	onte			8
1	Bas	sic Template and snippets	1	
<b>2</b>	$\mathbf{C}+$	+ Ref.	1	
	2.1	String	1	
	2.2	Algorithm	2	
3	Ma	x-Flow and Bipartite Matching	<b>2</b>	
	3.1	Dinic'c Flow algorithm	2	
	3.2	Maximum Bipartite Matching	2	
	3.3	Minimum cost Bipartite Matching	3	
	3.4	Konig's Theorem	4	H
	3.5	Minimum Edge Cover	4	1
4	Dat	ta Structures	4	
•	4.1	BIT	4	
	4.2	Segment Tree	4	;
	4.3	Suffix Array	5	ŀ
	4.4	· ·	6	
	4.4	Union Find	6	ŀ
	4.6	Z Function	7	ŀ
			•	1
5		aph Algorithms	7	1
	5.1	BFS	7	
	5.2	DFS	8	
	5.3	Dijkstra	8	1
	5.4	Krushkals	8	
	5.5	SCC	9	
	5.6	Eulerian Path	10	
	5.7	Bridges	10	
6	Nu	mber Theory	10	
Ū	6.1	Formulas	10	
	6.2	Math Library	10	
	6.3	Extended GCD	11	:
	6.4	Chineese Remainder Theorem	11	
	6.5	Heavy Light Decomposition	11	-
	6.6	Lucas	13	
	6.7	Primitive Root	13	
	6.8	Simplex Algorithm	13	
	6.9	Eulers Totient Function	14	
	0.5	Edicis Totiche Function	17	
7	Geometry			
	7.1	2-D Geometry	14	
	7.2	3-D Geometry	16	.
	7.3	Closest Pair of Points	18	Ŀ
	7.4	Convex Hull	19	

```
8 Miscellaneous
                               19
 19
 1 Basic Template and snippets
#include < bits / stdc++.h>
using namespace std;
typedef long long 11;
typedef vector<ll>v11;
typedef pair<11,11> pll;
#define xx first
#define yy second
#define rep(n) for(i=0;i<n;i++)</pre>
#define pb push_back
#define mp make_pair
#define clr(a) memset(a, 0, sizeof a)
#define reset(a) memset(a, -1, sizeof a)
#define Clr(a) fill(a.begin(),a.end(),0)
#define Reset(a) fill(a.begin(),a.end(),-1)
#define tr(c, it) \
 for(typeof(c.begin()) it=c.begin(); it!=c←
   .end(); it++)
 void debug(vector<11> v)
  for(int i=0;i<v.size();i++)</pre>
     cout << v[i] << " ";
  cout << "\n";
  // call debug({i,j,k})
int main()
  11 t,z,i,j,k,n,m,p,q,r,s,ans;
  scanf("%11d",&t);
  for (z=1;z<=t;z++)
  {
     scanf("%11d",&n);
     printf("Case %11d: \n",z);
  return 0;
```

```
ll steps[][2]=\{\{1,0\},\{-1,0\},\{0,1\},\{0,-1\}\};
bool isvalid(int i,int j,ll n,ll m)
         return (i>=0&&j>=0&&i<n&&j<m);
  struct node
    {
         11 n;
         ll cost;
         node(){}
         node(ll n, ll cost) \{this \rightarrow n = n; \leftarrow \}
             this->cost = cost:}
         bool operator < (const node &nd) ←
             const {
             if (cost!=nd.cost)
             return cost > nd.cost:
    }
    }:
```

### 2 C++ Ref.

### 2.1 String

- 1. to\_string(val) returns string equivalent of val, val is a number
- 2. sto i/l/ul/ll/ull stoi(string,nullptr,2); returns number; 2 is base default 10;
- 3. erase

```
str.erase (10,8); //start position, ←
    length
str.erase (str.begin()+9);
str.erase (str.begin()+5, str.end()←
    -9);
```

- 4. find
  - find\_first\_of
  - find\_first\_not\_of
  - find\_last\_of
  - find\_last\_not\_of

```
found=str.find("haystack",0);
if (found!=string::npos)
```

5. substr(start,length) make a substring

### 2.2 Algorithm

- 1. lower\_bound (v.begin(), v.end(), 20); returns iterator having element >=20
- 2. upper\_bound (v.begin(), v.end(), 20); returns iterator having element >20
- 3. binary\_search (v.begin(), v.end(), 3); returns true if 3 is present

### 3 Max-Flow and Bipartite Matching

# 3.1 Dinic'c Flow algorithm

```
// Dinic's blocking flow algorithm
// Running time:
// * general networks: O(|V|^2 |E|)
// * unit capacity networks: O(E min(V \leftarrow
   (2/3), E^{(1/2)}
// * bipartite matching networks: 0(E \ \text{sqrt}(\leftarrow))
   V))
const int INF = 2000000000;
struct Edge {
 int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, ←
       int index) :
    from(from), to(to), cap(cap), flow(flow←
        ), index(index) {}
};
struct Dinic {
 int N;
  vector < vector < Edge > > G;
  vector < Edge *> dad;
  vector < int > Q:
  // N = number of vertices
  Dinic(int N) : N(N), G(N), dad(N), Q(N) \leftarrow
     {}
  // Add an edge to initially empty network←
      . from. to are 0-based
  void AddEdge(int from, int to, int cap) {
    G[from].push_back(Edge(from, to, cap, \leftarrow)
        0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(Edge(to, from, 0, 0, G[\leftarrow
        from].size() - 1));
```

```
long long BlockingFlow(int s, int t) {
  fill(dad.begin(), dad.end(), (Edge *) ←
      NULL):
  dad[s] = &G[0][0] - 1;
  int head = 0, tail = 0;
  Q[tail++] = s:
  while (head < tail) {</pre>
    int x = Q[head++];
    for (int i = 0; i < G[x].size(); i++) \leftarrow
      Edge &e = G[x][i];
      if (!dad[e.to] && e.cap - e.flow > ←
        dad[e.to] = \&G[x][i];
        Q[tail++] = e.to;
      }
    }
  if (!dad[t]) return 0;
  long long totflow = 0;
  for (int i = 0; i < G[t].size(); i++) {</pre>
    Edge *start = &G[G[t][i].to][G[t][i].\leftarrow
        index1:
    int amt = INF;
    for (Edge *e = start; amt && e != dad←
        [s]; e = dad[e->from]) {
      if (!e) { amt = 0; break; }
      amt = min(amt, e->cap - e->flow);
    if (amt == 0) continue;
    for (Edge *e = start; amt && e != dad←)
        [s]: e = dad[e->from]) {
      e->flow += amt:
      G[e->to][e->index].flow -= amt;
    totflow += amt;
  return totflow;
// Call this to get the max flow. s, t \hookleftarrow
   are 0-based.
```

# 3.2 Maximum Bipartite Matching

```
// This code performs maximum bipartite \leftarrow
   matching.
11
// Running time: O(|E| |V|) -- often much \leftarrow
   faster in practice
// For larger input, consider Dinic, which \hookleftarrow
   runs in O(E sqrt(V))
   INPUT: w[i][j] = edge between row node←
    i and column node j
// OUTPUT: mr[i] = assignment for row ←
   node i, -1 if unassigned
              mc[j] = assignment for column \leftrightarrow
   node j, -1 if unassigned
             function returns number of \leftarrow
   matches made
typedef vector < vll> vvl;
bool FindMatch(ll i, const vvl &w, vl &mr, ←
   vl &mc. vl &seen) {
  for (11 j = 0; j < w[i].size(); j++) {</pre>
    if (w[i][j] && !seen[j]) {
      seen[i] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, ←</pre>
          mr, mc, seen)) {
        mr[i] = j;
        mc[i] = i;
        return true:
```

# 3.3 Minimum cost Bipartite Matching

```
// Min cost bipartite matching via shortest←
     augmenting paths
//
// This is an O(n^3) implementation of a \leftarrow
    shortest augmenting path
// algorithm for finding min cost perfect \hookleftarrow
   matchings in dense
// graphs. In practice, it solves 1000 \leftarrow
   x1000 problems in around 1
// second.
// cost[i][j] = cost for pairing left ←
   node i with right node j
// Lmate[i] = index of right node that \leftarrow
   left node i pairs with
// Rmate[j] = index of left node that ←
   right node j pairs with
// The values in cost[i][j] may be positive

     or negative. To perform
// maximization, simply negate the cost[][]\leftarrow
     matrix.
typedef vector < double > VD;
typedef vector < VD > VVD;
typedef vector<int> VI;
```

```
double MinCostMatching(const VVD &cost, VI ←
   &Lmate, VI &Rmate) {
  int n = int(cost.size());
  // construct dual feasible solution
  VD u(n):
  VD v(n):
  for (int i = 0; i < n; i++) {</pre>
    u[i] = cost[i][0]:
    for (int j = 1; j < n; j++) u[i] = min(\leftarrow
       u[i], cost[i][j]);
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(\leftarrow
        v[j], cost[i][j] - u[i]);
  // construct primal solution satisfying \hookleftarrow
      complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0:
  for (int i = 0; i < n; i++) {</pre>
    for (int j = 0; j < n; j++) {</pre>
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < ←</pre>
         1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break:
      }
  }
  VD dist(n):
  VI dad(n):
  VI seen(n);
  // repeat until primal solution is \hookleftarrow
      feasible
  while (mated < n) {
    // find an unmatched left node
    int s = 0;
```

```
while (Lmate[s] != -1) s++;
// initialize Dijkstra
fill(dad.begin(), dad.end(), -1);
fill(seen.begin(), seen.end(), 0);
for (int k = 0; k < n; k++)
  dist[k] = cost[s][k] - u[s] - v[k];
int j = 0;
while (true) {
  // find closest
  i = -1;
  for (int k = 0; k < n; k++) {</pre>
    if (seen[k]) continue;
    if (j == -1 || dist[k] < dist[j]) j←</pre>
        = k;
  seen[j] = 1;
  // termination condition
  if (Rmate[j] == -1) break;
  // relax neighbors
  const int i = Rmate[j];
  for (int k = 0; k < n; k++) {</pre>
    if (seen[k]) continue;
    const double new_dist = dist[j] + ←
        cost[i][k] - u[i] - v[k];
    if (dist[k] > new_dist) {
      dist[k] = new_dist;
      dad[k] = j;
// update dual variables
for (int k = 0; k < n; k++) {</pre>
  if (k == j || !seen[k]) continue;
  const int i = Rmate[k];
  v[k] += dist[k] - dist[j];
  u[i] -= dist[k] - dist[j];
u[s] += dist[j];
// augment along path
```

```
while (dad[j] >= 0) {
    const int d = dad[j];
    Rmate[j] = Rmate[d];
    Lmate[Rmate[j]] = j;
    j = d;
}
Rmate[j] = s;
Lmate[s] = j;

mated++;
}

double value = 0;
for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

return value;
}</pre>
```

