Financial Analytics

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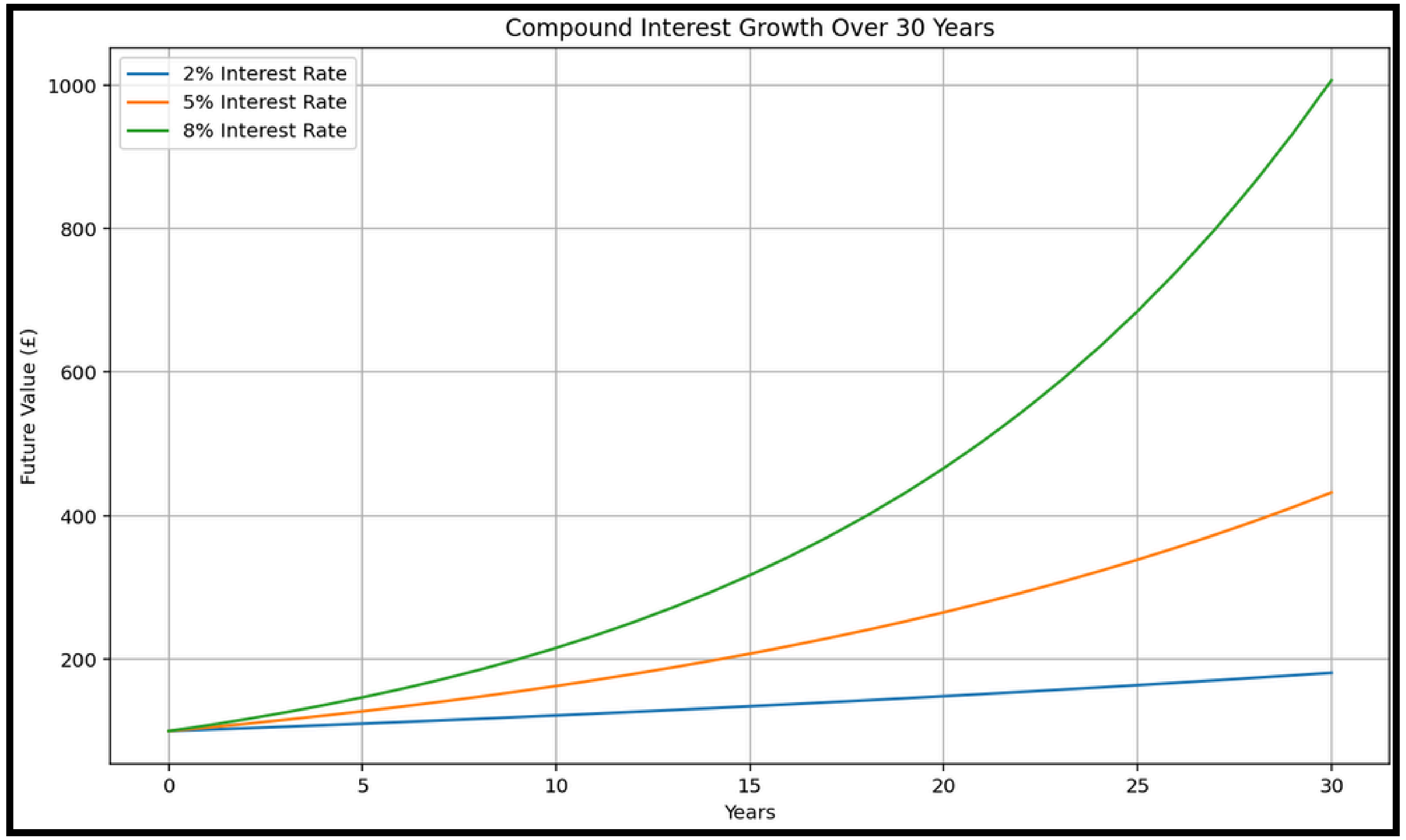
1.1 Compound Interest Modelling and Critical Analysis



Annual Compound Interest

Formula: $A = P(1 + r)^t$

A: Amount (Future Value)
P: 100 (initial investment £100)
r: Interest Rate (2%, 5%, 8%)
t: Time in Years (0–30)

Results & Key Observation

<u>Growth Over 30 years:</u>

- 2% → ~£181
- 5% → ~£432
- 8% → ~£1,006

<u>Exponential growth:</u>

Even a **small** difference in the interest rates is causing **large** differences in the final amount.

1.2 PV Framework vs IRR Framework

The two frameworks are not always equivalent.

<u>A Counter Example:</u>

Project A - Non-zero-coupon bond with £1000 principle, pays £300 annually for five years.
Project B - Zero-coupon bond with £1000 principle, repays £3000 in year five.
R = 10%

Time	0	1	2	3	4	5	PV	IRR
Project A	-1000	300	300	300	300	1300	758.15	30%
Project B	-1000	0	0	0	0	3000	862.78	24.57%

PV : Project A (758.15) < Project B (862.78)
IRR: Project A (30%) > Project B (24.57%)

Advantages of IRR

No pre-set discount rate needed

Advantages of PV

All cash flows and risk considered

1.3 Finding the 10-Year Spot Rate from Bond Prices

Bond A: $P_A = 98.7$ $\quad C_A = 100 * 10\% = 10$

Bond B: $P_B = 89.9$ $\quad C_B = 100 * 8\% = 8$

$$P_A: \; 98.7 = \sum_{k=1}^{9} \frac{10}{(1+S_k)^k} + \frac{110}{(1+S_{10})^{10}}$$

$$P_B: \; 85.9 = \sum_{k=1}^{9} \frac{8}{(1+S_k)^k} + \frac{108}{(1+S_{10})^{10}}$$

$$z = \frac{1}{(1+S_k)^k}$$

$$\longrightarrow \begin{cases} 98.7 = 10z + \dfrac{110}{(1+S_{10})^{10}} \\ 85.9 = 8z + \dfrac{108}{(1+S_{10})^{10}} \end{cases}$$

Solving the system yields the $10 - \text{year}$ spot rate: $\mathbf{11.2}\%$.

