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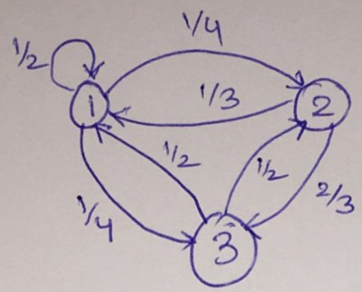
1.(a) STATE TRANSITION DIAGRAM:



(ii) $P(X1=3, X2=2, X3=1)$

Considering initial state as 0, we have the initial probability vector $\pi^{(0)} = [0.25 \quad 0.25 \quad 0.5]$

$$\pi^{(1)} = \cancel{P}\,\pi^{(0)} P$$

$$[0.25 \quad 0.25 \quad 0.5] \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} \cancel{0.25} \\ \cancel{0.25} \\ \cancel{0.5} \end{bmatrix} = [0.458 \quad 0.3125 \quad 0.229]$$

$$= \begin{bmatrix} \cancel{0.3125} \\ \cancel{0.4125} \\ \cancel{0.25} \end{bmatrix}$$

$\therefore P(X1=3) = 0.229$

$\therefore P(X1=3, X2=2, X3=1) = 0.229 \times P_{32} \times P_{21}$

$$= 0.229 \times 0.5 \times \frac{1}{3} = 0.0381$$

(iii) $P(X3=2)$

$$\pi^{(3)} = \pi^{(0)} P^3$$

$$= [0.25 \quad 0.25 \quad 0.5] \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}^3$$

$$= \begin{bmatrix} 0.25 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0.458 & 0.26 & 0.28 \\ 0.43 & 0.167 & 0.40 \\ 0.48 & 0.33 & 0.187 \end{bmatrix}$$

$$= \begin{bmatrix} 0.462 & 0.272 & 0.2635 \end{bmatrix}$$

$P(X3=2) =$ second element of $\pi^{(3)}$

$= 0.272$ Ans.

(iv) Steady State

At Steady State, $\pi_j = \sum_K \pi_k P_{kj}$

$$\frac{\pi_1}{2} + \frac{\pi_2}{3} + \frac{\pi_3}{2} = \pi_1 \Rightarrow 3\pi_1 + 2\pi_2 + 3\pi_3 = 6\pi_1$$

$$\Rightarrow 3\pi_1 = 2\pi_2 + 3\pi_3 \quad \text{.. eq i)}$$

$$\frac{\pi_1}{4} + \frac{\pi_3}{2} = \pi_2 \Rightarrow \pi_1 + 2\pi_3 = 4\pi_2 \quad \text{.. eq ii)}$$

$$\frac{\pi_1}{4} + \frac{2\pi_2}{3} = \pi_3 \Rightarrow 3\pi_1 + 8\pi_2 = 12\pi_3 \quad \text{.. eq iii)}$$

from eq i) & eq iii)

$$(2\pi_2 + 3\pi_3) + 8\pi_2 = 12\pi_3$$

$$\Rightarrow 10\pi_2 = 9\pi_3 \Rightarrow \pi_2 = \frac{9}{10}\pi_3$$

$$\pi_1 + 2\pi_3 = \frac{4\times9}{10}\pi_3 \Rightarrow \pi_1 + 2\pi_3 = 3.6\pi_3$$

$$\Rightarrow \pi_1 = 1.6\pi_3$$

also,

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$\Rightarrow 1.6\pi_3 + .9\pi_3 + \pi_3 = 1$$

$\Rightarrow 3.5\pi_3 = 1 \Rightarrow \pi_3 = \frac{1}{3.5}$, $\pi_1 = \frac{8}{5} \times \frac{2}{7} = \frac{16}{35}$

$\pi_2 = \frac{\frac{16}{35} + \frac{4}{7}}{4} = \frac{36}{140}$

$\therefore$ Steady state probabilities,

$\pi = [0.457 \quad 0.257 \quad 0.286]$

Q.2

$t_1^* = 1 + P_{11} t_1 + P_{12} t_2 + P_{13} t_3$

$t_1 = 1 + P_{11} t_1 + P_{12} t_2 + P_{13} t_3$

$= 1 + \frac{t_1}{4} + \frac{t_2}{2} + \frac{t_3}{4}$

$\Rightarrow \frac{3t_1}{4} = 1 + \frac{t_2}{2} + \frac{t_3}{4}$ ... eq i)

Similarly,

$t_2 = 1 + \frac{t_1}{3} + \frac{2}{3} t_3$ ... eq ii) $\because P_{21} = \frac{1}{3}$

$P_{23} = \frac{2}{3}$

$t_3 = 1 + \frac{t_1}{2} + \frac{t_3}{2}$ as $P_{31} = \frac{1}{2}$, $P_{33} = \frac{1}{2}$)

.. eq iii)

Using eq iii) $t_3 = 2 + t_1$

Putting in eq ii)

$t_2 = 1 + \frac{t_1}{3} + \frac{4 + 2t_1}{3}$

$\therefore t_2 = \frac{7 + 3t_1}{3}$ .. eq iv)

Putting t_3 & t_2 in eq i)

$\frac{3}{4} t_1 = 1 + \frac{7 + 3t_1}{6} + \frac{2 + t_1}{4} \Rightarrow t_1 = 0$

Putting in eqⁿ(i)

$$t_3 = 2+0 = 2$$

Putting in equⁿ(i)

$$t_2 = \frac{7}{3}$$

Now,

$$t_1^* = 1+0+P_{12}t_2 + P_{13}t_3$$

$$= 1+\frac{1}{2}\times\frac{7}{3}+\frac{1}{4}\times 2$$

$$= 1+\frac{7}{6}+\frac{1}{2} = \frac{8}{3} \quad \text{Ans.}$$