### 3.4 Konig's Theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover. To exhibit the vertex cover:

- 1. Find a maximum matching
- 2. Change each edge **used** in the matching into a directed edge from **right to left**
- 3. Change each edge **not used** in the matching into a directed edge from **left to right**
- 4. Compute the set T of all vertices reachable from unmatched vertices on the left (including themselves)
- 5. The vertex cover consists of all vertices on the right that are **in** T, and all vertices on the left that are **not in** T

### 3.5 Minimum Edge Cover

If a minimum edge cover contains C edges, and a maximum match- ing contains M edges, then C + M = |V|. To obtain the edge cover, start with a maximum matching, and then, for every vertex not matched, just select some edge incident upon it and add it to the edge set.

### 4 Data Structures

#### 4.1 BIT

```
// Binary indexed tree supporting binary \( \to \)
    search.
struct BIT {
    ll n;
    vector<ll> bit;
```

```
// BIT can be thought of as having \hookleftarrow
    entries f[1], ..., f[n]
// which are 0-initialized
BIT(ll n):n(n), bit(n+1) {}
// returns f[1] + ... + f[idx-1]
// precondition idx <= n+1</pre>
ll read(ll idx) {
11 \text{ res} = 0:
    while (idx > 0) res += bit[idx] . \leftarrow
        idx -= idx & -idx:
    return res;
// returns f[idx1] + ... + f[idx2]
// precondition idx1 <= idx2 <= n+1</pre>
ll read2(ll idx1, ll idx2) {
    return read(idx2) - read(idx1-1):
}
// adds val to f[idx]
// precondition 1 <= idx <= n (there is\leftarrow
     no element 0!)
void update(ll idx, ll val) {
    while (idx <= n) bit[idx] += val, \leftarrow
        idx += idx & -idx:
// returns smallest positive idx such \hookleftarrow
    that read(idx) >= target
ll lower_bound(ll target) {
    if (target <= 0) return 1;</pre>
    ll pwr = 1; while (2*pwr \le n) pwr\leftarrow
        *=2:
    11 idx = 0; 11 tot = 0;
    for (; pwr; pwr >>= 1) {
         if (idx+pwr > n) continue;
         if (tot + bit[idx+pwr] < target←</pre>
             tot += bit[idx+=pwr]:
        }
    }
    return idx+1;
// returns smallest positive idx such \leftarrow
    that read(idx) > target
ll upper_bound(ll target) {
    if (target < 0) return 1;</pre>
    ll pwr = 1; while (2*pwr \le n) pwr\leftarrow
        *=2:
```

```
11 idx = 0; 11 tot = 0;
         for (; pwr; pwr >>= 1) {
             if (idx+pwr > n) continue;
             if (tot + bit[idx+pwr] <= ←</pre>
                 target) {
                 tot += bit[idx+=pwr];
             }
        return idx+1;
    }
};
/*For range update and range query
To add v in range [a, b]: Update(a, v), \leftarrow
    Update(b+1, -v)first BIT and Update(a, \leftarrow
   v*(a-1)) and Update(b+1, -v*b) second \leftarrow
To get sum in range [0, x]: you simply do \leftarrow
    Query_BIT1(x)*x - Query_BIT2(x);
```

# 4.2 Segment Tree

```
class segtree{
public:
    ll size:
    const ll inf=0x7fffffff;
    ll MAX;
    vector<11>tree:
    vector < 11 > lazy;
    vector < 11 > arr :
    segtree(ll n)
        size=n;
        MAX=4*n;
            tree=vll(MAX,0);
            lazy=tree;
            arr=vll(size):
    void build_tree(ll node,ll a, ll b)
        if(a > b) return; // Out of range
        if(a == b) { // Leaf node
                tree[node] = arr[a]; // ←
                    Init value
            return:
```

```
build_tree(node*2, a, (a+b)/2); // \leftarrow
       Init left child
    build_tree(node*2+1, 1+(a+b)/2, b); \leftarrow
         // Init right child
    tree[node] = max(tree[node*2], tree←
        [node*2+1]); // Init root value
}
void update_tree(ll node, ll a, ll b, ←
   ll i, ll j, ll value)
    if(lazy[node] != 0) { // This node ←
       needs to be updated
        tree[node] += lazy[node]; // 
            Update it
        if(a != b) {
            lazy[node*2] += lazy[node]; ←
                 // Mark child as lazy
                lazy[node*2+1] += lazy[←
                    node]; // Mark ←
                    child as lazy
        }
        lazy[node] = 0; // Reset it
    }
    if(a > b || a > j || b < i) // ←
       Current segment is not within \hookleftarrow
       range [i, j]
        return;
    if(a >= i && b <= j) { // Segment ←
       is fully within range
            tree[node] += value;
        if(a != b) { // Not leaf node
            lazy[node*2] += value;
            lazy[node*2+1] += value;
        }
            return;
/* if(a == b) { // comment it out if \leftarrow
   lazy propagation
    tree[node] += value;
    return:
        }*/
```

```
update_tree(node*2, a, (a+b)/2, i, \leftarrow
        j, value); // Updating left ←
        child
    update_tree(1+node*2, 1+(a+b)/2, b,\leftarrow
         i, j, value); // Updating ←
        right child
    tree[node] = max(tree[node*2], tree←
        [node*2+1]); // Updating root ←
        with max value
}
ll query_tree(ll node, ll a, ll b, ll i \leftarrow
    , 11 i)
{
    if (a > b \mid | a > j \mid | b < i) return \leftarrow
        -inf; // Out of range
    if(lazy[node] != 0) { // This node ←
        needs to be updated
         tree[node] += lazy[node]; // ←
             Update it
         if(a != b) {
             lazy[node *2] += lazy[node]; ←
                  // Mark child as lazy
             lazy[node*2+1] += lazy[node↔
                 1: // Mark child as \leftarrow
                 lazv
        }
         lazy[node] = 0; // Reset it
    }
    if(a >= i && b <= j) // Current ←</pre>
        segment is totally within range\leftarrow
         [i, j]
        return tree[node];
    ll q1 = query_tree(node*2, a, (a+b)\leftarrow
        /2, i, j); // Query left child
    11 q2 = query_tree(1+node*2, 1+(a+b\leftarrow
        )/2, b, i, j); // Query right \leftarrow
        child
    ll res = max(q1, q2); // Return \leftarrow
        final result
```

### 4.3 Suffix Array

```
// Suffix array construction in O(L log^2 L←
   ) time. Routine for
// computing the length of the longest \leftarrow
   common prefix of any two
// suffixes in O(\log L) time.
//
// INPUT: string s
// OUTPUT: array suffix[] such that suffix←
   [i] = index (from 0 to L-1)
            of substring s[i...L-1] in the \leftarrow
   list of sorted suffixes.
            That is, if we take the inverse \leftarrow
    of the permutation suffix[],
            we get the actual suffix array.
struct SuffixArray {
  const int L;
  string s;
  vector < vector < int > > P;
  vector<pair<int,int>,int> > M;
  SuffixArray(const string &s) : L(s.length↔
      ()), s(s), P(1, vector < int > (L, 0)), M \leftarrow
```

```
(L) {
  for (int i = 0; i < L; i++) P[0][i] = \leftarrow
      int(s[i]):
  for (int skip = 1, level = 1; skip \langle L; \leftarrow \rangle
       skip *= 2, level++) {
    P.push_back(vector < int > (L, 0));
    for (int i = 0; i < L; i++)</pre>
            M[i] = mp(mp(P[level-1][i], i \leftarrow)
                 + skip < L ? P[level-1][i ←
                + skip] : -1000), i);
    sort(M.begin(), M.end());
    for (int i = 0; i < L; i++)</pre>
            P[level][M[i].vv] = (i > 0 && \leftarrow
                 M[i].xx == M[i-1].xx) ? P[\leftarrow
                 level][M[i-1].vv] : i;
  }
}
vector<int> GetRankArray() { return P. ←
    back(): }
vector < int > GetSuffixArray(){
  vector < int > & rank = P.back():
  int n=rank.size();
  vector < int > sa(n,0);
  for (int i=0; i < n; i++) sa [rank[i]] = i;</pre>
  return sa:
}
vector < int > LCP()
 {
       int n=s.size(),k=0;
       vector < int > lcp(n,0);
       vector < int > & rank = P.back();
       vector<int> sa=GetSuffixArray();
       for(int i=0; i<n; i++, k?k--:0)</pre>
           if(rank[i]==n-1) {k=0; continue ←
           int j=sa[rank[i]+1];
           while (i+k<n && j+k<n && s[i+k \leftarrow]
               ]==s[j+k]) k++;
           lcp[rank[i]]=k;
       return lcp;
  }
```

```
// returns the length of the longest \hookleftarrow
      common prefix of s[i...L-1] and s[j \leftarrow
      ...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0:
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i \leftrightarrow
        < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
         len += 1 << k;
      }
    return len;
  }
};
```