Therefore, an investor requires an annual return of 11.2% for a zero-coupon bond maturing in 10 years.

2.1 Markowitz's Portfolio Selection Model

Advantages

Diversification
Reduces unsystematic risk through asset combination.

Quantitative Framework
Uses mathematical approach for portfolio construction.

Efficient Portfolios
Identifies portfolios with optimal risk-return balance.

Customisable Risk Tolerance
Investors define their preferred risk-return targets.

Disadvantages

Data-Intensive
Needs precise return, variance, covariance estimations.

Input Sensitivity
Minor input changes drastically affect portfolio recommendations.

Assumptions
Assumes returns are normally distributed and markets are frictionless.

Scalability Challenges
Computationally complex for bigger portfolios to manage.

Application Steps

Identify Investment Assets
Select stocks, ETFs, or other financial instruments

Collect Historical Data
Gather past price or return data

Calculate Key Inputs
Compute expected returns, variances, and covariance matrix

Formulate Optimisation Problem
Define the objective with constraints

Solve for Optimal Weights
Use mathematical tools to determine asset allocation

Construct Efficient Frontier
Plot risk-return combinations of optimal portfolios

Choose Optimal Portfolio
Select based on risk tolerance or Sharpe Ratio

Monitor and Rebalance
Periodically review performance and adjust allocations

2.2.1 Mean-Variance Optimal Portfolio: Computation & Interpretation

Ticker	Company	Weight (%)
AZN.L	AstraZeneca	32.47%
BP.L	BP plc	~0.00%
HSBA.L	HSBC Holdings	21.55%
SHEL.L	Shell plc	9.27%
ULVR.L	Unilever	36.71%

Target Return: 7.00%

Volatility (Risk): 16.88%

Interpretations and Results

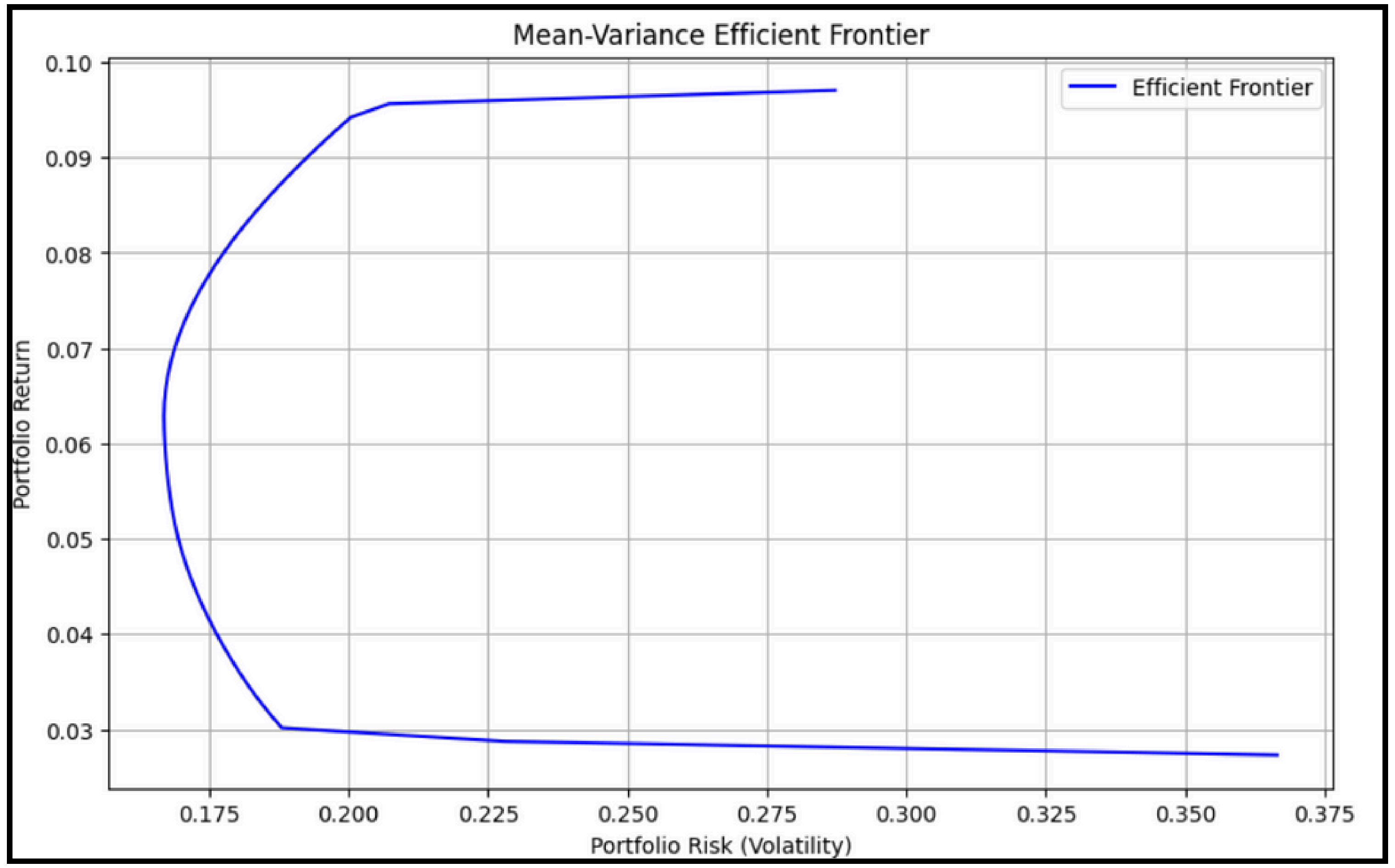
AZN.L (AstraZeneca) and **ULVR.L (Unilever)** have the **highest allocations** (32.47% and 36.71%) thereby Indicating **strong risk-adjusted returns** and **low correlation** with other assets.

