Starling Murmurations

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1 Problem Statement

Our objective is to design a simulation model for the starling murmurations and compute the average energy spend by each bird, the angular momentum and the force that each bird has to withstand in the phenomenon. We need to computationally simulate the phenomenon by modeling each bird as an independent agent communicating and cooperating with other neighboring agents.

2 Model

We have modeled each bird as separate identity which follows some basic rules that results in this amazing phenomenon. Apart from this, the group as a whole needs to make sure that its energy doesn't become infinite and its height is also limited by the ability of the birds to fly as high as possible.

2.1 Three basic rules followed by each bird separately

1. Cohesion - Each bird steer towards average position of neighbours (long range attraction). This means that boids try to fly towards the center of mass of the neighboring boids.

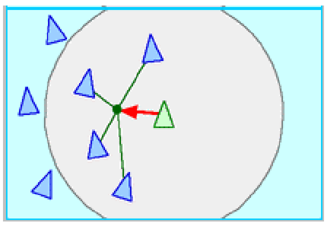

Figure 1: Boid moving towards center of mass

2. Alignment - Boids try to match velocity of neighboring boids. A boid steers itself towards the average heading of neighbors.

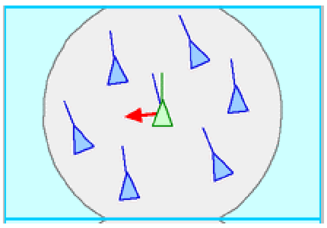

Figure 2: Boid moving towards average heading of flockmates

3. Separation - Boids try to keep a small distance away from other objects which includes other boids also. This is done in order to avoid crowding near the neighbors (short range repulsion).

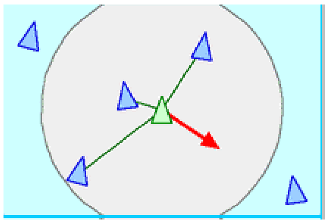

Figure 3: Boid steering away to avoid local crowding

2.2 Mathematical Analysis

Consider a bird represented by a point P_0 in $3D$ space whose position coordinates are given by (x, y, z), velocity at any time t is given by (v_x, v_y, v_z) and acceleration is (a_x, a_y, a_z). Each bird has some initial energy E_o and mass m_o. We call this entity representing a bird - "boid". When such a boid is flying in the air, it should satisfy the above three rules independently of others. Apart from this, the complete starling needs to follow the basic rules of physics, that is, the energy of the group as a whole can't be more than specified limit which is equal to the sum of the initial energies of each bird in the group.

Mathematical form of the above three rules is stated below -

1. Cohesion - Let their be a boid - P_0. It has N neighbours around it. The center of mass of the neighbors is denoted by CM with coordinates are (x_{cm}, y_{cm}, z_{cm}). According to the cohesion rule, it tries to move towards CM.Let the neighbours be represented by $P_1, P_2, \ldots, P_n$, CM is given by:

$$CM = \frac{P_1 + P_2 + \cdots + P_n}{n} = \frac{1}{n}\sum_i^n P_i$$

Direction of movement of the bird is given by:

$$\hat{\boldsymbol{v}} = \frac{CM - P_0}{\|CM - P_0\|}$$

Let the change in velocity due to cohesion be Δv_c give by:

$$\Delta v_c = \epsilon \, \|CM - P_0\| \, \hat{\boldsymbol{v}}$$

where, ϵ is some constant.

2. Separation - Consider a boid represented by point P_0. Let its position be $\vec{p_o}$ and its neighbours have position $\vec{p_i}$. Let $\vec{v}$ be a vector given by:

$$\vec{v} = \sum_i \vec{p_i} - \vec{p_o} \qquad \forall\, i,\ \|\vec{p_i} - \vec{p_o}\| < \epsilon'$$

$$\hat{\boldsymbol{v}} = \frac{\vec{v}}{\|\vec{v}\|}$$

where, ϵ' is some constant. Let the change in velocity due to separation be Δv_s given by:

$$\Delta v_s = -\epsilon \hat{\boldsymbol{v}}$$

3. Alignment - The boid steers itself towards the motion of its neighbors. Let the velocity of the neighbors of a boid having velocity v_0 be defined by $v_1, v_2, \ldots, v_n$. Let the change in velocity due to alignment be Δv_a given by:

$$\Delta v_a = \epsilon \frac{\sum_{i=1}^{n} (v_i \vec{-} \vec{v_0})}{\left\| \sum_{i=1}^{n} (v_i \vec{-} \vec{v_0}) \right\|}$$

where, ϵ is some constant.

The new velocity of each boid is given by:

$$\vec{v_0} = \vec{v_0} + \Delta \vec{v_c} + \Delta \vec{v_s} + \Delta \vec{v_a}$$

The new position of each boid is given by:

$$\vec{x_0} = \vec{x_0} + \vec{v_0} \Delta t$$

2.3 Energy Calculations

Let each bird have an initial energy E_0 and mass m_0. The simulation starts with n birds each having initial position coordinates given by :

$$\boldsymbol{X_i} = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T$$

Let μ and Σ be given by:

$$\boldsymbol{\mu} = \begin{bmatrix} x_0 & y_0 & h_0 \end{bmatrix}^T$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_h^2 \end{bmatrix}$$

$\boldsymbol{X_i}$ comes from a normal distribution:

$$\boldsymbol{X_i} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

Now we have:

$$CM_{system_z} = \frac{\sum h_i}{n} = h_0$$

Similarily, Let the velocity be initialized randomly such that:

$$v_{system} = \frac{\|\sum v_i\|}{n} = v_0$$

At any time t, the mechanical energy of boid i is given by:

$$E_{mech,i,t} = \frac{1}{2} m \|v_{i,t}\|^2 + mgh_{i,t}$$

At time $t + 1$:

$$E_{mech,i,t+1} = \frac{1}{2} m \|v_{i,t+1}\|^2 + mgh_{i,t+1}$$

Let the energy stored with a boid i at time t be $E_{str,i,t}$, then $E_{str,i,t+1}$ is given by:

$$E_{str,i,t+1} = E_{str,i,t} - (E_{mech,i,t+1} - E_{mech,i,t}) \qquad (Using\ Energy\ Conservation)$$

If at any time t, $E_{str,i,t} < \epsilon$ for some constant ϵ, then $v_{i,t+1}$ is given by:

$$v_{i,t+1} = \epsilon' \|v_{i,t}\| \, [cosa\boldsymbol{\hat{i}} + cosb\boldsymbol{\hat{j}} + cosa\boldsymbol{\hat{k}}] - v_z\boldsymbol{\hat{k}},$$

such that,

$$\|v_{i,t+1}\| < \|v_{i,t}\|$$

where cosa, cosb, cosc are direction cosines