### 4.4 Union Find

```
ll par[100];
void Union(ll x,ll y)
{
  par[x]=y;
}
ll find(ll x)
{
  if(par[x]==x) return x;
  else return par[x]=find(par[x]);
}
```

### 4.5 Palindrome Tree

```
Palindrome tree. Useful structure to 
    deal with palindromes in strings. 0←
    (N)/

#include <iostream>
#include <cstdio>
#include <cstdlib>
#include <algorithm>
#include <vector>
#include <set>
```

```
#include <map>
#include <string>
#include <utility>
#include <cstring>
#include <cassert>
#include <cmath>
#include <stack>
#include <queue>
using namespace std;
const int MAXN = 105000;
struct node {
    int next[26];
    int len:
    int sufflink;
    int num:
};
int len:
char s[MAXN];
node tree[MAXN];
int num:
                     // node 1 - root with \leftrightarrow
   len -1, node 2 - root with len 0
                     // max suffix ↔
int suff:
   palindrome
long long ans;
bool addLetter(int pos) {
    int cur = suff, curlen = 0;
    int let = s[pos] - 'a';
    while (true) {
        curlen = tree[cur].len;
        if (pos - 1 - curlen >= 0 && s[pos \leftarrow
            - 1 - curlen] == s[pos])
            break:
        cur = tree[cur].sufflink;
    if (tree[cur].next[let]) {
        suff = tree[cur].next[let];
        return false:
    num++;
```

```
suff = num;
    tree[num].len = tree[cur].len + 2;
    tree[curl.next[let] = num:
    if (tree[num].len == 1) {
        tree[num].sufflink = 2;
        tree[num].num = 1;
        return true;
    while (true) {
        cur = tree[cur].sufflink:
        curlen = tree[cur].len;
        if (pos - 1 - curlen >= 0 && s[pos \hookleftarrow
           - 1 - curlen] == s[pos]) {
            tree[num].sufflink = tree[cur].←
                next[let]:
            break:
       }
    }
    tree[num].num = 1 + tree[tree[num]. ←
       sufflink 1. num:
    return true;
void initTree() {
    num = 2; suff = 2;
    tree[1].len = -1; tree[1].sufflink = 1;
    tree[2].len = 0; tree[2].sufflink = 1;
}
int main() {
    //assert(freopen("input.txt", "r", ←
       stdin)):
    //assert(freopen("output.txt", "w", ←
       stdout)):
    gets(s);
    len = strlen(s);
    initTree();
    for (int i = 0; i < len; i++) {</pre>
        addLetter(i);
```

```
ans += tree[suff].num;
    }
    cout << ans << endl;</pre>
    return 0;
}
4.6 Z Function
#include < bits / stdc++.h>
using namespace std;
const int MAXN = 1000100;
string s;
int n;
int z[MAXN];
int 1, r;
int main() {
    getline(cin, s);
    n = (int) s.length();
    1 = r = 0:
    for (int i = 2; i <= n; i++) {</pre>
        int cur = 0:
        if (i <= r)
             cur = min(r - i + 1, z[i - 1 + \leftarrow
                 11):
         while (i + cur <= n && s[i + cur - \leftarrow
            1] == s[cur])
             cur++;
        if (i + cur - 1 > r) {
             1 = i; r = i + cur - 1;
        z[i] = cur;
    z[1] = n;
    for (int i = 1; i <= n; i++)</pre>
        printf("%d ", z[i]);
    return 0;
}
```

# 5 Graph Algorithms

# 5.1 BFS

```
vector < vector < 11 > > graph (110);
void bfs(ll root,ll n)
    vll visited(n,0);
    11 v,i;
    visited[root]=1;
    queue <11> q;
    q.push(root);
    while(!q.empty())
        11 u=q.front();
        q.pop();
        for(i=0;i<graph[u].size();i++)</pre>
            v=graph[u][i];
            if(!visited[v])
                 visited[v]=1:
                 q.push(v);
        }
    }
```

```
bool valid(ll x,ll y,ll p, ll q)
{
    return (x>=0&&x<p&&y>=0&&y<q);
}
int steps
    [][2]={{1,0},{0,1},{-1,0},{0,-1},{0,0}};

char mat[15][21][21];
ll bfs(ll p,ll q,ll r)
{
    queue<ll> st,end,tm;
    int x,y,j,k;
    bool visited[15][21][21]={0};
    st.push(0);
    end.push(0);
    tm.push(0);
    visited[0][0][0]=='1')return -1;
    while(!st.empty())
```

```
{
        x=st.front();
        y=end.front();
        k=tm.front();
        st.pop();
        end.pop();
        tm.pop();
        if(x==p-1&&y==q-1)
            return k:
        k++:
        j=(k)%r;
        for(int i=0;i<5;i++)</pre>
            x += steps[i][0];
            v+=steps[i][1];
            if(valid(x,y,p,q)&&!visited[j][←
                x][y]&&mat[j][x][y]=='0')
            {
                visited[j][x][y]=1;
                st.push(x);
                end.push(y);
                tm.push(k);
            x-=steps[i][0];
            y-=steps[i][1];
        }
    }
    return -1;
ans=bfs(p,q,r);
```

#### 5.2 DFS

```
bool visited[1010];
void dfs(ll num,ll ind)
{
     //cout << ind << endl;
     visited[ind] = true;
     cnt[num] ++;
     coins[num] += arr[ind];
     for(int i=0;i < graph[ind].size();i++)
     {
          if(!visited[graph[ind][i]])
          dfs(num,graph[ind][i]);
     }
}</pre>
```

# 5.3 Dijkstra

```
#define SIZE 10010
struct node
    11 n;
    11 cost:
    node(){}
    node(ll n,ll cost) {this->n = n;this->\leftarrow
        cost = cost:}
    bool operator < (const node &nd) const \hookleftarrow
        if(cost!=nd.cost)
        return cost > nd.cost;
};
  struct edge
    11 u;
    11 v:
    ll w:
    edge(){}
    edge(ll u,ll v,ll w) {this->u = u; this\leftarrow
        ->v=v; this ->w=w;
    bool operator < (const edge &nd) const ↔
        return this->w>nd.w;
}:
ll dist[505][505];
vector < vector <pll> > graph;
11 anscnt, ansdist,s;
bool visited[SIZE];
11 weight[SIZE];
void dijkstra(ll st)
{
    priority_queue < node > q;
    clr(visited):
    reset(weight);
    weight[st]=0;
    q.push(node(st,0));
    node x,z;
    pll y;
    while(!q.empty())
        x=q.top();
        q.pop();
        if(visited[x.n])
```

```
continue;
        else visited[x.n]=true;
        dist[st][x.n]=x.cost;
        for(int i=0;i<graph[x.n].size();i←</pre>
             y=graph[x.n][i];
             if (weight[y.yy] == -1|| weight[y. ←
                 yy]>x.cost+y.xx)
                 weight[y.yy]=x.cost+y.xx;
                 q.push(node(y.yy,weight[y.←
                     vv]));
            }
        }
    }
scanf("%11d %11d %11d",&n,&m,&s);
for(i=0;i<s;i++)</pre>
    scanf("%11d",&arr[i]);
11 wt:
fill(&dist[0][0],&dist[505][0],1e10);
graph.clear();
graph.resize(n+1);
for(i=0;i<n;i++)dist[i][i]=0;</pre>
for (i = 0: i < m: i++)</pre>
    scanf("%11d %11d %11d",&p,&q,&wt);
    graph[p].push_back(mp(wt,q));
dijkstra(0);
```

### 5.4 Krushkals

```
vector < 11 > par;
void make_set(ll n)
{
    par.clear();
    par.resize(n);
    for(ll i=0;i<n;i++)
        par[i]=i;
}
ll find(ll ch)
{
    if(ch==par[ch])
    {
        return ch;
    }
}</pre>
```

```
}else{
        par[ch]=find(par[ch]);
        return par[ch];
    }
11 xunion(ll x,ll y)
    par[x]=y;
vector<pair<11,pll> > edges;
//main
edges.clear();
edges.push_back(mp(p,mp(i,j)));
sort(edges.begin(),edges.end());
11 size=edges.size();
11 cnt=0:
make_set(n);
for(i=0:i<size:i++)</pre>
    if (find(edges[i].yy.xx)!=find(edges[i].←
        уу.уу))
    ł
        //cout <<find(edges[i].yy.xx) <<find(←
            edges[i].vv.vv) << endl;
        xunion(find(edges[i].yy.xx),find(←
            edges[i].yy.yy));
        ans-=edges[i].xx;
        cnt++;
        //cout << i << " " << edges [i] .xx << endl;
        if(cnt==n-1)
            break;
    }
if(cnt==n-1)
printf("Case %1ld: %1ld\n",z,ans);
else
printf("Case %lld: -1\n",z);
```

