End Term

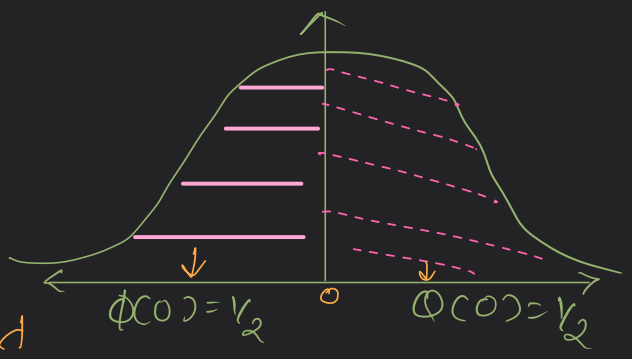
To prove → $\phi(0) = Q(0) = 1/2$

$$\phi(x) + Q(x) = 1$$

$$\phi(0) + Q(0) = 1$$

$$\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda$$

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda$$



$$\Rightarrow \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda + \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} = 1 = \text{proved}$$

$$\phi(0) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda = 1/2$$

$$Q(0) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2}\, d\lambda = 1/2$$

CDF of Gaussian distribution function

$$F_x(x) = P[X \le x] = \int_{-\infty}^{x} f_x(\lambda)\, d\lambda$$

$$F_x(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma_x^2} \cdot e^{\frac{-(\lambda - m_x)^2}{2\sigma_x^2}}\, d\lambda$$

let, $\tau = \dfrac{\lambda - m_x}{\sigma_x} \Rightarrow \dfrac{\lambda}{\sigma_x} - \dfrac{m_x}{\sigma_x} = \tau$

$$\frac{d\lambda}{\sigma_x} = d\tau \Rightarrow d\lambda = \sigma_x d\tau$$

when $\lambda = -\infty$, $\tau = -\infty$

$\lambda = x$, $\tau = \frac{x - m_x}{\sigma_x}$

$$F_x(x) = \int_{-\infty}^{\frac{x-m_x}{\sigma_x^2}} \frac{1}{\sqrt{2\pi}\sigma_x^2} \cdot e^{-\frac{\tau^2}{2}} \sigma_x d\tau$$

$$F_x(x) = \int_{-\infty}^{\frac{x-m_x}{\sigma_x^2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}} d\tau$$

comparing it to $\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\lambda^2/2} d\lambda$

$$\boxed{F_x(x) = \phi\left(\frac{x - m_x}{\sigma_x}\right)}$$

Ques If x is a random variable (x) with guass-ian PDF with $m_x = 2$ and $\sigma_x^2 = 4$ then find $P[x \le 1]$

$$F_x(x) = \phi\left[\frac{x - m_x}{\sigma_x}\right]$$

put $x = 1$, $m_x = 2$ and $\sigma_x = \sqrt{4} = 2$

$$\Rightarrow \phi\left(\frac{1-2}{2}\right)$$

$$\Rightarrow \phi(0.5) = 0.30854 \quad \text{Ans}$$

Q] X is a guassian random variable

$\mu = 5$, $\sigma_x = 2$

$$P(x > 8) = 1 - \phi\left(\frac{8-5}{2}\right)$$

$$= 1 - \Phi(3/2)$$

$$= 1 - (1 - Q(3/2))$$

$$= 0.06687$$

Information theory & coding: Source coding & channel coding.

Information: $I \propto \frac{1}{P}$ → Probability.

↓ Info content

eg → Sun rises in the east. $P = 1 \Rightarrow I = 0$

element of surprise ↑ (Prob ↓) → Information content ↑

Engineering view:

- Message with high prob. = less information = less no. of bits
- " " low " = more " = more " " "

Example: Moarse code → A = .— = 01

D = —.. = 100

Coding → a] fixed length coding

→ b] Variable " " (Eg: Moarse code)

↳ (Source coding)

↓ Data compress

	Symbol	Prob.	fixed	Variable
A	$x_1 = 500$	$P_1 = 1/2$	00	0
B	$x_2 = 250$	$P_2 = 1/4$	01	10
C	$x_3 = 125$	$P_3 = 1/8$	10	110
D	$x_4 = 125$	$P_4 = 1/8$	11	111