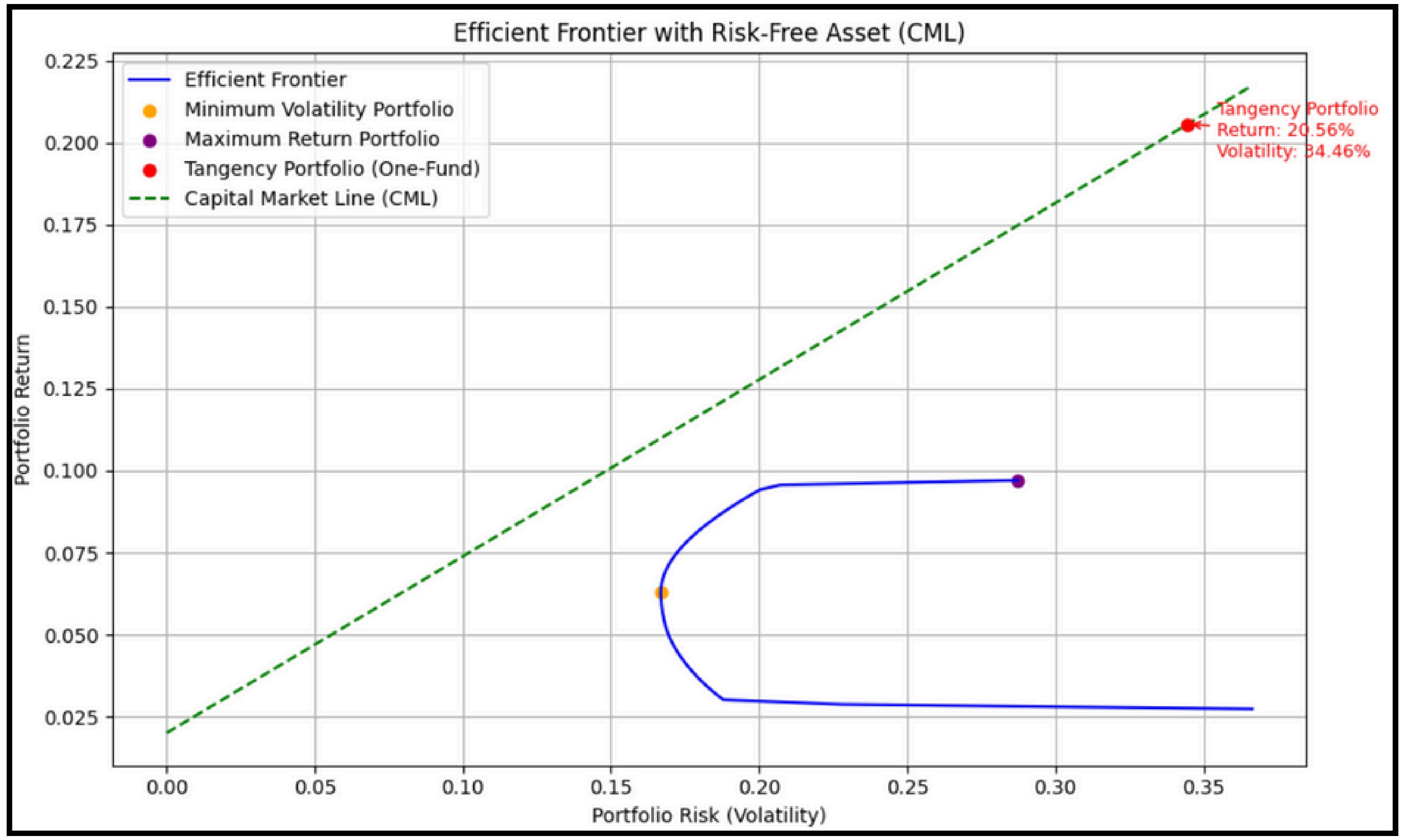
BP.L (BP) is assigned a **near-zero weight (~0.00%)** thereby suggesting **unfavorable return-to-risk ratio** or **redundancy** (e.g., overlap with **SHEL.L**).

HSBA.L (HSBC) and **SHEL.L (Shell)** hold **moderate weights** (21.55% and 9.27%). Contribute to **diversification** and help **reduce portfolio volatility**.

Hence, it is suitable for moderate risk investors seeking a balanced risk-return profile.

2.2.2 Mean-Variance Efficient Frontier with Risk-Free Asset and Capital Market Line (CML)





Interpretations of Results

Tangency Portfolio:
Highest risk-adjusted return
(Sharpe ≈ 1.2).

Risk-free rate (~2%):
Enables Capital Market Line
(CML), improving portfolio
efficiency.

Efficient frontier
(Max return ≈ 16%, risk ≈
18%) shows risky portfolios
only.

