2b) Show that $\phi_1 = \delta_1 + \epsilon_1 - \alpha_{11}$

Multiplying $A = LDL^T$, $A = UEU^T$,

where

$$LDL^T = \begin{bmatrix} L_{00} & 0 \\ \lambda_{10} & E_{22} \end{bmatrix} \begin{bmatrix} D_{00} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} L_{00} \\ \lambda_{10} & E_{22} \end{bmatrix}^T$$

$$UDU^T = \begin{bmatrix} L_{00} & 0 \\ \lambda_{10} & E_{22} \end{bmatrix} \begin{bmatrix} U_{00} & a_{10} & a_{21} \\ 0 & a_{11} & a_{21} \end{bmatrix} \begin{bmatrix} L_{00} \\ \lambda_{10} & E_{22} \end{bmatrix}^T$$

Multiplying both,

$$A = LDL^T * UDU^T$$

$$= \begin{bmatrix} L_{00} D_{00} L_{00}^T + 0 + 0 & L_{00} U_{00} L_{00}^T & L_{00} a_{10} L_{00}^T \\ \lambda_{10} D_{00} L_{00}^T + E_{22} \lambda_{10} E_{22}^T & 0 & \lambda_{10} U_{00} L_{00}^T + E_{22} a_{11} E_{22}^T \end{bmatrix}$$

$$= \begin{bmatrix} \delta_1 + 0 + 0\epsilon_1 + a_{11} \\ a_{10}\delta_2 + \theta_{12}\theta_{12}^T \epsilon_2 + \alpha_{21}\alpha_{21}^T \end{bmatrix}$$

- Cost ϕ_1: Since the twisted factorization contains $\theta_{12}\theta_{12}^T \epsilon_2$, the cost is $O(n^2)$

- Cost of all: For computing cost of all, To compute the remaining unknowns, it would take another cost $O(n^3)$. Thus the total cost would be $O(n^3 + n^2)$