Total symbols = 1000 ↳ 2000 bits ↳ 1750 bits

Total bits → a] fixed : (500 x 2) + (250 x 2) + (125 x 2) + (125 x 2)
⇒ 2000 bits

b] variable : (500 x 1) + (250 x 2) + (125 x 3) + (125 x 3)
⇒ 1750 bits

Expression of information

$I(I) = \log_2\left(\frac{1}{p}\right)$ Bits

Let's take a example of equiprobable message

No. of Messages

$n = 2$ | $m_1 \rightarrow 0$, $m_2 \rightarrow 1$

$P_i = \frac{1}{2} \rightarrow P_1 = \frac{1}{2}$, $P_2 = \frac{1}{2}$

$\log_2(n) = \log_2(2) = 1$ bit

$n = 4$ | $m_1 \rightarrow 00$, $m_2 \rightarrow 01$, $m_3 \rightarrow 10$, $m_4 \rightarrow 11$

$P_i = \frac{1}{4}$ $P_1 = 1/4$, $P_2 = 1/4$, $P_3 = 1/4$, $P_4 = 1/4$

$\log_2(n) = \log_2(4) = 2$ bit

Generalisation: $I_i = \log_2\left(\frac{1}{P_i}\right)$

Ques-1 Prove that if the reciever knows the message being T_x, the amount of info. carried is 0.

$P = 1$ → This means it is a definite event.

$I = \log_2(1) = 0$

Ques-2 Calculate the amt. of info. if $P_k = \frac{1}{4}$

$P_k = \frac{1}{4}$; $I = \log_2(4) = 2$ bits.

Ques - 3] In binary PCM if $V_1 \to 0$ occurs with probability $\frac{1}{4}$ and $V_2 \to 1$ occurs with prob. $\frac{3}{4}$ then find the informat? conveyed by each bit.

$V_1 \to 0 \longrightarrow P_1 = \frac{1}{4} \Rightarrow \log_2\left(\frac{1}{P_1}\right) = \log_2(4) = 2$ bits

$V_2 \to 1 \longrightarrow P_2 = \frac{3}{4} \Rightarrow \log_2\left(\frac{4}{3}\right) = 0.4151$ bits.

ENTROPY: Average information (bits) per symbol (messages)

H(S)
↓ ↳ Source
Entropy

→ Consider a memoryless source transmitting messages $(m_1, m_2, \cdots, m_n)$ with prob. $(P_1, P_2, \cdots P_n)$

- Memoryless → Messages are independent of previous messages.
- Entropy of source is given by -

$$H(S) = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right) \quad \text{bits/symbol}$$

TUT

Ques - 1] Let us consider a binary source with $S = \{S_1, S_2\}$ with probabilities $\frac{1}{256}, \frac{255}{256}$ then calculate entropy of source.

Ans - $P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right)$

$$= \frac{1}{256}\log_2(256) + \frac{255}{256}\log_2\left(\frac{256}{255}\right)$$

$$= \frac{1}{256} \times 0.8 + 0.996 \times 0.006$$

$$= 0.3721 \text{ bits/symbol.}$$

Ques-2 $P = \left\{\frac{7}{16}, \frac{9}{16}\right\}$

Ans- $$P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right)$$

$$= \frac{7}{16}\log_2\left(\frac{16}{7}\right) + \frac{9}{16}\log_2\left(\frac{16}{9}\right)$$

$$= \frac{7}{16} \times 1.1926 + \frac{9}{16} \times 0.830$$

$$= 0.9885 \text{ bits/symbol}$$

Ques-3 $P_i \left\{\frac{1}{2}, \frac{1}{2}\right\}$

$$= \frac{1}{2}\log(2) + \frac{1}{2}\log(2)$$

$$= \frac{1}{2} \times 1 + \frac{1}{2} \times 1$$

$$= 1 \text{ bit/symbol.}$$

Properties of entropy

Maximum entropy occurs when all messages are equiprobable.

Ques- If there are M equilikely messages such

that $(m = 2^n)$ then find entropy of source.