### 5.5 SCC

```
vector < vll > graph , graph2;
vector < bool > visited;
stack<ll>nodes, nodes2;
ll scc[20010];
void dfs(ll num)
    visited[num]=1;
    for(int i=0;i<graph[num].size();i++)</pre>
```

```
if(!visited[graph[num][i]])
             dfs(graph[num][i]);
    nodes.push(num);
void dfs2(11 num,11 cnt)
    visited[num]=1:
    scc[num]=cnt:
    for(int i=0;i<graph[num].size();i++)</pre>
        if(!visited[graph[num][i]])
             dfs2(graph[num][i],cnt);
    }
int main()
    ll t,z,i,j,k,n,m,p,q,r,s,ans;
    scanf("%11d",&t);
    for(z=1;z<=t;z++)
         scanf("%11d %11d",&n,&m);
         graph.clear();
         graph2.clear();
         graph.resize(n+1);
         graph2.resize(n+1);
         nodes=stack<ll>();
         visited.resize(n+1);
        fill(visited.begin(), visited.end() \leftarrow
             ,0);
             for ( j = 0; j < m; j ++)</pre>
                 scanf("%11d %11d",&p,&q);
                 p--;q--;
                graph[p].push_back(q);
                graph2[q].push_back(p);
             }
        11 cnt=0:
        for (i=0;i<n;i++)</pre>
             if(!visited[i])
                 {dfs(i);
                 }
         nodes2=nodes;
         swap(graph, graph2);
```

```
fill(visited.begin(), visited.end()←
    cnt=0:
    while(!nodes2.empty())
         p=nodes2.top();
         nodes2.pop();
         if(!visited[p])
             dfs2(p,cnt);
             cnt++;
    swap(graph,graph2);
    graph2.clear();
    graph2.resize(cnt+2);
    11 src[20010]={0}, sink[20010]={0};
    for(int i=0:i<n:i++)</pre>
         for(int j=0;j<graph[i].size();j</pre>
             ++)
             11 v=graph[i][j];
             //cout <<scc[i] << " " << scc[v ←
                 1<<endl:</pre>
             if(scc[i]!=scc[v])
                 //graph2[scc[i]]. ←
                     push_back(scc[v]);
                 src[scc[v]]=1:
                 sink[scc[i]]=1;
             }
        }
    11 \text{ sc=0,sk=0};
    for (i = 0; i < cnt; i++)</pre>
         if (!src[i])sc++;
        if(!sink[i])sk++;
    if (cnt==1) sc=sk=0;
    printf("Case %11d: %11d\n",z,max(sc←
        .sk)):
return 0:
```

#### 5.6 Eulerian Path

```
// Eulerian path/circuit in an undirected \leftarrow
   graph. TODO: Does this handle self-\leftarrow
   edges?
// NOTE(Brian): This looks like it could \leftarrow
   theoretically degrade to quadratic time\leftarrow
    in, say, a graph where we keep going \leftarrow
   back and forth between two vertices; in←
     this case a lot of time could be \hookleftarrow
   wasted searching for an unused edge.
struct EulerianPath {
    int n:
    vector<vector<int> > adj;
    vector<pair<int, int> > edges;
    vector < int > valid;
    vector<int> circuit;
    EulerianPath(int n): n(n), adj(n) {}
    // Call this to construct the graph.
    // Edges are zero-based and undirected \hookleftarrow
        (only add each edge once!)
    void add_edge(int x, int y) {
        adj[x].push_back(edges.size());
        adj[y].push_back(edges.size());
        edges.push_back(make_pair(x, y));
        valid.push_back(1);
    }
    void find_path(int x){
      for(int i = 0; i < adj[x].size(); i \leftarrow
          ++){
        int e = adj[x][i];
        if(!valid[e]) continue;
        int v = edges[e].first;
        if(v == x) v = edges[e].second;
        valid[e] = 0;
        find_path(v);
      circuit.push_back(x);
    // Call this to find the path/circuit (\leftarrow
        autodetects)
    // Returns the path/circuit itself in "\hookleftarrow
        circuit" variable
```

### 5.7 Bridges

```
// Finds bridges and cut vertices
// Receives:
// N: number of vertices
// 1: adjacency list
// Gives:
// vis, seen, par (used to find cut \leftarrow
   vertices)
// ap - 1 if it is a cut vertex, 0 \leftarrow
   otherwise
// brid - vector of pairs containing the \hookleftarrow
   bridges
typedef pair<int, int> PII;
int N:
vector <int> 1[MAX];
vector <PII> brid;
int vis[MAX], seen[MAX], par[MAX], ap[MAX];
int cnt, root;
void dfs(int x){
  if(vis[x] != -1)
    return;
  vis[x] = seen[x] = cnt++;
  int adj = 0;
  for(int i = 0; i < (int)1[x].size(); i++)\leftarrow
    int v = l[x][i];
    if(par[x] == v)
      continue;
    if(vis[v] == -1){
```

```
adi++;
      par[v] = x;
      dfs(v):
      seen[x] = min(seen[x], seen[v]);
      if (seen[v] >= vis[x] && x != root)
        ap[x] = 1;
      if(seen[v] == vis[v])
        brid.push_back(make_pair(v, x));
    elsef
      seen[x] = min(seen[x], vis[v]);
      seen[v] = min(seen[x], seen[v]);
 if(x == root) ap[x] = (adj>1);
void bridges(){
 brid.clear();
 for(int i = 0; i < N; i++){</pre>
    vis[i] = seen[i] = par[i] = -1;
    ap[i] = 0;
 }
  cnt = 0;
 for(int i = 0; i < N; i++)</pre>
    if(vis[i] == -1){
      root = i:
      dfs(i);
    }
```

# 6 Number Theory

# 6.1 Formulas

- 1. Eulers Formulas  $a^{\varphi(m)} \equiv 1 \pmod{m}$  where  $\varphi(m)$  is Euler's totient function
- 2. Cayley's Formula: There are n n2 spanning trees of a complete graph with n labeled vertices. 2.Derangement: der(n) = (n-1)(der(n-1) + der(n-2))
- 3. Euler's Formula for Planar Graph 6 : V E + F = 2, where F is the number of faces 7 of the Planar Graph.
- 4. Pick's theorem: the area A of a polygon in terms of the number i of lattice points in the interior located in the polygon and the number b of lattice points on the boundary placed on the polygon's perimeter:  $A = i + \frac{b}{2} 1$
- 5. A complete bipartite graph K(m,n) has m^(n-1) n^(m-1) spanning trees.

# 6.2 Math Library

```
ll mod_inverse ( ll a , ll n ) {
11 x , y ;
11 d = extended_euclid (a , n , x , y );
if ( d > 1) return -1;
return mod (x , n );
i64 nCr[MAX][MAX], fact[MAX];
fact[0] = nCr[0][0] = 1;
    for(i = 1; i < MAX; i++) {</pre>
        fact[i] = (fact[i-1] * i) % MOD;
        nCr[i][0] = nCr[i][i] = 1;
    for(i = 1; i < MAX; i++)</pre>
        for(k = 1; k < i; k++)
            nCr[i][k] = (nCr[i-1][k] + nCr[\leftarrow
                i-1][k-1]) % MOD;
ll modpow(ll a,ll b,ll mod)
    ll res=1;
    while(b)
    { if(b\&1)res=(res*a)\%mod;
        a=(a*a)\%mod;
        b>>=1:
    return res;
11 func(11 x.11 mod)
    return ((x)%mod+mod)%mod;
long long inv(ll n, ll MOD=md)
    return modpow(n,MOD-2,MOD);
void sieve(vll &imap,ll u)
    11 j;
    vector < bool > prime (u+1, true);
    for(11 i = 2; i <=u; i++)</pre>
        if(prime[i])
            j=i;
            imap.push_back(i);
            while(j<=u)</pre>
```

### 6.3 Extended GCD

```
int extGcd(int a, int b, int &x, int &y){
    if(b == 0){
        x = 1;
        y = 0;
        return a;
}

int g = extGcd(b,a % b,y,x);
    y -= a / b * x;
    return g;
}

int modInv(int a, int m){
    int x,y;
    extGcd(a, m, x, y);
    return (x % m + m) % m;
}
```