Better risk-adjusted returns
than the efficient frontier
thereby preferred by
rational investors.

3.1 CAPM Application: Estimating Risk and Return in a Two-Asset Market

Given : Two risky assets A and B and a risk-free asset F

Market portfolio: $M = \frac{1}{2}(A + B)$, Risk-free rate: $r_F = 0.10$

Variance of A: $\sigma_A^2 = 0.04$, Variance of B: $\sigma_B^2 = 0.02$

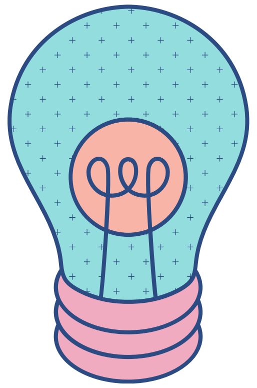
Covariance: $\sigma_{AB} = 0.01$, Expected return of the market: $r_M = 0.18$

Step 1: Find the general expression (without substituting values) for σ^2_m and β_a

- $\sigma_M^2 = \frac{1}{4}\sigma_A^2 + \frac{1}{4}\sigma_B^2 + \frac{1}{2}\sigma_{AB}$
- $\beta_A = \frac{0.5\sigma_A^2 + 0.5\sigma_{AB}}{\sigma_M^2}$

Step 2 : According to the CAPM, what is the numerical value of r_a?

$$r_A = r_F + \beta_A(r_M - r_F) = 0.1 + 1.25(0.18 - 0.1) = 0.20$$



Interpretation of Results

1. Asset A has $\beta = 1.25$ → More sensitive to market risk
2. Expected return of 20% reflects higher risk premium
3. CAPM supports A as attractive for high-risk, high-return portfolios

3.2 CAPM Analysis of Stock Returns: 2004 - 2006

Metric	GM	F	CAT	PFE
CAPM - Estimated Excess Return	-0.002417	-0.002425	-0.00262	-0.002354
Sample Mean Excess Return	-0.002994	-0.003278	-0.0017	-0.00276
Difference	0.000576	0.000854	-0.00092	0.000406
Alpha	1.0295	1.0326	1.1160	1.0025
P-value	1.6288e-72	2.3055e-104	7.7917e-175	2.5448e-152
Reject H0 (α = 0 at 1% level)	True	True	True	True

Key Findings

CAPM Fit

CAPM shows a strong fit for all. For CAT, larger differences imply deviation.

Outperformance

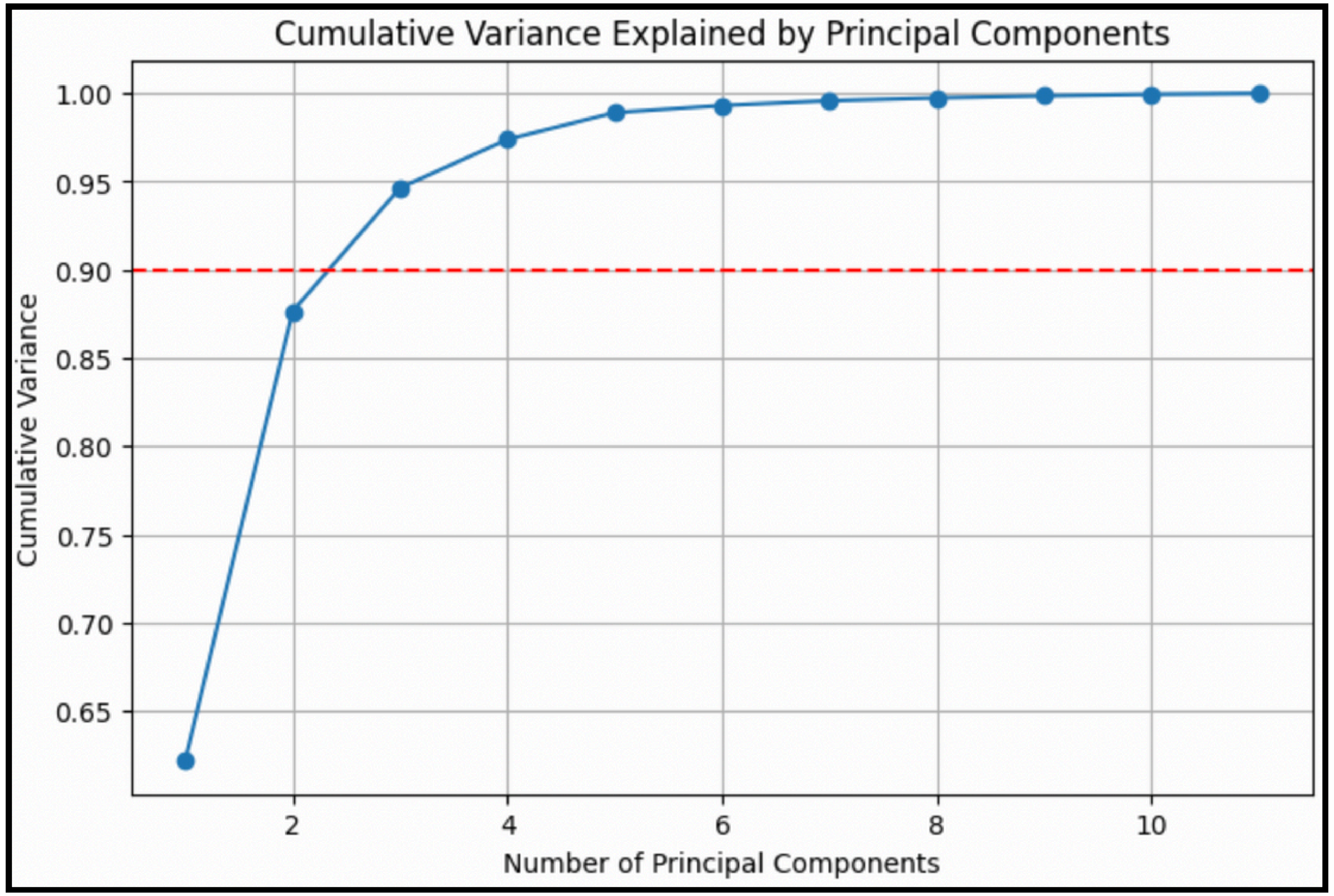
All firms have positive Alpha, outperforming the market.