Solut^n — $P = m = 2^n$

$$S = \{ S_1, S_2, \dots S_m \}$$

$$P = \{ P_1, P_2, \dots P_m \}$$

$$P_i \Rightarrow P_1 = P_2 = P_3 = \dots = P_m = \frac{1}{m}$$

$$H(S) = \sum_{i=1}^{n} P_i \log_2 \left(\frac{1}{P_i}\right)$$

$$= M \cdot \frac{1}{M} \log_2 (M)$$

$$= \log_2 (M) \text{ OR } \log_2 (2^n)$$

$$= n \text{ bits/symbol.}$$

Proof : $S = \{ m_1, m_2, \dots, m_n \}$

$$P = \{ P_1, P_2, \dots, P_n \}$$

Suppose sequence of L messages are transmitted then $P_1 \times L$ messages of m_1 are transmitted.

$P_2 \times L$ " " m_2 " "

$P_3 \times L$ " " m_3 " "

→ no. of symbols in L length of messges m_3.

$$I_n (\text{total}) = P_n \times L \times I_n \qquad \because I_n = \log_2 \left(\frac{1}{P_n}\right)$$

$$= P_n \times L \times \log_2 \left(\frac{1}{P_n}\right)$$

$$H(S) = \frac{\text{Total Info.}}{\text{Total Symbol}}$$

$$= \frac{P_1 L \log_2\left(\frac{1}{P_1}\right) + P_2 L \log_2\left(\frac{1}{P_2}\right) + \cdots P_n L \log_2\left(\frac{1}{P_n}\right)}{L}$$

$$H(S) = P_1 \log_2\left(\frac{1}{P_1}\right) + P_2 \log_2\left(\frac{1}{P_2}\right) + \cdots P_n \log\left(\frac{1}{P_n}\right)$$

$$\boxed{H(S) = \sum_{i=1}^{n} P_i \log\left(\frac{1}{P_i}\right)}$$ Ans

Variable length coding (Source Encoding)

- Shannon fano coding
- Huffman coding.

(1) Shanon fano coding (Source Encoder)

Steps → Algo

(i) The messages are first written in order of decreasing / non increasing probabilities.

(ii) Then devide the message set in two most equiprobable subsets.

(iii) The message in the first set is given bit 0, and message in the second set is given bit 1.

(iv) The procedure is now applied for each set saperately and continued untill no furthur division is possible.

(v) finally, we get the code word for each symbol.

Q-1] Find the code word for the symbols – S_1, S_2, S_3 & S_4 with probabilities $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\}$ using shanon fano coding. Find the efficiency & redundancy of the code.

Ans –

Symbol	Prob.		
S_1	$\frac{1}{2}$	] 0	
S_2	$\frac{1}{4}$	] 0	
S_3	$\frac{1}{8}$	1] 1	] 0
S_4	$\frac{1}{8}$		] 1

Symbol	Code
S_1	0
S_2	1 0
S_3	1 1 0
S_4	1 1 1

Efficiency of code = $\frac{H(S)}{\bar{L}}$ → Entropy, → avg. length of code

$$H(S) = \sum_{i=1} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$= \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log_2(8) + \frac{1}{8}\log_2(8)$$

$$= \frac{1}{2} + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \frac{1}{8}\cdot 3$$

$$= \frac{4+4+3+3}{8}$$

$$H(S) = \frac{14}{8} = 1.75 \text{ bits / symbol.}$$

$$\bar{L} = \sum_{i=1}^{n} P_i \cdot l_i = P_1 L_1 + P_2 L_2 + P_3 L_3 + P_4 L_4$$ (length of code.)

$$= \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{8}\times 3$$

$$= 14/8 = 1.75$$

Efficiency = $1.75/1.75 = 1$ → 100% efficiency.

Efficiency is a measure of compact-ness of the code, since it is 1, the code cannot be further reduced.

Huffman Coding

(i) Symbols are arranged in order of decreasing (non increasing prob., then the 2 symbols of lowest probabilities are assigned 0 & 1.

(ii) Combine last two symbols and move the combined symbol as high as possible.