### 6.4 Chineese Remainder Theorem

# 6.5 Heavy Light Decomposition

```
/∗←
    Heavy-light decomposition with segment \hookleftarrow
        trees in paths.
    Used for finding maximum on the path \hookleftarrow
       between two vertices.
    O(N) on building, O(\log N^2) on query.
    Based on problem 1553 from acm.timus.ru
    http://acm.timus.ru/problem.aspx?space←
       =1&num=1553
#include <iostream>
#include <fstream>
#include <cmath>
#include <algorithm>
#include <vector>
#include <set>
#include <map>
#include <stack>
#include <queue>
#include <cstdlib>
#include <cstdio>
#include <string>
#include <cstring>
#include <cassert>
#include <utility>
#include <iomanip>
using namespace std;
const int MAXN = 105000;
struct SegmentTree {
    int * tree:
    int size;
    void init(int sz) {
        tree = new int [4 * sz]:
```

```
memset(tree, 0, 4 * sz * sizeof(int ← | char q;
        size = sz:
    }
    int getMax(int v, int from, int to, int←)
         1. int r) {
        if (1 > to | | r < from)
            return 0:
        if (from == 1 && to == r)
            return tree[v];
        int mid = (from + to) / 2;
        int res = getMax(v * 2, from, mid, ←
            1, min(r, mid));
        res = max(res, getMax(v * 2 + 1, \leftarrow)
            mid + 1, to, max(1, mid + 1), r \leftarrow
            ));
        return res:
    }
    int getMax(int 1, int r) {
        return getMax(1, 1, size, l, r);
    }
    void update(int v, int from, int to, \leftarrow
        int pos, int val) {
        if (pos > to || pos < from)</pre>
            return:
        if (from == pos && to == pos) {
             tree[v] = val:
             return;
        }
        int mid = (from + to) / 2;
        update(v * 2, from, mid, pos, val);
        update(v * 2 + 1, mid + 1, to, pos\leftarrow | }
            , val);
        tree[v] = max(tree[v * 2], tree[v *\leftarrow
             2 + 11):
    }
    void update(int pos, int val) {
        update(1, 1, size, pos, val);
    }
};
int n, qn;
```

```
int a, b;
int timer;
int sz[MAXN];
int tin[MAXN], tout[MAXN];
int val[MAXN];
vector <int> g[MAXN];
int p[MAXN];
int chain[MAXN], chainRoot[MAXN];
int chainSize[MAXN], chainPos[MAXN];
int chainNum:
SegmentTree tree[MAXN];
bool isHeavy(int from, int to) {
    return sz[to] * 2 >= sz[from];
void dfs(int v, int par = -1) {
    timer++;
    tin[v] = timer;
    p[v] = par;
    sz[v] = 1;
    for (int i = 0; i < (int) g[v].size(); \leftarrow
       i++) {
        int to = g[v][i];
        if (to == par)
            continue;
        dfs(to, v);
        sz[v] += sz[to];
    }
    timer++;
    tout[v] = timer;
int newChain(int root) {
    chainNum++:
    chainRoot[chainNum] = root;
    return chainNum;
}
void buildHLD(int v, int curChain) {
    chain[v] = curChain;
    chainSize[curChain]++:
    chainPos[v] = chainSize[curChain];
```

```
for (int i = 0; i < g[v].size(); i++) {</pre>
        int to = g[v][i];
        if (p[v] == to)
            continue:
        if (isHeavy(v, to))
            buildHLD(to, curChain);
            buildHLD(to, newChain(to));
   }
bool isParent(int a, int b) {
    return tin[a] <= tin[b] && tout[a] >= ←
       tout[b];
}
int getMax(int a, int b) {
    int res = 0;
    while (true) {
        int curChain = chain[a];
        if (isParent(chainRoot[curChain], b←
           ))
            break:
        res = max(res, tree[curChain]. ←
            getMax(1, chainPos[a]));
        a = p[chainRoot[curChain]];
    while (true) {
        int curChain = chain[b]:
        if (isParent(chainRoot[curChain], a←
            break;
        res = max(res, tree[curChain]. ←
            getMax(1, chainPos[b]));
        b = p[chainRoot[curChain]];
    int from = chainPos[a], to = chainPos[b↔
       1:
    if (from > to)
        swap(from, to);
    res = max(res, tree[chain[a]].getMax(←
       from. to)):
    return res;
```

```
int main() {
    //assert(freopen("input.txt","r",stdin)←
    //assert(freopen("output.txt","w", ←
       stdout)):
    scanf("%d", &n);
    for (int i = 1; i < n; i++) {</pre>
        int from, to;
        scanf("%d %d", &from, &to);
        g[from].push_back(to);
        g[to].push_back(from);
   }
    dfs(1):
    buildHLD(1, newChain(1));
    for (int i = 1; i <= chainNum; i++) {</pre>
        tree[i].init(chainSize[i]);
    scanf("%d\n", &qn);
    for (int i = 1; i <= qn; i++) {
        scanf("%c %d %d\n", &q, &a, &b);
        if (q == 'I') {
            val[a] += b:
            tree[chain[a]].update(chainPos[←
                a], val[a]);
       }
        else {
            printf("%d\n", getMax(a, b));
        }
    }
    return 0:
```

### 6.6 Lucas

```
else
      ans=0:
  return ans;
6.7 Primitive Root
int getPriRoot(int p) {
  if (p==2) return 1;
  int phi = p - 1;
  getFactor(phi);
  for (int g = 2; g < p; ++g) {
    bool flag=1;
    for (int i = 0; flag && i < N; ++i)</pre>
      if (power(g, phi/fac[i], p) == 1)
        flag=0;
    if (flag)
      return g;
 }
}
```

### 6.8 Simplex Algorithm

```
// Two-phase simplex algorithm for solving \hookleftarrow
   linear programs of the form
//
       maximize
                      c^T x
//
       subject to Ax <= b
11
                      x >= 0
// INPUT: A -- an m x n matrix
          b -- an m-dimensional vector
//
          c -- an n-dimensional vector
          x -- a vector where the optimal \hookleftarrow
    solution will be stored
// OUTPUT: value of the optimal solution (\hookleftarrow
   infinity if unbounded
11
           above, nan if infeasible)
11
// To use this code, create an LPSolver \leftarrow
    object with A, b, and c as
// arguments. Then, call Solve(x).
typedef long double DOUBLE;
typedef vector < DOUBLE > VD;
typedef vector < VD > VVD;
typedef vector<int> VI;
```

```
const DOUBLE EPS = 1e-9:
struct LPSolver {
 int m, n;
  VI B, N;
  VVD D:
  LPSolver(const VVD &A, const VD &b, const←
       VD &c):
    m(b.size()), n(c.size()), N(n+1), B(m), \leftarrow
         D(m+2, VD(n+2)) {
    for (int i = 0; i < m; i++) for (int j \leftarrow
        = 0; i < n; j++) D[i][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n+ \leftarrow
        i: D[i][n] = -1: D[i][n+1] = b[i]: \leftrightarrow
    for (int j = 0; j < n; j++) { N[j] = j; \leftarrow
         D[m][i] = -c[i]; }
    N[n] = -1; D[m+1][n] = 1;
  void Pivot(int r, int s) {
    for (int i = 0; i < m+2; i++) if (i != \leftarrow
      for (int j = 0; j < n+2; j++) if (j \leftarrow
        D[i][j] = D[r][j] * D[i][s] / D[r \leftarrow
            ][s];
    for (int j = 0; j < n+2; j++) if (j != \leftarrow
        s) D[r][i] /= D[r][s];
    for (int i = 0; i < m+2; i++) if (i != \leftarrow
        r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
  bool Simplex(int phase) {
    int x = phase == 1 ? m+1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j \le n; j++) {
        if (phase == 2 && N[j] == -1) \leftarrow
             continue:
        if (s == -1 \mid | D[x][j] < D[x][s] \mid | \leftarrow
              D[x][j] == D[x][s] && N[j] < N \leftrightarrow
```

```
[s]) s = j;
      if (D[x][s] >= -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {</pre>
         if (D[i][s] <= 0) continue;</pre>
         if (r == -1 || D[i][n+1] / D[i][s] \leftrightarrow
             < D[r][n+1] / D[r][s] ||
             D[i][n+1] / D[i][s] == D[r][n \leftarrow
                 +1] / D[r][s] && B[i] < B[r↔
                 ]) r = i;
      }
      if (r == -1) return false;
      Pivot(r. s):
    }
  }
  DOUBLE Solve(VD &x) {
    int r = 0:
    for (int i = 1; i < m; i++) if (D[i][n \leftarrow
        +1] < D[r][n+1]) r = i;
    if (D[r][n+1] <= -EPS) {</pre>
      Pivot(r, n);
      if (!Simplex(1) || D[m+1][n+1] < -EPS \leftarrow
          ) return -numeric_limits < DOUBLE \hookleftarrow
          >::infinity();
      for (int i = 0; i < m; i++) if (B[i] \leftarrow
          == -1) {
         int s = -1;
         for (int j = 0; j <= n; j++)</pre>
           if (s == -1 || D[i][j] < D[i][s] ←
               || D[i][j] == D[i][s] && N[j]↔
                < N[s]) s = i;
         Pivot(i. s):
      }
    }
    if (!Simplex(2)) return numeric_limits <←</pre>
        DOUBLE >::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < \leftarrow
        n) \times [B[i]] = D[i][n+1];
    return D[m][n+1];
  }
};
```

### 6.9 Eulers Totient Function

```
// This code took less than 0.5s to 
    calculate with MAX = 10^7
#define MAX 10000000

int phi[MAX];
bool pr[MAX];

void totient(){
  for(int i = 0; i < MAX; i++){
    phi[i] = i;
    pr[i] = true;
}

for(int i = 2; i < MAX; i++)
    if(pr[i]){
    for(int j = i; j < MAX; j+=i){
        pr[j] = false;
        phi[j] = phi[j] - (phi[j] / i);
    }
    pr[i] = true;
}
</pre>
```

