Project 1: Problem Statement Analysis

Nimish Goel (2015EE10463) and Shubham Singla (2015ME10383)

March 11, 2018

1 Problem Statement

To design and implement a software package for *Engineering Drawing*. The package should have the following functionalities:

1. To be able to interactively input or read from a file either i) an isometric drawing and a 3D object model or ii) projections on to any cross section.
2. To be able to generate projections on to any cross section or cutting plane given the 3D model description.
3. To be able to interactively recover the $3D$ description and produce an isometric drawing from any view direction given two or more projections.

2 Model

We model the problem as a graph where the nodes represent the vertices and the edges represent the actual edges in the projection or the $3D$ object, hidden edges are ignored and are drawn as solid lines.

$2D$ orthographic views: 4 files of nodes and edges corresponding to 2 views

$3D$ model: 2 files of nodes and edges.

Input file: Set of nodes and edges.

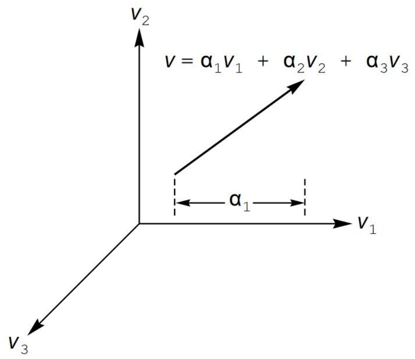
Output file: Set of nodes and edges.

- How many views are necessary? - 2 views (any two of top view, front view and right hand side view)
- How many are sufficient? - 2 views are sufficient but this may not necessarily yield a unique $3D$ model (explanations are provided in the analysis further).

3 Mathematical Analysis

3.1 3D to 2D model

Every node or vertex is represented by a coordinate system (x, y, z). We need to project each of these points to a 2 dimensional surface. So these triplets are mapped to a pair.



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{to} \begin{bmatrix} x \\ y \end{bmatrix}$$

To obtain the orthographic projections of each vertex, co-ordinate triplet is multiplied with the transformation matrix corresponding to that view.

$\underline{TopView}$: The axes assumed are x and y.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\underline{LeftHandSideView}$: The axes assumed are y and z.

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\underline{FrontView}$: The axes assumed are x and z.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Projected edges can be generated by joining the projected points. If an edge exists between two points in the $3D$ model then the line joining the corresponding projected points is an edge in the orthographic projected view.

$\underline{Isometric}$: Assuming that isometric view is given after rotating at some angle θ_1 from x - axis, θ_2 from y - axis and θ_3 from z - axis. To obtain the orthographic projections, we need to transform the co-ordinate triplets into normal view direction.

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Since working with angles can lead to more approximation errors, we assume the input in the following way -

Input: We are given a normalized vector

$$v = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix}$$

Therefore, $v = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3$ which represents the direction normal to the cutting plane We will rotate the co-ordinate axes in such a way that this vector now corresponds the z axis in the transformed system, this enables us to get the required plane parallel to the transformed x-y plane.
Let M correspond to the transformation parameters to the new system.

$$M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{32} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Therefore

$u_1 = \alpha_{11} v_1 + \alpha_{12} v_2 + \alpha_{13} v_3$
$u_2 = \alpha_{21} v_1 + \alpha_{22} v_2 + \alpha_{23} v_3$
$u_3 = \alpha_{31} v_1 + \alpha_{32} v_2 + \alpha_{33} v_3$