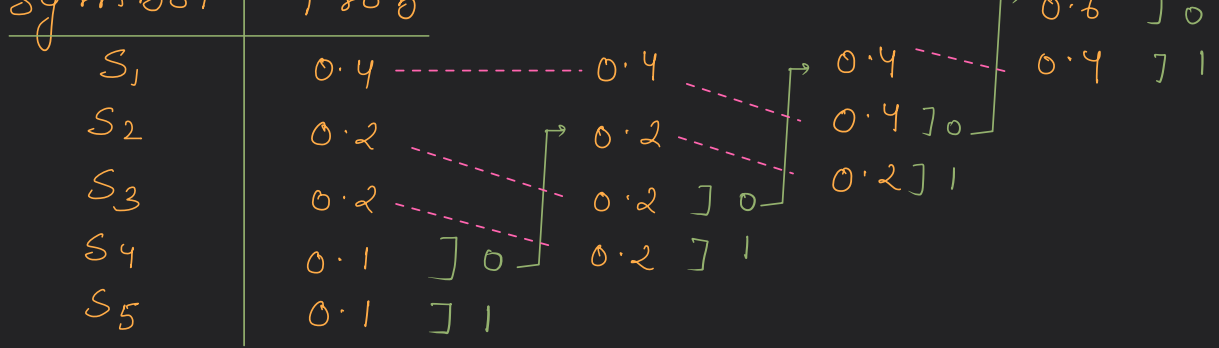
(iii) Repeat the above step until we are left with the final list of source symbols of only 2 for which 0 & 1 are assigned.

(iv) Code for each symbole is formed by moving backward & forward.

Ques - Five alphabet with probabilities $\{0.4, 0.2, 0.2, 0.1, 0.1\}$ for source $S = \{S_1, S_2, S_3, S_4, S_5\}$ find huffman code for these symbols & find efficiency and redundancy of code.

Ans -

Symbol	Prob
S_1	0.4
S_2	0.2
S_3	0.2
S_4	0.1
S_5	0.1



Symbol	Code	L
S_1	0 0	2
S_2	1 0	2
S_3	1 1	2
S_4	0 1 0	3
S_5	0 1 1	3

$$H(S) = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$= 0.4 \log_2\left(\frac{10}{4}\right) + 0.2 \log_2\left(\frac{10}{2}\right) + 0.2 \log_2\left(\frac{10}{2}\right) + 0.1 \log_2\left(\frac{10}{1}\right) + 0.1 \log_2\left(\frac{10}{1}\right)$$

$$= 2.12 \text{ bits/symbol}$$

$$\bar{L} = P_1L_1 + P_2L_2 + P_3L_3 + P_4L_4 + P_5L_5$$

$$= 0.4(2) + 0.2(2) + 0.2(2) + 0.1(3) + 0.1(3)$$

$$= 2.2$$

$$\text{efficiency } \eta = \frac{H(S)}{\bar{L}} = \frac{2.12}{2.2} = 0.96$$

$$\text{redundancy} = 1 - \eta \Rightarrow 1 - 0.96 = 0.04 \text{ Ans.}$$

TUT

Ques - Find the code word using shanon fano coding for prob. $P = \{1/4, 1/4, 1/8, 1/8, 1/16, 1/16, 1/16, 1/16\}$ for the symbols $S = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$. Also find efficiency & redundancy of code.

Ans -

Symbol	Prob
S_1	1/4

0 0

Symbol	Code
S_1	0 0

$$H(S) = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$= \frac{1}{4}\log_2(4) + \frac{1}{4}\log_2(4) + \frac{1}{8}\log_2(8) + \frac{1}{8}\log_2(8)$$

$$+ \frac{1}{16}\log_2(16) + \frac{1}{16}\log_2(16) + \frac{1}{16}\log_2(16) + \frac{1}{16}\log_2(16)$$

$$= \frac{1}{4}(2) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(3) + \frac{1}{16}(4)$$

$$+ \frac{1}{16}(4) + \frac{1}{16}(4) + \frac{1}{16}(4)$$

$$= \frac{2}{4} + \frac{2}{4} + 2 \cdot \frac{3}{8} + 4 \cdot \frac{4}{16}$$

$$= 1 + 1.75$$

$$H(S) = 2.75$$

$$L' = \sum_{i=1}^{n} P_i l_i$$

$$= 2.75$$

$$\text{Efficiency} = \frac{H(S)}{L'} = \frac{2.75}{2.75} = 1$$

$$\text{redundancy} = 1 - \eta = 1 - 1 = 0$$ Ans.