# 7 Geometry

# 7.1 2-D Geometry

+p.y\*q.y; }

```
// C++ routines for computational geometry.
double INF = 1e100:
double EPS = 1e-12;
struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT \&p) : x(p.x), y(p.y)
  PT operator + (const PT &p) const { \hookleftarrow
      return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p) const { \hookleftarrow
      return PT(x-p.x, y-p.y); }
  PT operator * (double c)
                                 const { ←
      return PT(x*c, y*c ); }
  PT operator / (double c)
                              \mathtt{const} \ \ \{ \ \ \hookleftarrow
     return PT(x/c, y/c ); }
};
double dot(PT p, PT q)
                             { return p.x*q.x↔
```

```
double dist2(PT p, PT q) { return dot(p-q←
    ,p-q); }
double cross(PT p, PT q) { return p.x*q.y←
    -p.v*q.x; }
ostream &operator << (ostream &os, const PT &←
  os << "(" << p.x << "," << p.y << ")";
// rotate a point CCW or CW around the \hookleftarrow
    origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x↔
   ); }
PT RotateCW90(PT p) { return PT(p.y,-p.x↔
   ); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(←)
      t)+p.y*cos(t));
}
// project point c onto line through a and \hookleftarrow
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
 return a + (b-a)*dot(c-a, b-a)/dot(b-a, b \leftarrow
      -a):
// project point c onto line segment ←
   through a and b
// if the projection doesn't lie on the \leftarrow
    segment, returns closest vertex
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a.b-a):
  if (fabs(r) < EPS) return a;</pre>
  r = dot(c-a, b-a)/r:
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
// compute distance from c to segment \leftarrow
    between a and b
double DistancePointSegment(PT a, PT b, PT \hookleftarrow
    c) {
```

```
return sqrt(dist2(c, ProjectPointSegment(← | // segments intersect first
      a, b, c)));
// determine if lines from a to b and c to \leftarrow
    d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) ←
 return fabs(cross(b-a, c-d)) < EPS:
bool LinesCollinear(PT a, PT b, PT c, PT d)\leftarrow
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS:
// determine if line segment from a to b \leftarrow
    intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT\leftarrow
     d) {
 if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS \mid | dist2(a, d) < \leftarrow | \}
        EPS II
      dist2(b, c) < EPS \mid | dist2(b, d) < \leftarrow
          EPS) return true;
    if (dot(c-a, c-b) > 0 \&\& dot(d-a, d-b) \leftarrow
        > 0 \&\& dot(c-b, d-b) > 0)
      return false;
    return true:
  if (cross(d-a, b-a) * cross(c-a, b-a) > \leftarrow
      0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > \leftarrow
      0) return false;
  return true;
// compute intersection of line passing \hookleftarrow
    through a and b
// with line passing through c and d, \leftarrow
    assuming that unique
// intersection exists; for segment \hookleftarrow
    intersection, check if
```

```
PT ComputeLineIntersection(PT a, PT b, PT c\leftarrow
    , PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS\leftarrow
  return a + b*cross(c, d)/cross(b, d);
// determine if c and d are on same side of \leftarrow
     line passing through a and b
bool OnSameSide(PT a, PT b, PT c, PT d) {
  return cross(c-a, c-b) * cross(d-a, d-b) \leftarrow
// compute center of circle given three \hookleftarrow
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2:
  c = (a+c)/2:
  return ComputeLineIntersection(b, b+←
      RotateCW90(a-b), c, c+RotateCW90(a-c)\leftarrow
      );
// determine if point is in a possibly non-\leftarrow
    convex polygon (by William
// Randolph Franklin); returns 1 for \leftarrow
    strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for\leftarrow
     the remaining points.
// Note that it is possible to convert this\leftarrow
     into an *exact* test using
// integer arithmetic by taking care of the\hookleftarrow
     division appropriately
// (making sure to deal with signs properly \leftarrow
    ) and then by writing exact
// tests for checking point on polygon \hookleftarrow
    boundary
bool PointInPolygon(const vector <PT> &p, PT←
  bool c = 0:
  for (int i = 0; i < p.size(); i++){</pre>
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||</pre>
```

```
p[j].y \le q.y \&\& q.y \le p[i].y) \&\&
      q.x < p[i].x + (p[j].x - p[i].x) * (q \leftarrow
          .y - p[i].y) / (p[j].y - p[i].y))
      c = !c;
  return c;
// determine if point is on the boundary of \leftarrow
     a polygon
bool PointOnPolygon(const vector <PT> &p, PT←
  for (int i = 0; i < p.size(); i++)</pre>
    if (dist2(ProjectPointSegment(p[i], p[(\leftarrow
        i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
// compute intersection of line through \hookleftarrow
    points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a. PT \leftarrow
   b, PT c, double r) {
  vector < PT > ret:
  b = b-a:
  a = a-c:
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret:</pre>
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret:
// compute intersection of circle centered \leftarrow
    at a with radius r
// with circle centered at b with radius R
vector \langle PT \rangle CircleCircleIntersection (PT a, \leftarrow
   PT b, double r, double R) {
  vector < PT > ret;
  double d = sqrt(dist2(a, b));
```

```
if (d > r+R \mid | d+min(r, R) < max(r, R)) \leftarrow
      return ret:
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d:
  ret.push_back(a+v*x + RotateCCW90(v)*y);
 if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y)\leftarrow
  return ret;
// This code computes the area or centroid \hookleftarrow
   of a (possibly nonconvex)
// polygon, assuming that the coordinates \leftarrow
   are listed in a clockwise or
// counterclockwise fashion. Note that the\leftarrow
     centroid is often known as
// the "center of gravity" or "center of \leftarrow
   mass".
double ComputeSignedArea(const vector < PT > & ←
   } (a
  double area = 0:
 for(int i = 0; i < p.size(); i++) {</pre>
   int j = (i+1) % p.size();
   area += p[i].x*p[j].y - p[j].x*p[i].y;
  return area / 2.0;
double ComputeArea(const vector <PT> &p) {
 return fabs(ComputeSignedArea(p));
PT ComputeCentroid(const vector <PT > &p) {
 PT c(0.0):
  double scale = 6.0 * ComputeSignedArea(p)←
  for (int i = 0; i < p.size(); i++){</pre>
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[\leftarrow
        i].x*p[i].y);
  return c / scale;
```

### 7.2 3-D Geometry

```
#define LINE O
#define SEGMENT 1
#define RAY 2
struct point{
    double x, y, z;
    point(){};
    point(double _x, double _y, double _z){←
        x=_x; v=_v; z=_z; 
    point operator+ (point p) { return ←
       point(x+p.x, y+p.y, z+p.z); }
    point operator- (point p) { return \hookleftarrow
       point(x-p.x, y-p.y, z-p.z); }
    point operator* (double c) { return ←
       point(x*c, y*c, z*c); }
};
double dot(point a, point b){
    return a.x*b.x + a.y*b.y + a.z*b.z;
point cross(point a, point b) {
    return point(a.y*b.z-a.z*b.y,
                 a.z*b.x-a.x*b.z,
                 a.x*b.y-a.y*b.x);
double distSq(point a, point b){
    return dot(a-b, a-b);
```

```
// compute a, b, c, d such that all points \leftarrow
   lie on ax + bv + cz = d. TODO: test \leftarrow
    this
double planeFromPts(point p1, point p2, ←
    point p3, double& a, double& b, double&←
    c. double& d) {
    point normal = cross(p2-p1, p3-p1);
    a = normal.x: b = normal.v: c = normal.↔
    d = -a*p1.x-b*p1.y-c*p1.z;
// project point onto plane. TODO: test \leftarrow
point ptPlaneProj(point p, double a, double ←
    b, double c, double d) {
    double 1 = (a*p.x+b*p.v+c*p.z+d)/(a*a+b \leftarrow
        *b+c*c):
    return point(p.x-a*1, p.y-b*1, p.z-c*1) ←
// distance from point p to plane aX + bY +\leftarrow
    cZ + d = 0
double ptPlaneDist(point p, double a, \leftarrow
    double b. double c. double d){
    return fabs(a*p.x + b*p.y + c*p.z + d) ←
        / sqrt(a*a + b*b + c*c);
// distance between parallel planes aX + bY\leftarrow
    + cZ + d1 = 0 and
// aX + bY + cZ + d2 = 0
double planePlaneDist(double a, double b, \leftrightarrow
    double c, double d1, double d2){
    return fabs(d1 - d2) / sgrt(a*a + b*b +←
         c*c):
}
// square distance between point and line, \leftarrow
    ray or segment
double ptLineDistSq(point s1, point s2, ←
   point p, int type){
    double pd2 = distSq(s1, s2);
    point r;
```

```
if(pd2 == 0)
    r = s1:
    else{
    double u = dot(p-s1, s2-s1) / pd2;
    r = s1 + (s2 - s1)*u;
    if(type != LINE && u < 0.0)</pre>
        r = s1:
   if(type == SEGMENT && u > 1.0)
        r = s2:
   return distSq(r, p);
// Distance between lines ab and cd. TODO: \leftarrow
   Test this
double lineLineDistance(point a, point b, \leftarrow
   point c, point d) {
   point v1 = b-a;
   point v2 = d-c;
    point cr = cross(v1, v2);
    if (dot(cr, cr) < EPS) {</pre>
        point proj = v1*(dot(v1, c-a)/dot(←
            v1. v1)):
        return sqrt(dot(c-a-proj, c-a-proj)↔
   } else {
        point n = cr/sqrt(dot(cr, cr));
        point p = dot(n, c - a);
        return sqrt(dot(p, p));
   }
// Distance between line segments ab and cd \leftarrow
    (translated from Java)
double segmentSegmentDistance(point a, ←
   point b, point c, point d) {
   point u = b - a, v = d - c, w = a - c;
    double a = dot(u, u), b = dot(u, v), c \leftarrow
       = dot(v, v), d = dot(u, w), e = dot \leftarrow
       (v, w);
    double D = a*c-b*b;
    double sc, sN, sD = D;
    double tc, tN, tD = D;
    // compute the line parameters of the \leftarrow
       two closest points
```

```
if (D < EPS) { // the lines are almost \leftarrow
    parallel
    sN = 0.0;
                   // force using \hookleftarrow
        point PO on segment S1
                    // to prevent \hookleftarrow
    sD = 1.0:
        possible division by 0.0 later
    tN = e:
    tD = c:
} else {
                        // get the \leftrightarrow
    closest points on the infinite \leftarrow
   lines
    sN = (b*e - c*d):
    tN = (a*e - b*d);
    if (sN < 0.0) {
                           // sc < 0 => ←
        the s=0 edge is visible
        sN = 0.0:
        tN = e;
        tD = c:
    else if (sN > sD) { // sc > 1 => \leftarrow
        the s=1 edge is visible
        sN = sD:
        tN = e + b;
        tD = c;
    }
}
if (tN < 0.0) {
                           // tc < 0 => ←
    the t=0 edge is visible
    tN = 0.0;
    // recompute sc for this edge
    if (-d < 0.0)
        sN = 0.0;
    else if (-d > a)
        sN = sD:
    else {
        sN = -d:
        sD = a:
    }
else if (tN > tD) {
                         // tc > 1 => ←
   the t=1 edge is visible
    tN = tD:
   // recompute sc for this edge
    if ((-d + b) < 0.0)
        sN = 0:
```

```
else if ((-d + b) > a)
             sN = sD:
        else {
            sN = (-d + b);
            sD = a:
    // finally do the division to get sc \leftarrow
        and tc
    sc = (abs(sN) < EPS ? 0.0 : sN / sD);
    tc = (abs(tN) < EPS ? 0.0 : tN / tD);
    // get the difference of the two \leftarrow
        closest points
    point dP = w + (sc * u) - (tc * v); //\leftarrow
         = S1(sc) - S2(tc)
    return sqrt(dot(dP, dP)); // return ←
        the closest distance
double signedTetrahedronVol(point A, point \hookleftarrow
    B, point C, point D) {
    double A11 = A.x - B.x:
    double A12 = A.x - C.x;
    double A13 = A.x - D.x:
    double A21 = A.y - B.y;
    double A22 = A.y - C.y;
    double A23 = A.v - D.v;
    double A31 = A.z - B.z;
    double A32 = A.z - C.z;
    double A33 = A.z - D.z;
    double det =
        A11*A22*A33 + A12*A23*A31 +
        A13*A21*A32 - A11*A23*A32 -
        A12*A21*A33 - A13*A22*A31;
    return det / 6:
// Parameter is a vector of vectors of \leftarrow
   points - each interior vector
// represents the 3 points that make up 1 \leftrightarrow
   face, in any order.
// Note: The polyhedron must be convex, \leftarrow
   with all faces given as triangles.
double polyhedronVol(vector < vector < point > →
    poly) {
```