Here we have 9 unknowns The equations to find these are as follows

1. $a_{31} = \alpha_1$ (By assigning normal to the plane as new z axis)
2. $a_{32} = \alpha_2$ (By assigning normal to the plane as new z axis)
3. $a_{33} = \alpha_3$ (By assigning normal to the plane as new z axis)
4. $a_{11}a_{31} + a_{12}a_{32} + a_{13}a_{33} = 0$ (The new x and z axes are perpendicular)
5. $a_{21}a_{31} + a_{22}a_{32} + a_{23}a_{33} = 0$ (The new y and z axes are perpendicular)
6. $a_{11}a_{21} + a_{12}a_{22} + a_{13}a_{23} = 0$ (The new x and y axes are perpendicular)
7. $a_{11}^2 + a_{12}^2 + a_{13}^2 = 1$ (Normalization x axis)
8. $a_{21}^2 + a_{22}^2 + a_{23}^2 = 1$ (Normalization y axis)

So we have 8 independent equations for 9 unknowns. Therefore we cannot uniquely determine the x and y axes from this information. This is intuitively explainable. Since we have only fixed the cutting plane and the normal the x and y axes can be placed anywhere on the plane.

Therefore once either of x or y is fixed then the other can be determined. Now all the points can be mapped to this transformed co-ordinate system. Projection on the cutting plane can now be obtained.

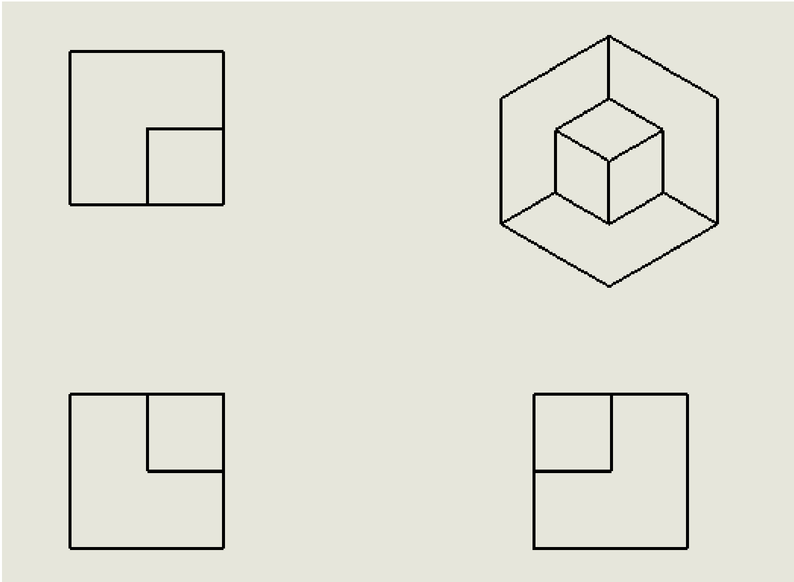
3.2 2D to 3D Model

Assuming that we are provided with any 2 orthographic views necessary for a unique $3D$ model where every node or vertex is represented by a coordinate system (x,y,z). We need to project each of these points to a 3 dimensional surface. So these pairs are mapped to a triplet. Apart from the two views, it is assumed that the correspondence of each point is given in all of the views.

To map any node or vertex to $3D$ surface, consider the following views of a point A(x, y, z):

Top View - (x, y)
Front View - (x, z)
Left Hand Side View - (y, z)
The point in $3D$ is given by (x, y, z).

2 views are sufficient since we have assumed that all the vertices and edges are shown in the orthographic views even if they overlap with some other vertex or edge. Since, the correspondences are assumed, 2 views are sufficient to obtain $3D$ representation of a vertex.



This figure shows a sample $3D$ object with its front, top and side views shown. Each point will have a correspondence in another view which would enable us to determine all the 3 coordinates of that point hence its location in the $3D$ space.

To map any edge to $3D$ surface - If there exists an edge between two points in any of the views, then corresponding edges are made in $3D$ model as well.

(Not included as of now in the design specification but a rough estimate for hidden edges) To identify the hidden edges, vertices are projected in descending order of their x co-ordinates in case of front view and z co-ordinates in top view and then, proceeded further.

4 References

Lecture 3: Co-ordinate Systems and Transformations

http://www.uio.no/studier/emner/matnat/ifi/INF3320/h03/undervisningsmateriale/lecture3.pdf