Ques – Given the messages $\{S_1, S_2, S_3, S_4, S_5, S_6\}$ with respective probabilities $\{0.4, 0.2, 0.2, 0.1, 0.07, 0.03\}$ construct a binary code by applying shanon fano coding procedure and hence find efficiency & redundancy

Symbol	Prob
S_1	0.4
S_2	0.2
S_3	0.2
S_4	0.1
S_5	0.07
S_6	0.03

Symbol	Code
S_1	0
S_2	100
S_3	101
S_4	110
S_5	1110
S_6	1111

$$H(S) = \sum_{i=1}^{n} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$\Rightarrow 0.4 \log_2\left(\frac{10}{4}\right) + 0.2 \log_2\left(\frac{10}{2}\right) + 0.2 \log_2\left(\frac{10}{2}\right) + 0.1 \log_2\left(\frac{10}{1}\right) + 0.07 \log_2\left(\frac{100}{7}\right) + 0.03 \log_2\left(\frac{100}{3}\right)$$

$$\Rightarrow 0.4 \times 1.322 + (0.2 \times 2.322)2 + 0.1(3.322) + 0.07(3.8365) + 0.03(5.059)$$

$$\Rightarrow 2.21 \text{ bits/symbol}$$

$$L' = \sum_{i=1}^{n} P_i l_i \Rightarrow 0.4(1) + 0.2(3) + 0.2(3) + 0.1(3) + 0.07(4) + 0.03(4)$$

$$= 0.4 + 1.2 + 0.3 + 0.28 + 0.12$$

$$= 2.3$$

$$\eta = \frac{2.21}{2.3} = 0.96 \rightarrow 96\%$$

$$\text{Redundancy} = \eta - 1 = 0.04 \rightarrow 4\%$$

Ques – Construct the huffman code for symbol $\{x_1, x_2, x_3, x_4, x_5\}$ with Prob. $\{0.3, 0.15, 0.25, 0.05, 0.25\}$ find code efficiency and redundancy

Ans → Rearranging in decreasing order

Symbol	Prob
x_1	0.3
x_3	0.25
x_5	0.25
x_2	0.15]0
x_4	0.05]1

0.3 + 0.25 → 0.55]0
0.3]0 + 0.25]1 → 0.45]1
0.25]0 + 0.15]1 → 0.2]1

Symbol	Code
x_1	0 0
x_3	0 1
x_5	1 0
x_2	1 1 0
x_4	1 1 1

$$H(S) = \sum_{i=1}^{n} P_i \log\left(\frac{1}{P_i}\right)$$

$$\Rightarrow 0.3 \log_2\left(\frac{1}{0.3}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.15 \log_2\left(\frac{1}{0.15}\right) + 0.05 \log_2\left(\frac{1}{0.05}\right)$$

$$\Rightarrow 2.14775 \text{ bits/symbol}$$

$$L' = \sum_{i=1}^{n} P_i l_i \Rightarrow 0.3(2) + 2(0.25(2)) + 0.15(3) + 0.05(3)$$

$$\Rightarrow 0.6 + 1 + 0.45 + 0.15$$

$$\Rightarrow 2.2$$

$$\eta = \frac{2.14}{2.2} = 0.97 \rightarrow 97\%$$

redundancy $= 1 - \eta = 0.03 \rightarrow 3\%$ Ans.

Shanon's channel capacity theorem

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

C → channel capacity (Kbps, mbps)
B → Bandwidth (Hz, KHz, MHz)
$SNR = \frac{\text{Signal power}}{\text{Noise power}}$

Side note → for numerical SNR → 12 dB → if SNR is in dB convert it into actual value for numerical.

Special case: (I) $\frac{S}{N} = 1 = 0\,dB$.

$$C = B\log_2(1+1)$$

$$C = B\log_2(2)$$

$$\boxed{C = B}$$

(II) $\frac{S}{N} > 1 \Rightarrow C > B$

(III) $\frac{S}{N} < 1 \Rightarrow C < B$

(IV) $\frac{S}{N} \to \infty \Rightarrow C \to \infty$

(V) $B \to \infty \Rightarrow C = 1.44\frac{S}{N_0}$

(S → Signal power (watt), N_0 → Noise power spectral density ($\frac{watt}{Hz}$), ($N \neq N_0$))

$$C = \lim_{B\to\infty} B\log_2\left(1+\frac{S}{N_0}\right)$$

$N = N_0 B$.