# 7.3 Closest Pair of Points

```
/*←
    Finding the closest pair of points. O(\leftarrow)
       NlogN), divide-and-conquer.
    Based on http://www.spoj.com/problems/←
       CI.OPPATR/
   */
#include <iostream>
#include <fstream>
#include <cmath>
#include <algorithm>
#include <vector>
#include <set>
#include <map>
#include <stack>
#include <queue>
#include <cstdlib>
#include <cstdio>
#include <string>
#include <cstring>
#include <cassert>
#include <utility>
#include <iomanip>
using namespace std;
#define sqr(x) ((x) * (x))
```

```
const double inf = 1e100;
const int MAXN = 105000;
struct point {
    double x, y;
    int ind:
};
bool cmp(point a, point b) {
    return (a.x < b.x || (a.x == b.x && a.y←
        < b.v));
}
double dist(point a, point b) {
    return sgrt(sgr(a.x - b.x) + sgr(a.v - \leftrightarrow
******************************
int n;
int a[MAXN];
point p[MAXN], tmp[MAXN];
double ans = inf;
int p1, p2;
  ·************************
void updateAnswer(point a, point b) {
    double d = dist(a, b);
    if (d < ans) {
        ans = d:
        p1 = a.ind; p2 = b.ind;
}
void closestPair(int 1. int r) {
    if (1 >= r)
        return:
    if (r - 1 == 1) {
        if (p[1].y > p[r].y)
            swap(p[1], p[r]);
        updateAnswer(p[1], p[r]);
        return:
    int m = (1 + r) / 2:
    double mx = p[m].x;
```

```
closestPair(1, m);
    closestPair(m + 1, r);
    int lp = 1, rp = m + 1, sz = 1;
    while (lp <= m || rp <= r) {</pre>
        if (lp > m || ((rp <= r && p[rp].y ←</pre>
            < p[lp].y))) {
            tmp[sz] = p[rp];
            rp++;
        }
        else {
            tmp[sz] = p[lp];
            lp++;
        sz++:
    for (int i = 1; i <= r; i++)</pre>
        p[i] = tmp[i - l + 1];
    sz = 0:
    for (int i = 1; i <= r; i++)</pre>
        if (abs(p[i].x - mx) < ans) {
            sz++:
            tmp[sz] = p[i];
    for (int i = 1; i <= sz; i++) {</pre>
        for (int j = i - 1; j >= 1; j--) {
            if (tmp[i].y - tmp[j].y >= ans)
                 break:
            updateAnswer(tmp[i], tmp[j]);
   }
int main() {
    //assert(freopen("input.txt", "r", stdin) ←
    //assert(freopen("output.txt","w", ←
        stdout));
    scanf("%d", &n);
    for (int i = 1; i <= n; i++) {</pre>
        scanf("%lf %lf", &p[i].x, &p[i].y);
```

```
p[i].ind = i - 1;
}

sort(p + 1, p + n + 1, cmp);

closestPair(1, n);

printf("%d %d %.6lf\n", min(p1, p2), \cdot max(p1, p2), ans);

return 0;
}
```

### 7.4 Convex Hull

```
// O(N log N) Monotone Chains algorithm for\hookleftarrow
     2d convex hull.
// Gives the hull in counterclockwise order←
     from the leftmost point, which is \leftarrow
   repeated at the end. Minimizes the \hookleftarrow
   number of points on the hull when \leftarrow
   collinear points exist.
long long cross(pair<int, int> A, pair<int, ←
    int> B, pair<int, int> C) {
    return (B.first - A.first)*(C.second - ←
        A.second)
         - (B.second - A.second)*(C.first -←
              A.first);
// The hull is returned in param "hull"
void convex_hull(vector<pair<int, int> > ←
   pts, vector<pair<int, int> >& hull) {
    hull.clear(); sort(pts.begin(), pts.end←
        ()):
    for (int i = 0; i < pts.size(); i++) {</pre>
        while (hull.size() >= 2 && cross(←
            hull[hull.size()-2], hull.back←
            (), pts[i]) <= 0) {
            hull.pop_back();
        }
        hull.push_back(pts[i]);
    int s = hull.size();
    for (int i = pts.size()-2; i \ge 0; i--) \leftarrow
        while (hull.size() >= s+1 && cross(\leftarrow
            hull[hull.size()-2], hull.back←
            (), pts[i]) <= 0) {
```

```
hull.pop_back();
}
hull.push_back(pts[i]);
}
```

### 8 Miscellaneous

### 8.1 Dates

```
// Routines for performing computations on \hookleftarrow
    dates. In these routines,
// months are expressed as integers from 1 \leftrightarrow
    to 12, days are expressed
// as integers from 1 to 31, and years are \leftarrow
    expressed as 4-digit
// integers.
#include <iostream>
#include <string>
using namespace std;
string dayOfWeek[] = {"Mon", "Tue", "Wed", ←
    "Thu", "Fri", "Sat", "Sun"};
// converts Gregorian date to integer (\hookleftarrow
    Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (v + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12↔
    3 * ((y + 4900 + (m - 14) / 12) / 100) \leftarrow
        / 4 +
    d - 32075;
}
// converts integer (Julian day number) to \hookleftarrow
    Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int←
     &y){
  int x, n, i, j;
  x = id + 68569;
  n = 4 * x / 146097;
  x = (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
```

```
x = 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
 x = j / 11;
 m = j + 2 - 12 * x;
 y = 100 * (n - 49) + i + x;
// converts integer (Julian day number) to \hookleftarrow
   day of week
string intToDay (int jd){
 return dayOfWeek[jd % 7];
int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);
  // expected output:
        2453089
        3/24/2004
        Wed
  cout << jd << endl
    << m << "/" << d << "/" << y << endl
    << day << endl;
```

#### 8.2 KMP

```
/*
Searches for the string w in the string s (
    of length k). Returns the
0-based index of the first match (k if no 
    match is found). Algorithm
runs in O(k) time.
*/
void buildTable(string& w, vll& pre)
{
  pre = vll(w.length());
  ll i = 2, j = 0;
  pre[0] = -1; pre[1] = 0;

  while(i < w.length())
  {
    if(w[i-1] == w[j]) {pre[i] = j+1; i++; \leftarrow
        j++; }</pre>
```

```
else if(j > 0) j =pre[j];
    else {pre[i] = 0; i++; }
}

ll KMP(string& s, string& w)
{
    ll m = 0, i = 0;
    vll pre;
    buildTable(w,pre);
    while(m+i < s.length())
    {
        if(w[i] == s[m+i])
        {
            i++;
            if(i == w.length()) return m;
        }
        else
        {
            m += i-pre[i];
            if(i > 0) i =pre[i];
        }
    }
    return s.length();
}
```

