on that page or he could stop clicking on a link on that page due to boredom or any other cause and select

another page at random, this probability is also called the damping factor. Hence the formula for calculating the page rank is proportional to the sum of fraction of page rank of all the pages which has a outbound link to that page with respect to the number of outgoing links multiplied by the probability of him choosing a link on that page or damping factor. Larry Page and Sergey Brin in their research paper published that on an average, the damping factor comes out to be 0.85 . Page Rank is also improved when a page has a lot of inbound links from pages with higher page ranks.

### **An Analogy**

Page Ranking algorithm can also be used as Social Ranking Algorithm on a social networking site. As we are ranking web pages, one can also rank people's profile depending on the number of friends they have. Social Rank not only depends upon the number of friends rather it would depend upon the probability of one clicking on one's profile from the profile of their friends. A Friend with high rank will increase this person's rank. Due to the similarity with page ranking algorithm, this technique had been effectively used by the sites like Facebook to give ranking to the people's profile. As one searches for a particular keyword on Facebook, the search results are in decreasing order of rank.

### **Formula for calculating Page Ranking**

$$\text{Pr}(A) = (1-d) + d[\text{Pr}(T1)/L(T1) + \text{Pr}(T2)/L(T2) + \dots + \text{Pr}(Tn)/L(Tn)]$$

Pr(A) – Page Rank of page A

d – Damping factor

Pr(Ti) – Page Rank of the ith page which has a link to page A

L(Ti) - Total number of outbound links on the ith page

### **Most Interesting part of Page Ranking**

The most interesting part of Page Ranking is the introduction of PR0 or Zero page rank. The pages with PR0 is not thrown out of index but rather they are at the end of the search results. The causes behind a page having PR0 is

mainly not having many inbound links from the pages with higher ranks.

Google has used this concept of PR0 to detect spams.

Spams have always been the biggest problem that the search engines had to deal with. When spam is detected, the usual proceeding is the banishment of the websites which is the difficult task on its own. If a page has many outbound links to the less rank pages then its rank will surely decrease. Many bad neighbors could lead to PR0 with a maximum chance to be termed as a spam.

### **Significance of Algorithm**

Finding useful information on the World Wide Web is something many of us take for granted. According to Internet research firm Netcraft, there are nearly 150,000,000 active Web sites on the Internet today. The task of sifting through all those sites to find helpful information is monumental. That's why search engines use complex algorithms that tell computers how to complete assigned tasks.

Google's algorithm does the work for you by searching out Web pages that contain the keywords you used to search, then assigning a rank to each page based several factors, including how many times the keywords appear on the page. Higher ranked pages appear further up in Google's **search engine results page (SERP)**, meaning that the best links relating to your search query are theoretically the first ones Google lists.

### **Additional Factor influencing Page Rank**

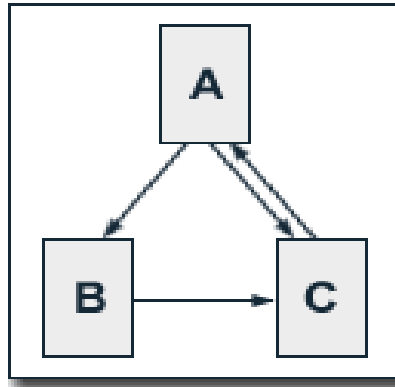
Larry Page himself pointed out later that there can be some more factors which can influence the page rank. Some of them are :

- ◆ Visibility of a link
- ◆ Position of the link on the page
- ◆ Distance between web pages
- ◆ Importance of a linking page

These factors can have certain effect on the page's Rank like less visibility of the link decreases the probability of the random surfer making the choice of

the link to that page thereby decreasing the page rank of that page. Position of the link also marks the visibility of that link. Larger distance between the web pages decreases the probability of that link to be picked. Importance of a linking page is one of the major factor in deciding the page's rank as a link from the more important page increases the page rank of the page of which's link is present on this page.

### **Example**



We have a small example to illustrate the algorithm consisting of three pages A, B and C whereby page A links to B and C, page B links to page C and page C links to page A. To simplify calculation, let damping factor be 0.5. According to the equation,

$$\text{Pr}(A) = (0.5) + 0.5\text{Pr}(C)$$

$$\text{Pr}(B) = (0.5) + 0.5[\text{Pr}(A)/2]$$

$$\text{Pr}(C) = (0.5) + 0.5[\text{Pr}(A)/2 + \text{Pr}(B)]$$

Solving these equation, we get

$$\text{Pr}(A) = 14/13 = 1.07692308$$

$$\text{Pr}(B) = 10/13 = 0.76923077$$

$$\text{Pr}(C) = 15/13 = 1.15384615$$

It is obvious that sum of the page ranks is equal to the number of web pages, in this case 3.

### **Implementation**

In practice, calculation of the solution to these equations is a difficult task

even on a computer. So there is even a simple way to get the solution which is also known as the “Iterative approach”. Due to large number of pages on the web, this is an approximative approach which assigns a starting value to all the pages and calculate the corresponding page rank values using these equations iteratively for 100 iterations. According to publications of Larry Page and Sergey Brin, about 100 iterations are necessary to get a good approximation of the page rank values of the whole web. Implementation of this approach for the above mentioned three page example is as follows:

Iteration	PR(A)	PR(B)	PR(C)
0	1	1	1
1	1	0.75	1.125
2	1.0625	0.765625	1.1484375
3	1.07421875	0.76855469	1.15283203
4	1.07641602	0.76910400	1.15365601
5	1.07682800	0.76920700	1.15381050
6	1.07690525	0.76922631	1.15383947
7	1.07691973	0.76922993	1.15384490
8	1.07692245	0.76923061	1.15384592
9	1.07692296	0.76923074	1.15384611
10	1.07692305	0.76923076	1.15384615
11	1.07692307	0.76923077	1.15384615
12	1.07692308	0.76923077	1.15384615

As we can see in 12 iterations, we have got the same solution as above.

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