Statistical Significance

Extremely low P - values confirm Alpha's reliability and allow rejecting H0 (α = 0 at 1% level).

3.3. Principal Component Analysis: Daily Treasury Yield Changes

PC	Proportion of Variance	Cumulative Variance
PC1	0.6218	0.6218
PC2	0.2547	0.8765
PC3	0.0699	0.9464
PC4	0.0275	0.9739
PC5	0.0151	0.989
PC6	0.0041	0.9931
PC7	0.0027	0.9958
PC8	0.0016	0.9974
PC9	0.0012	0.9986
PC10	0.0009	0.9995
PC11	0.0005	1.0000



Three principal components explain more than 90% of the total variance.

4.1 Analysis of Simulated 8-Year Monthly Stock Returns

Month	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Overall
Jan	3.15	0.4	3.8	7.59	-0.01	-0.01	7.84	4.32	
Feb	-1.03	3.35	-1.01	-1.02	2.05	-7.28	-6.47	-1.43	
Mar	-3.39	2.36	-2.93	-5.12	7.35	0.02	1.29	-5.17	
Apr	-1.36	1.48	-3.98	2.63	-1.6	-0.26	-1.61	9.02	
May	0.94	-3.58	4.56	-4.29	1.9	-7.49	-4.75	1.85	
Jun	4.2	1.74	0.5	-0.3	-5.4	-2.12	-0.99	5.58	
Jul	2.49	-6.63	2.4	-0.67	-1.93	3.65	5.46	5.03	
Aug	-2.63	-0.34	2.43	5.22	-1.07	0.2	-3.79	-4.18	
Sep	4.52	6.87	0.69	5.35	2.57	-1.79	2.56	7.66	
Oct	0.84	7.77	-10.34	4.56	1.38	-0.29	1.4	-7.61	
Nov	0.05	2.55	7.4	-1.24	-2.5	-1.17	4.96	2.42	
Dec	-1.29	3.22	1.42	5.19	-2.04	-0.42	-0.7	-5.34	
Mean	0.54	1.6	0.41	1.49	0.06	-1.41	0.43	1.01	0.52
Std Dev	2.62	3.97	4.62	4.15	3.27	3.13	4.33	5.6	3.99

High Yearly Variability

- Despite identical assumptions, returns varied from −1.41% to 1.60%, highlighting small - sample misleads.

B) Unstable Short-Term Volatility

- Yearly standard deviations varied from 2.62% to 5.60%, underscoring how short-term risk is unstable.

C) Long-Term Stability

- The 8-year average return (0.52%) and volatility (3.99%) closely match the model's 1% mean and 4.33% std dev.

D) Key Insight: Time Smooths Out Uncertainty

- Longer periods reduce randomness and align outcomes with theoretical expectations.

4.2 CAPM and Historical Data

CAPM: $\bar{r}_i^e = r_f + \beta_i(\bar{r}_M - r_f)$

CAPM + Historical Data: $$\bar{r}_i = \frac{\frac{\bar{r}_i^h}{(\sigma_i^h)^2} + \frac{\bar{r}_i^e}{(\sigma_i^e)^2}}{\frac{1}{(\sigma_i^h)^2} + \frac{1}{(\sigma_i^e)^2}}$$

	Stock1	Stock2	Stock3	Stock4
Historic Average	15.00%	14.34%	10.90%	15.09%
CAPM	13.031%	13.9898%	17.026%	11.2732%
CAPM + Historical Data	13.8033%	14.1347%	14.1745%	12.5264%

Interpretation

Stock 1,2,4:

CAPM + historical outperformed CAPM slightly, hinting CAPM may miss assets' full value, underestimating returns.

Stock 3:

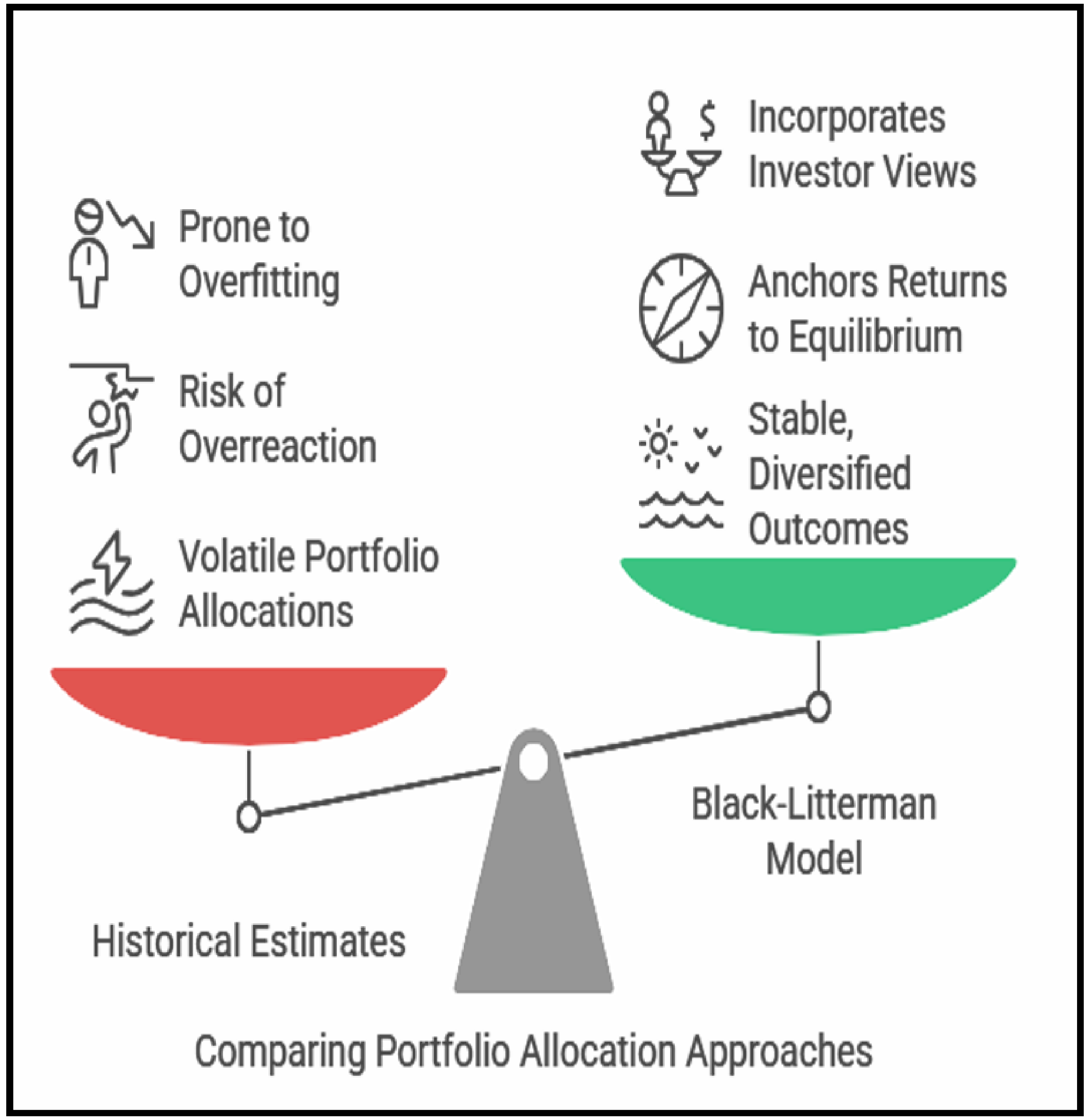
CAPM - calculated return exceeds that of CAPM with historical data, implying its β may overstate return potential.

4.3 Exploration on the Black-Litterman Model

The resulting **π** = (6%, 7%, 9%, 8%, 17%, 10%) appears to be reasonable, as it represents the market-implied equilibrium returns derived from $\pi = \lambda \Sigma w_{\mathrm{m}}$.

The high expected return for the US (17%) is justified by its significant market weight (71%) and relatively higher volatility compared to other assets.

Good:
Relies on objective market data rather than subjective or potentially noisy inputs, providing a more stable, data-driven foundation for portfolio optimization.



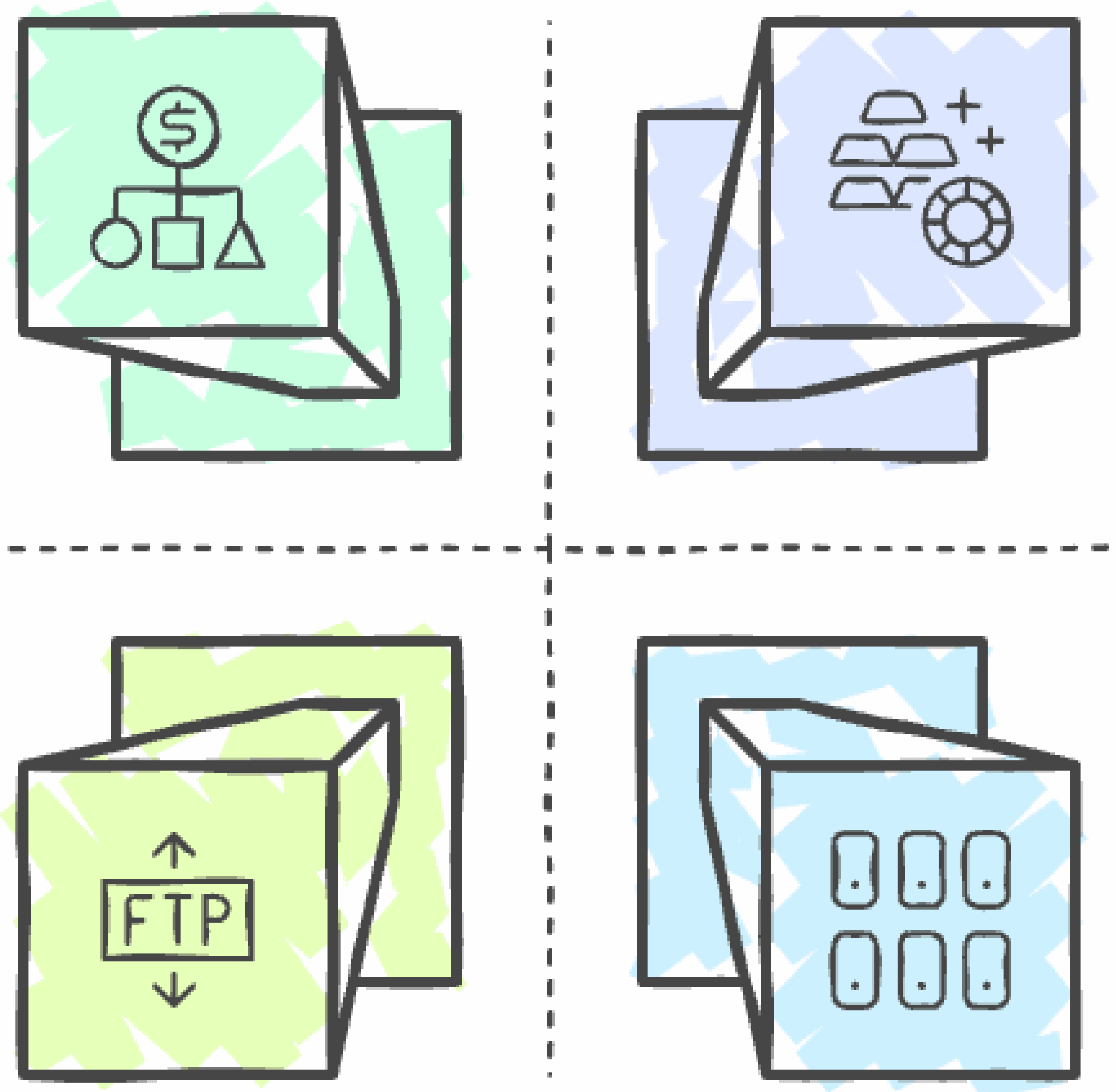
5.1.1 Comparative Analysis of Betterment and Wealthfront