### 8.3 2-SAT

```
// 2-SAT solver based on Kosaraju's ←
    algorithm.
// Variables are 0-based. Positive \leftarrow
    variables are stored in vertices 2n. \leftarrow
    corresponding negative variables in 2n \leftarrow
   +1
// TODO: This is quite slow (3x-4x slower \hookleftarrow
    than Gabow's algorithm)
struct TwoSat {
    int n:
    vector < vector < int > > adj, radj, scc;
    vector<int> sid, vis, val;
    stack<int> stk:
    int scnt;
    // n: number of variables, including \leftarrow
        negations
    TwoSat(int n): n(n), adj(n), radj(n), \leftarrow
        sid(n), vis(n), val(n, -1) {}
    // adds an implication
```

```
void impl(int x, int y) { adj[x].
    push_back(y); radj[y].push_back(x);←
// adds a disjunction
void vee(int x, int y) { impl(x^1, y); \leftarrow
    impl(v^1, x); }
// forces variables to be equal
void eq(int x, int y) { impl(x, y); \leftarrow
    impl(y, x); impl(x^1, y^1); impl(y \leftarrow
    ^1, x^1); }
// forces variable to be true
void tru(int x) { impl(x^1, x); }
void dfs1(int x) {
    if (vis[x]++) return;
    for (int i = 0; i < adj[x].size(); \leftarrow
        i++) {
        dfs1(adj[x][i]);
    stk.push(x);
void dfs2(int x) {
    if (!vis[x]) return; vis[x] = 0;
    sid[x] = scnt; scc.back().push_back←
        (x):
    for (int i = 0; i < radj[x].size(); \leftarrow
         i++) {
        dfs2(radj[x][i]);
    }
// returns true if satisfiable, false ←
    otherwise
// on completion, val[x] is the \leftarrow
    assigned value of variable x
// note, val[x] = 0 implies val[x^1] = \leftarrow
bool two_sat() {
    scnt = 0:
    for (int i = 0; i < n; i++) {</pre>
         dfs1(i);
    while (!stk.empty()) {
        int v = stk.top(); stk.pop();
        if (vis[v]) {
```

```
scc.push_back(vector<int>()←
                     ):
                  dfs2(v):
                  scnt++;
        }
        for (int i = 0; i < n; i += 2) {</pre>
             if (sid[i] == sid[i+1]) return ←
                 false:
        vector < int > must(scnt);
        for (int i = 0; i < scnt; i++) {</pre>
             for (int j = 0; j < scc[i].size \leftarrow
                 (); j++) {
                  val[scc[i][j]] = must[i];
                  must[sid[scc[i][j]^1]] = ! \leftarrow
                     must[i];
             }
        }
        return true;
};
```

# 8.4 Binary Search

```
// Binary search. This is included because \hookleftarrow
   binary search can be tricky.
// n is size of array A, c is value we're \leftarrow
    searching for. Semantics follow those \hookleftarrow
    of std::lower_bound and std::\leftarrow
    upper bound
int lower_bound(int A[], int n, int c) {
    int 1 = 0;
    int r = n;
    while (1 < r) {</pre>
        int m = (r-1)/2+1; //prevents \leftarrow
            integer overflow
        if (A[m] < c) 1 = m+1; else r = m;
    }
    return 1:
int upper_bound(int A[], int n, int c) {
    int 1 = 0;
    int r = n;
    while (1 < r) {
        int m = (r-1)/2+1;
        if (A[m] \le c) l = m+1; else r = m;
```

```
}
return 1;
}
```

#### 8.5 FFT

```
struct cpx
 () (Xq)
 cpx(double aa):a(aa){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b:
  double modsq(void) const
   return a * a + b * b;
 cpx bar(void) const
   return cpx(a, -b);
cpx operator +(cpx a, cpx b)
return cpx(a.a + b.a, a.b + b.b);
cpx operator *(cpx a, cpx b)
return cpx(a.a * b.a - a.b * b.b, a.a * b\leftarrow
     .b + a.b * b.a):
cpx operator /(cpx a, cpx b)
 cpx r = a * b.bar();
 return cpx(r.a / b.modsq(), r.b / b.modsq←
cpx EXP(double theta)
return cpx(cos(theta),sin(theta));
const double two_pi = 4 * acos(0);
```

```
// in: input array
// out: output array
// step: {SET TO 1} (used internally)
// size: length of the input/output {MUST\leftarrow
    BE A POWER OF 2}
// dir:
           either plus or minus one (\leftarrow
    direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size} - 1} \leftrightarrow
   in[j] * exp(dir * 2pi * i * j * k / ←
    size)
void FFT(cpx *in, cpx *out, int step, int ←
    size. int dir)
  if(size < 1) return;</pre>
  if(size == 1)
 ł
    out[0] = in[0];
    return:
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, \hookleftarrow
      size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)</pre>
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two_pi * i / ←
        size) * odd:
    out[i + size / 2] = even + EXP(dir * ←
        two_pi * (i + size / 2) / size) * \leftarrow
        odd:
  }
}
// Usage:
// f[0...N-1] and g[0...N-1] are numbers
// Want to compute the convolution h, \hookleftarrow
    defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N \leftarrow
// Here, the index is cyclic; f[-1] = f[N \leftarrow
   -1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, \leftarrow
    define G and H.
// The convolution theorem says H[n] = F[n] \leftarrow
   G[n] (element-wise product).
```

```
// To compute h[] in O(N log N) time, do 
    the following:
// 1. Compute F and G (pass dir = 1 as 
    the argument).
// 2. Get H by element-wise multiplying F
    and G.
// 3. Get h by taking the inverse FFT (
    use dir = -1 as the argument)
// and *dividing by N*. DO NOT FORGET 
    THIS SCALING FACTOR.
// To compute an *acyclic* convolution, pad
    f and g to the right with zeroes.
```

### 8.6 All nearest smaller values

```
// Linear time all nearest smaller values, \leftarrow
   standard stack-based algorithm.
// ansv_left stores indices of nearest ←
   smaller values to the left in res. -1 \leftrightarrow
   means no smaller value was found.
// ansv_right likewise looks to the right. \hookleftarrow
   v.size() means no smaller value was ←
   found.
void ansv_left(vector<int>& v, vector<int>& ↔
    stack<pair<int, int> > stk; stk.push(←
        make_pair(INT_MIN, v.size()));
    for (int i = v.size()-1; i >= 0; i--) {
        while (stk.top().first > v[i]) {
            res[stk.top().second] = i; stk.←
                pop();
        stk.push(make_pair(v[i], i));
    while (stk.top().second < v.size()) {</pre>
        res[stk.top().second] = -1; stk.pop\leftrightarrow
            ();
    }
void ansv_right(vector<int>& v, vector<int←</pre>
   >& res) {
    stack <pair <int, int> > stk; stk.push (←
        make_pair(INT_MIN, -1));
    for (int i = 0; i < v.size(); i++) {</pre>
        while (stk.top().first > v[i]) {
            res[stk.top().second] = i; stk.←
                pop();
```

```
}
    stk.push(make_pair(v[i], i));
}
while (stk.top().second > -1) {
    res[stk.top().second] = v.size(); 
        stk.pop();
}
```

# 8.7 Manacher's Algorithm

```
// Manacher's algorithm: finds maximal ←
    palindrome lengths centered around each
// position in a string (including \hookleftarrow
    positions between characters) and \hookleftarrow
// them in left-to-right order of centres. \hookleftarrow
    Linear time.
// Ex: "opposes" -> [0, 1, 0, 1, 4, 1, 0, \leftarrow
   1, 0, 1, 0, 3, 0, 1, 0]
vector<ll> fastLongestPalindromes(string \leftarrow
    str) {
    11 i=0,j,d,s,e,lLen,palLen=0;
    vector<ll> res;
    while (i < str.length()) {</pre>
         if (i > palLen && str[i-palLen-1] ←
            == str[i]) {
             palLen += 2; i++; continue;
        res.push_back(palLen);
        s = res.size()-2;
         e = s-palLen;
         bool b = true;
         for (j=s; j>e; j--) {
             d = j-e-1;
             if (res[j] == d) { palLen = d; \leftarrow
                b = false; break; }
             res.push_back(min(d, res[j]));
        }
         if (b) { palLen = 1; i++; }
    res.push_back(palLen);
    lLen = res.size();
    s = 1Len-2;
    e = s-(2*str.length()+1-lLen);
    for (i=s; i>e; i--) { d = i-e-1; res.
        push_back(min(d, res[i])); }
    return res;
```

```
}
```

# 8.8 Matrix Library

```
struct Matrix{
    ll r,c;
    vector < vector < ll> > matrix;
    Matrix(ll r, ll c, ll def=0):r(r),c(c), \leftarrow
        matrix(r,vll(c,def)){}
    void unit(){for(ll i=0;i<r;i++)matrix[i←</pre>
        ][i]=1;}
    void add(Matrix &b) //a=a+b
         for(ll i=0;i<r;i++)</pre>
             for(11 j=0;j<c;j++)</pre>
                  matrix[i][j]+=b.matrix[i][j \leftarrow
                     ];
    }
    Matrix mult(Matrix &b) //return this*b
         Matrix a(r,b.c,0);
         for(11 i=0;i<r;i++)</pre>
             for(ll j=0;j<b.c;j++)</pre>
                 for(11 k=0; k<c; k++)
                      a.matrix[i][j]+=matrix[←
                          i][k]*b.matrix[k][j↔
                          ];
         return a:
    void pow(ll b)
                           //a=a^b
         Matrix a(r,c,0);
         a.unit();
         while(b)
             if (b&1) a=a.mult(*this);
             *this=this->mult(*this);
             b>>=1;
         *this=a;
    void display()
    {
         for(ll i=0;i<r;i++)</pre>
```