2c) Ex 7.1

$$\left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10}e_i^T & 1 & v_{12}e_F^T \\ \hline 0 & 0 & v_{22} \end{array}\right)\left(\begin{array}{c|c|c} D_{00} & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & F_{22} \end{array}\right) \quad \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10}e_i^T & 1 & v_{12}e_F^T \\ \hline 0 & 0 & v_{22} \end{array}\right)^T \left(\begin{array}{c} x_0 \\ \hline x_1 \\ \hline x_2 \end{array}\right) = \left(\begin{array}{c} 0 \\ \hline 0 \\ \hline 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{c|c|c} L_{00} & 0 & 0 \\ \hline \lambda_{10}e_L^T & 1 & v_{12}e_F^T \\ \hline 0 & 0 & u_{22} \end{array}\right)^T \left(\begin{array}{c} x_0 \\ \hline x_1 \\ \hline x_2 \end{array}\right) = \left(\begin{array}{c} 0 \\ \hline 1 \\ \hline 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc} L_{00} & \lambda_{10}e_L^T & 0 \\ 0 & 1 & 0 \\ 0 & v_{12}e_F^T & u_{22} \end{array}\right)_{3\times 3} \left(\begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \\ 0 \end{array}\right)_{3\times 1}$$

$$\Rightarrow \left(\begin{array}{c} L_{00}\,x_0 + \lambda_{10}e_i^T x_1 \\ x_1 \\ v_{12}e_F^T + u_{22}x_2 \end{array}\right) = \left(\begin{array}{c} 0 \\ 1 \\ 0 \end{array}\right)$$

$$\underline{x_1 = 1}$$

Substituting x_1 in $\overline{1st}$ element,

$$L_{00}\,x_0 + \lambda_{10}e_L^T (1) = 0$$
$$L_{00}\,x_0 + \lambda_{10}e_L^T = 0$$
$$x_0 = \dfrac{-\lambda_{10}e_L^T}{L_{00}}\,/\!/$$

Substituting x_0 in 3rd element,

$$v_{12}e_F^T + u_{22}x_2 = 0$$
$$x_2 = -\dfrac{v_{12}e_F^T}{u_{22}}$$

Cost of computing revolves around x_0, x_2 which is $O(n^2)$