$$= \lim_{B\to\infty} \frac{N_0 B}{S} \times \frac{S}{N_0}\log_2\left(1+\frac{S}{N_0 B}\right)$$

$x = \frac{N_0 B}{S}$, $B \to \infty$, $x \to \infty$

$$= \lim_{x\to\infty} x\log_2\left(1+\frac{1}{x}\right)\cdot\frac{S}{N_0} \qquad x = \frac{1}{t}$$

$$= \lim_{t\to 0} \frac{\log_2(1+t)}{t}\cdot\frac{S}{N_0}$$

applying L-Hospital rule

$$= \lim_{t \to 0} \frac{\frac{d}{dt}(\log_2(1+t))}{\frac{d}{dt}(t)} \cdot \frac{S}{N_0}$$

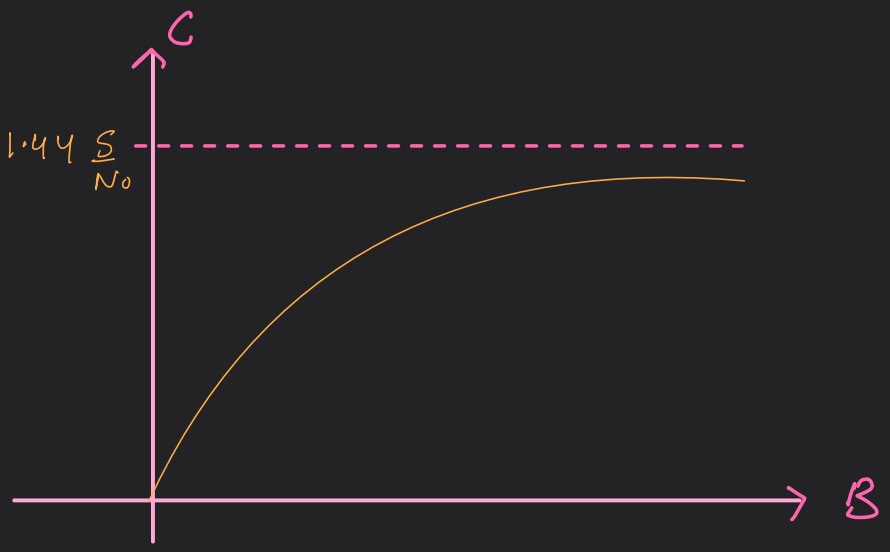
$\frac{d}{dx}\log_a x = \frac{1}{x \log_e a}$

$\frac{d}{dt}(t) = 1$

$$= \lim_{t \to 0} \frac{1}{(1+t)\log_e 2} \cdot \frac{S}{N_0}$$

$$= \frac{1}{\log_e 2} \cdot \frac{S}{N_0}$$

$$\boxed{C = 1.44\left(\frac{S}{N_0}\right)}$$



Ques – A radio channel has a bandwidth of 10KHz & SNR of 15 dB. find the maximum data rate that can be transmitted over this channel.

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

$SNR = 15\ dB \Rightarrow 10^{1.5} \Rightarrow 31.62\ Hz$

$$C = 10 \times 10^3 \times \log_2(1 + 31.62)$$

$$\boxed{C = 50.27\ kbps}$$ Ans

Ques - find the channel capacity for $\frac{S}{N} = 15$ for the following 3 cases -

a) B = 1 KHz (twisted pairs)

b) B = 1 MHz (Co-axel cable)

c) B = 1 GHz (optical fibre)

d) B = ∞

Ans → a) $C = B \log_2 (1 + \frac{S}{N})$

$= 1 \log_2 (1 + 15)$

$C = 4$ Kbps.

b) $C = 1 \log_2 (16)$

$= 4$ Mbps

c) $C = 1 \log_2 (16)$

$= 4$ Gbps.

d) $C = 1.44 (\frac{S}{N_0})$

TUT -

Que In communication with AWGN operating at SNR >> 1 & BW = B has capacity C_1 If SNR is doubled by keeping B constant is less than C_1.