Portfolio Construction

Betterment applies Modern Portfolio Theory while Wealthfront uses Mean-Variance Optimisation with Black-Litterman adjustments.

Asset Classes

Betterment uses 13 diversified asset classes while Wealthfront includes broader classes like TIPS, commodities, and REITs.



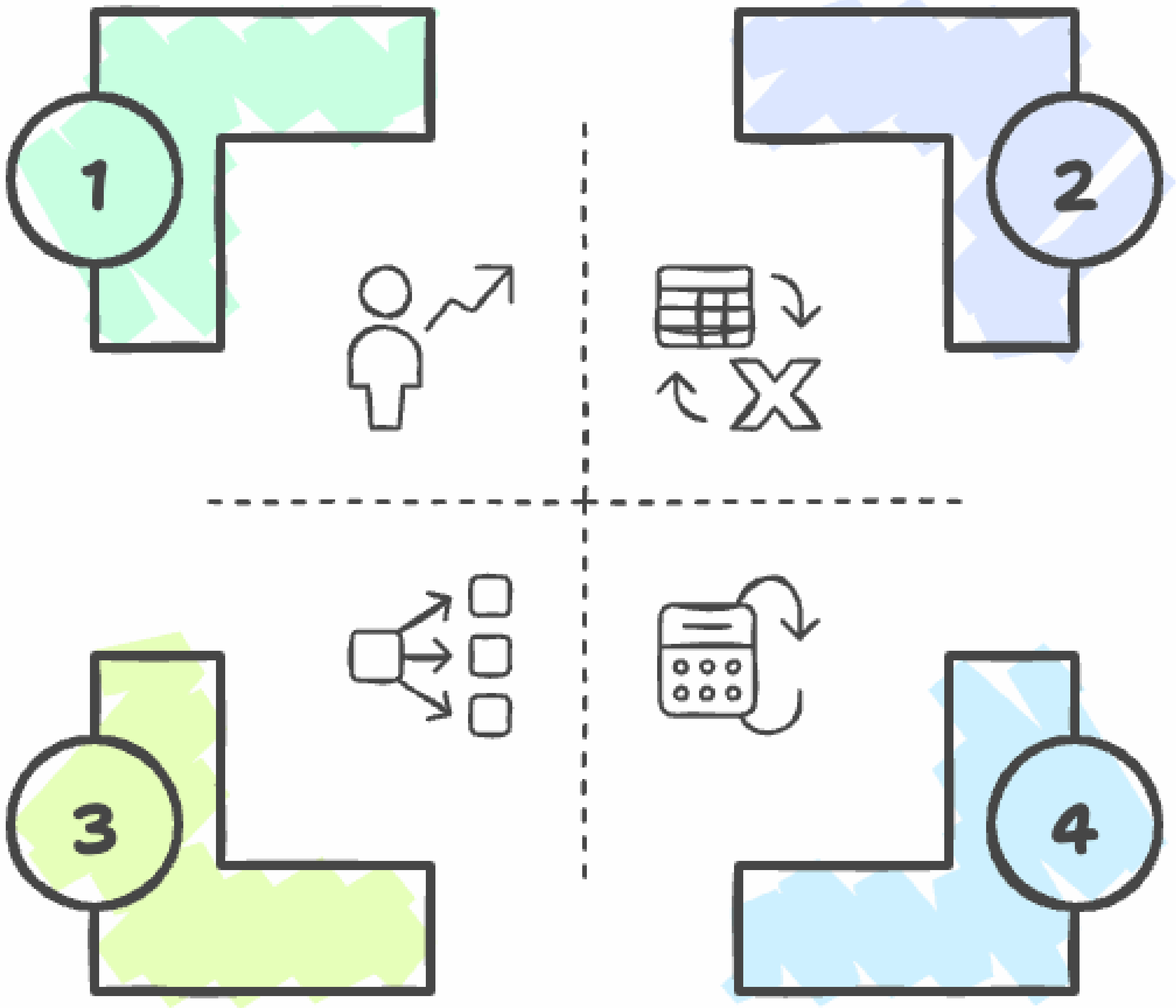
Investment Vehicles

Betterment chooses low-cost ETFs with minimal tracking error while Wealthfront prioritises low-cost, liquid index ETFs.

