HW 5 Solution

Problem 1.

a) If Y is the number of arrival in (2 4], then Y is a Poisson($\lambda = 0.4 \times 2) = e^{-0.8} = 0.449$
b) The two time intervals have no overlaps. So, it is equivalent to have one arrival in the union of the two which is a 3 sec time interval. Thus, Poisson($\lambda = 0.4 \times 3) = e^{-1.2} = 0.301$
c) Note that the two intervals (0,1] and (0,5] are not disjoint. Thus, we cannot multiply the probabilities for each interval to obtain the desired probability.
   Let X and Y be the number of arrivals in (0 1] and (1 5]. Then X and Y are independent and,
   X ~ Poisson($\lambda = 0.4 \times 1$) and Y ~ Poisson($\lambda = 0.4 \times 4$)
   Let A be then event that there is one arrival in (0 1] and two arrivals in (1 5] then
   P(A)=P(X=1,Y=2)=P(X=1)P(Y=2)= $(0.4\, e^{-0.4})(0.8\, e^{-1.6}) = 0.32e^{-2} = 0.043$

Problem 2.

The rate of N(t) is equal to $\lambda = \lambda_1 + \lambda_2 = 3$.

a) N(1)=2 is the Probability of two arrivals in (0 1] and N(2)=5 is the probability of 5 arrivals in (0 2].
   Let X and Y be the number of arrivals in (0 1] and (1 2], respectively and
   X ~ Poisson($\lambda = 3 \times 1$) and Y ~ Poisson($\lambda = 3 \times 1$)

   P(N(1)=2, N(2)=5)=P(X=2,Y=3)=P(X=2)P(Y=3)= $\left(\frac{3^2}{2!}\, e^{-3}\right)\left(\frac{3^3}{3!} e^{-3}\right) = \frac{81}{4} e^{-6} = 0.050$

b) P$P(N_1(1) = 1 | N(1) = 2) = \frac{P(N_1(1)=1\,,N(1)=2)}{P(N(1)=2)} = \frac{P(N_1(1)=1\,,N_2(1)=1)}{P(N(1)=2)} = \frac{P(N_1(1)=1)\, P(N_2(1)=1)}{P(N(1)=2)} =$
   $\frac{e^{-1}.2e^{-2}}{\frac{3^2}{2!}e^{-3}} = \frac{4}{9}$