$C = B \log_2 (1 + \frac{S}{N})$

$C_1 = B \log_2 (1 + \frac{S}{N})$

$C^* = B \log_2 [1 + \frac{2S}{N}]$

$\frac{C_2^*}{C_1} = \frac{C \log_2 [1 + 2\frac{S}{N}]}{\log_2 [1 + \frac{S}{N}]}$

$C_2 = C_1 \Rightarrow \frac{\log_2 [\frac{2S}{N}]}{\log [\frac{S}{N}]} = \frac{1 + \log_2 (\frac{S}{N})}{\log_2 (\frac{S}{N})}$

$C_2 = C_1 \left[\left(\frac{1}{\log_2 S/N} + 1 \right) \right]$ $C_1 = C_2$

$B + C_1 = C_2$

Que A communication channel has BW of 3 MHz and SNR of 20dB find channel capacity 19.97 mbps

Que BW is 10 KHz and SNR is 30 dB find channel C.? 99.07 Kbps

Que Find the Required SNR in dB to achieve 10 Mbps with BW of 1 MHz

Sol'n $C = B \log_2 (1 + \frac{S}{N})$

$10 \times 10^6 = 1 \times 10^6 \log_2 (1 + \frac{S}{N})$

$10 = \log_2 (1 + \frac{S}{N}) = \log_2 2^{10} = 1 (1 + \frac{S}{N})$

$\Rightarrow 2^{10} - 1 = \frac{S}{N} \approx 2^{10} \Rightarrow 30$dB.

Que A channel width capacity 5 Mbps has SNR of 15dB find the Requirement Bw.

$C = B \log_2 (1 + \frac{S}{N})$

$5 \times 10^6 = B \log_2 (1 + 31.62)$

$5 \times 10^6 = B \log_2 (32.62)$

$5 \times 10^6 = B \times 5$

$B = 1$ MHz

SNR = 15dB

$10^{\frac{15}{10}}$

$10^{1.5}$

31.62

Channel coding (Error control codes)

- Block codes
  - Linear block code
    - cyclic codes
- Convolution codes

Block codes: Every block of k data digits is encoded into a longer codeword of n digits

k data $\longrightarrow$ n $\Rightarrow$ (n, k) $\Rightarrow$ Block code

(n: total no. of bits after coding; k: total no. of bits before coding)

- redundant bits / extra bits $= n-k$ (parity bits (PB))

Properties:

a) Unique sequence of k data digits results in unique code word of n digits

b) Wait untill k digits are accumulated

c) Not suitable for burst error

Convolution code: Coded sequence of n digits depends on the k data digits and also depend on previous (n−1) data digits as well.

Properties:

a) It has a memory.

b) Coded sequence for a curtain k data digits

may not be unique.

c] Continuous Encoding (no wait, bit by bit operat?).

d] It is comparatively better for burst error.

Block code (n, k):

Example:

Parity code = Parity check block code.

- Simplest form of error control code
- Single parity bit is used to generate codeword with even/odd parity.
- It can only detect the error but can't correct the order.

→ format $(n, k) \Rightarrow (k+1, k)$, $n = k+1$, $p = n-k = 1$

Single bit parity code $(n, k) = (k+1, k)$ $n = k+1$, $p = n-k = 1$

Information bits = $i = [i_1, i_2, i_3 \ldots]$

for even parity = $P = i_1 \oplus i_2 \oplus i_3 \oplus \ldots \oplus i_k$

for odd parity = $P = i_1 \oplus i_2 \oplus i_3 \oplus \ldots \oplus i_k \oplus 1$

final code word = $[i_1 i_2 i_3 \ldots i_k P]$

Ques – given [5, 4] even parity block code find the code word corresponding to 1011 and 1010.

$n = 5$

$k = 4$

for code [1011]

$i = [1011] = [i_0 \; i_1 \; i_2 \; i_3]$

for even parity → $p = 1$

$p = i_1 \oplus i_2 \oplus i_3 \oplus i_4$

$p = 1 \oplus 0 \oplus 1 \oplus 1 = 1$

code = [1 0 1 1 1] Ans.

for code [1 0 1 0]

$p = i_1 \oplus i_2 \oplus i_3 \oplus i_4$

$p = 1 \oplus 0 \oplus 1 \oplus 0 = 0$

code = [1 0 1 0 0] Ans.

