ASSIGNMENT 4 NAME: SHUBHAM SHARMA EAS 595

PROFESSOR: DR. ESFAHANI

PERSON #: 50290293

Q.1 Given: Maximum limit of the van = 3900 lbs

No. of boxes that the customer asks to transport = 40

Distribution for the weight of the boxes is given by :

$\mu = 97$ lbs , $\sigma = 5$ lbs

We can estimate the μ and σ for the entire order using the mean and standard deviation of individual distributions.

$$\mu_F = n\mu = 40 \times 97 = 3880$$
$$\sigma_F = \sqrt{n}\,\sigma_{\text{box}} = \sqrt{40} \times 5 = 31.6$$

If the cargo van has to transport all the boxes, the total weight (x) should be < 3900 lbs

∴ Calculating the z score :

$$z = \frac{x - \mu}{6} = \frac{3900 - 3880}{31.6}$$

$$\Rightarrow z = 0.632$$

$$\therefore P(X < 3900) \simeq 0.735 \quad [\phi(0.632)]$$

Ans. 0.735

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2. $\mu = 2.4$, $\sigma = 20$ days, $n = 100$ items

If we consider X to be the joint distribution of all the 100 products → (time taken for 100 As)

$\therefore \mu_x = 100 \times 2.4 = 240$

$\sigma_x = \sqrt{100} \times 2 = 20$

Case 1: $P(X < 200)$ → Profit of \$ 10,000

z score $= \frac{x - \mu}{6} = \frac{200 - 240}{20} = -2$

$\therefore P = 0.022$

Case 2: $P(250 > X > 200)$ which means profit = \$6,000

$z(X = 250) = \frac{250 - 240}{20} = 0.5$

$\therefore P = 0.7$

$\therefore P(250 > X > 200) = 0.7 - 0.0228$
$= 0.67$

Case 3: $P(X > 400)$ which means loss \$400

$z = \frac{400 - 240}{20} = 8$

Probability for $z = 8 \cong 0$

Expected value of profit | loss:

$$\frac{0.0228 \times 10{,}000 + 0.67 \times 6{,}000 + 0}{1}$$

$= \$ 4248$

The expected value of profit is \$ 4248

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3(a) Average accuracy = 80% $\Rightarrow \mu = 0.8$

Standard Deviation $(\sigma) = 16\% = 0.16$

$N = 100$ samples

P (that classifiers' accuracy is between 79 and 81)

$\therefore P_{range} = P(S_n < 81) - P(S_n < 79)$

$P(S_n < 81) \Rightarrow \quad x_n = 81$

$$z = \frac{81 - 0.8 \times 100}{0.16 \times \sqrt{100}} = \frac{1}{1.6} = 0.625$$

$z = 0.625$

$\therefore P(S_n < 81) \cong \phi(0.625) = 0.7324$

$P(S_n < 79)$: $\quad x_n = 79$

$$z = \frac{79 - 0.8 \times 100}{0.16 \times \sqrt{100}} = \frac{-1}{1.6} = -0.625$$

$z = -0.625$

$P(S_n < 79) \cong \phi(-0.625) = 1 - \phi(0.625)$

$= 1 - 0.7324 = 0.2676$

$\therefore P_{range} = P(S_n < 81) - P(S_n < 79) = 0.7324 - 0.2676$

$= 0.4648$ Ans.

(b) $\therefore$ P(of the interval (95%)) $= 0.95$

$\therefore P(|x - \mu| \leq d) = 0.95$

Dividing LHS by $\frac{\sigma}{\sqrt{\mu}}$

$$P\left(\frac{|x - \mu|}{\sigma/\sqrt{\mu}} \leq \frac{d}{\sigma}\sqrt{n}\right) = 0.95$$

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$$= P\left(|Z| \le \frac{10d}{16}\right) = 0.95$$

Thus, we can write,

$$1 - 2P\left(Z \le -\frac{10}{16}d\right) = 0.95$$

or $P\left(Z \le -\frac{10}{16}d\right) = 0.025$

$$\Rightarrow -\frac{10}{16}d = -1.96$$

$$\Rightarrow d = \frac{1.96 \times 16}{10} = 3.13$$

Thus, 95% confidence interval is

$$[80 - 3.13,\ 80 + 3.13]$$

$$= [76.87,\ 83.13]$$

4. (a) $N = 50,000$

$\sigma = 10,000$

Weekly supply = 47000 gallon

Supply for the time till 11th week

$= 74000 + 11 \times 47000 = 591000$ gallons

For supply to be below 20,000 g total gasoline purchased in these 11 weeks should be more than $591000 - 20000 = 571000$ gallon

Now $P(X > 571000)$

Finding z : $\frac{571000 - 11(50000)}{\sqrt{11} \times 10,000} = 0.6332$

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$\therefore P(X > 571{,}000) = 1 - 0.7357 = 0.2643 \quad [\because \Phi(0.6332) = 0.7357]$

(b) Let the required weekly delivery be A.

The probability of total purchase to be more than $74000 + 11 \times A - 20000$, should be less than 0.005

$$P(X > 74000 + 11A - 20000) = 0.005$$

z-Score for 0.005 is +2.575

Thus,

$$+2.575 = \frac{74000 + 11A - 20000 - 11(50000)}{\sqrt{11} \times 10{,}000}$$

$$\Rightarrow 2.575 = \frac{-496000 + 11A}{3.3166 \times 10000}$$

$\Rightarrow A = 52854.8$ gallons Ans.

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5. Given: The hedge fund manager invests half of her current fortune into the stock each day.

After n days her fortune is Y_n

Initial fortune $Y_0 = 100$

We can see expected fortune as Y_n or 50% drop in price or 70% hike in price

$$= \frac{Y_{n-1}}{2} + Y_{n-1} \times 1.7x \times 0.5_{n-1-x}$$

$$\therefore Y_n = 50 + 50 \times (1.7 - 0.5)$$

$$= \$110$$

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