

2a) A in terms of bidiagonal form can be written as

$$A = QBQ^T$$

$B \rightarrow$ bidiagonal matrix

$Q \rightarrow$ orthogonal matrix

$$A = Q(D-U)(D+U)^T Q^T \quad \text{--- (1)}$$

Here, $D \rightarrow$ diagonal matrix with entries of B

$U \rightarrow$ diagonal matrix with off diagonal of B

$$\begin{aligned} \text{Here, } (D+U)^T &= D^T + U^T \\ &= D + U^T \end{aligned}$$

Thus (1) becomes

$$A = Q(D+U^T)Q^T$$

Considering a new matrix U ,

$$U = Q(D+U^T)Q^T$$

$$A = U(D+U^T)U^T$$

Algorithm

Set $U = T$, traverse through the matrix,
 \rightarrow compute d_k of D ,

$$d_k = \frac{a_{kk} - u_{(k-1),k}^2}{d_{k-1}}$$

$$\rightarrow U_{kk} = \sqrt{d_k}$$

$$\rightarrow U_{k,k+1} = \frac{u_{k+1,k}}{U_{kk}}$$

Iterate through matrix to get

$$U_{kk} = U_{kk}$$

$$\begin{aligned} A &= UDU^T U^T \\ &= UU^T \end{aligned}$$