Single Bit Parity code – Decoding

Recieved code word = $[v_1\ v_2 \dots v_n]$

checkSum $S = v_1 \oplus v_2 \oplus \dots \oplus v_n$

if $S = \begin{cases} 0 & \text{even parity with correct recieved data} \\ 1 & \text{odd} \quad " \quad " \quad " \quad " \end{cases}$

Ques - Given the [8,7] even parity block code, determine weather $v_1 = \{1\,0\,1\,1\,0\,1\,1\,0\}$ & $v_2 = \{0\,1\,1\,0\,1\,0\,0\,1\}$ give parity failures.

→ for $v_1 = (1\,0\,1\,1\,0\,1\,1\,0)$

$S = 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 = 1$

as, $S = 1$ means parity failure is here

→ for $v_2 = (0\,1\,1\,0\,1\,0\,0\,1)$

$S = 0 \oplus 1 \oplus 1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 1 = 0$

as $S = 0$ means parity failure is not here

Even parity Hamming code = Hamming code

→ They are special type of parity codes.

→ Even parity Hamming codes are very popular and by default called as hamming code.

→ Odd parity hamming codes also exist but they are rarely used.

→ Hamming code uses multiple no. of parity bits at different positions, mixed with informat^n bits.

→ On contrary to single bit parity code which can only detect the error, the hamming code can detect + correct the error.

$(n, k) = p = n - k$

n → total bits after coding
k → no. of message bits.
p → no. of parity bits.

Eg → Popular hamming code $(7, 4)$ (n, k)

What is p ?

To identify the no. of parity bits the following condition should be satisfied.

$$2^p \geq p + k + 1$$

The min. value of p which satisfies the above eqn is the answer of p.

Qus - find the value of p for $k = 5$

$$2^p \geq p + k + 1$$

put $p = 4$

$$16 \geq 4 + k + 1$$

$$16 \geq 9$$

$$=$$

therefore, $n = k + p = 5 + 4$

$$n = 9$$

$$[n, k] = [9, 4]$$ Ans.

Ques – Generate the hamming code for data (1011)

→ Data (1011), $k = 4$

$$2^p \geq p + k + 1$$

$$(2)^3 \geq 8$$

$p = 3$ $\quad n = k + p = 3 + 4 = 7$

⇒ hamming code (n, k)

⇒ $(7, 4)$

Position of parity bit: $P_1 = 2^0 = 1$ position

$P_2 = 2^1 = 2$ "

$P_3 = 2^2 = 4$ "

$P_4 = 2^3 = 8$ "

code =

D_7	D_6	D_5	P_4	D_3	P_2	P_1
1	0	1		1		

$$P_1 = D_3 \oplus D_5 \oplus D_7 = 1 \oplus 1 \oplus 1 = 1$$

$$P_2 = D_3 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 1 = 0$$

$$P_4 = D_5 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 1 = 0$$

code = (1 0 1 0 1 0 1)

Ques- Decode (1 0 0 0 0 1 0 1)

Received code (1 0 0 0 0 1 0 1)
$D_7\ D_6\ D_5\ P_4\ D_3\ P_2\ P_1$

Check Bit -

$C_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$

$C_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 = 0 \oplus 1 \oplus 0 \oplus 1 = 0$

$C_4 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 = 0 \oplus 0 \oplus 0 \oplus 1 = 1$

$\Rightarrow C_4 C_2 C_1 = 1\ 0\ 1 \Rightarrow 5^{th}$ bit

correct word = (1 0 1 0 1 0 1)

Ques- Decode (1 1 1 0 1 0 1)
$D_7\ D_6\ D_5\ P_4\ D_3\ P_2\ P_1$

$C_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7 = 0$

$C_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 = 1$

$C_4 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 = 1$

$\Rightarrow C_4 C_2 C_1 \Rightarrow 1\ 1\ 0 \Rightarrow 6^{th}$ Bit

correct word = (1 0 1 0 1 0)

Ques- Generate the hamming code for the data 101

$K = 3$

$2^p \geq p + K + 1$

$2^3 \geq 3 + 3 + 1$

$8 \geq 7$

$n = K + p = 6.$

format $(n, K) = (6, 3)$

1	0	P_4	1	P_2	P_1

Check Bit -

$P_1 = 1 \oplus 0 = 1$

$P_2 = 1 \oplus 1 = 0$

$P_4 = 0 \oplus 1 = 1$

code →

1	0	1	1	0	1