Fee Structure

Betterment has a flat-fee model while Wealthfront charges 0.25% and offers advanced features for larger accounts.

5.1.2 Comparative Analysis of Betterment and Wealthfront



Allocation Strategy

Betterment uses goal-based glide paths while Wealthfront blends subjective and objective risk profiling.

Tax Optimisation

Betterment applies asset location strategies while Wealthfront offers Tax-Loss Harvesting and Monte Carlo tax modelling.

Rebalancing

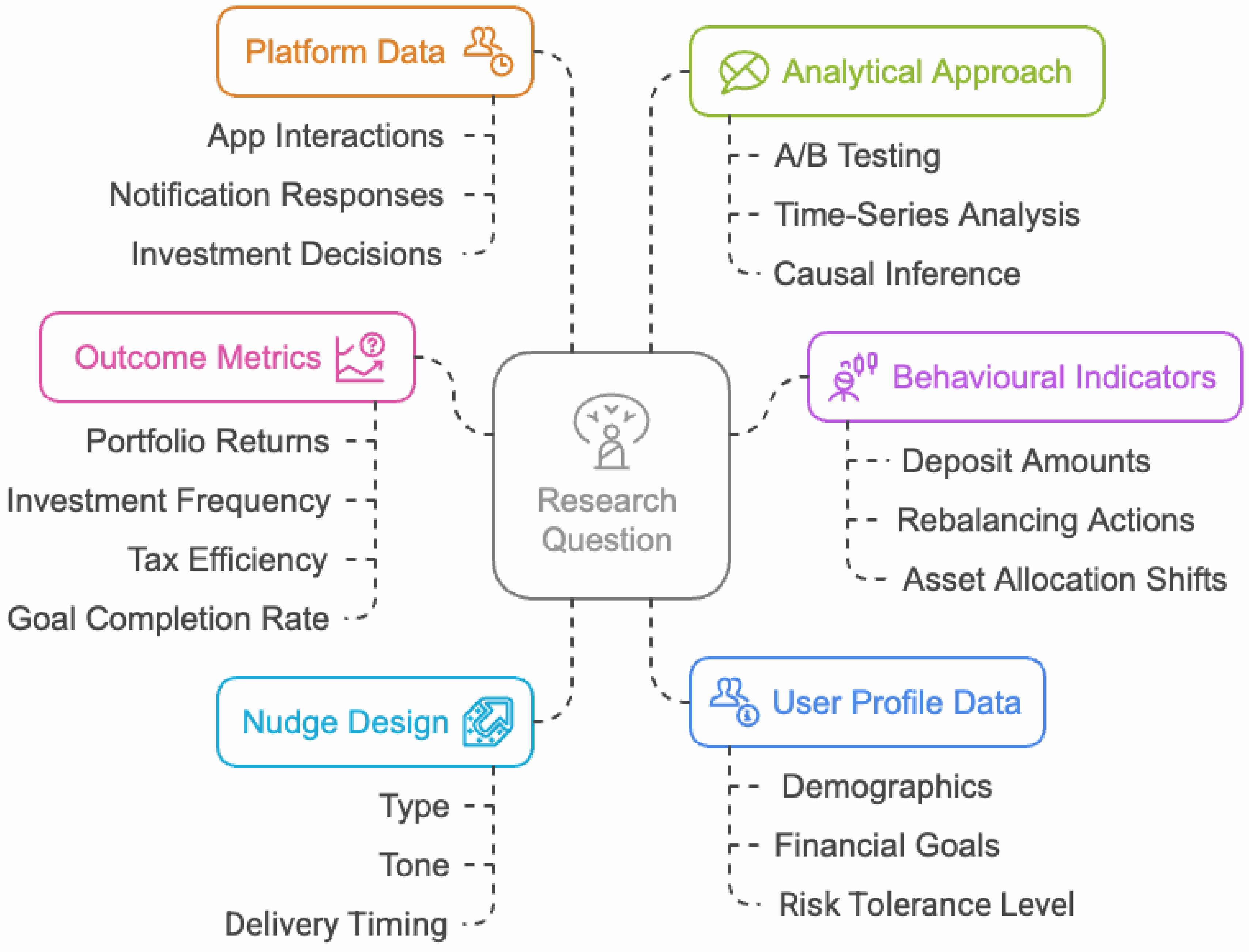
Betterment rebalances via cash flows and time triggers while Wealthfront uses threshold-based rebalancing for tax efficiency.

Customisation

Betterment emphasises simplicity and goals while Wealthfront enables deep personalisation and tax tools for high-net-worth users.

5.2 Integrating Behavioral Finance in Robo-Advising: Key Data & Insights

<u>Research Question</u>: To what extent do behavioural nudges used by robo-advisors contribute to correlated investor behaviours, and could this increase systemic risk in periods of market stress?



This study explores whether behavioural nudges used by robo-advisors to guide individual investors could unintentionally cause herding behaviour at scale. When many users receive similar prompts to rebalance or invest, their collective actions may become synchronised, increasing market volatility during periods of stress and potentially contributing to systemic risk.

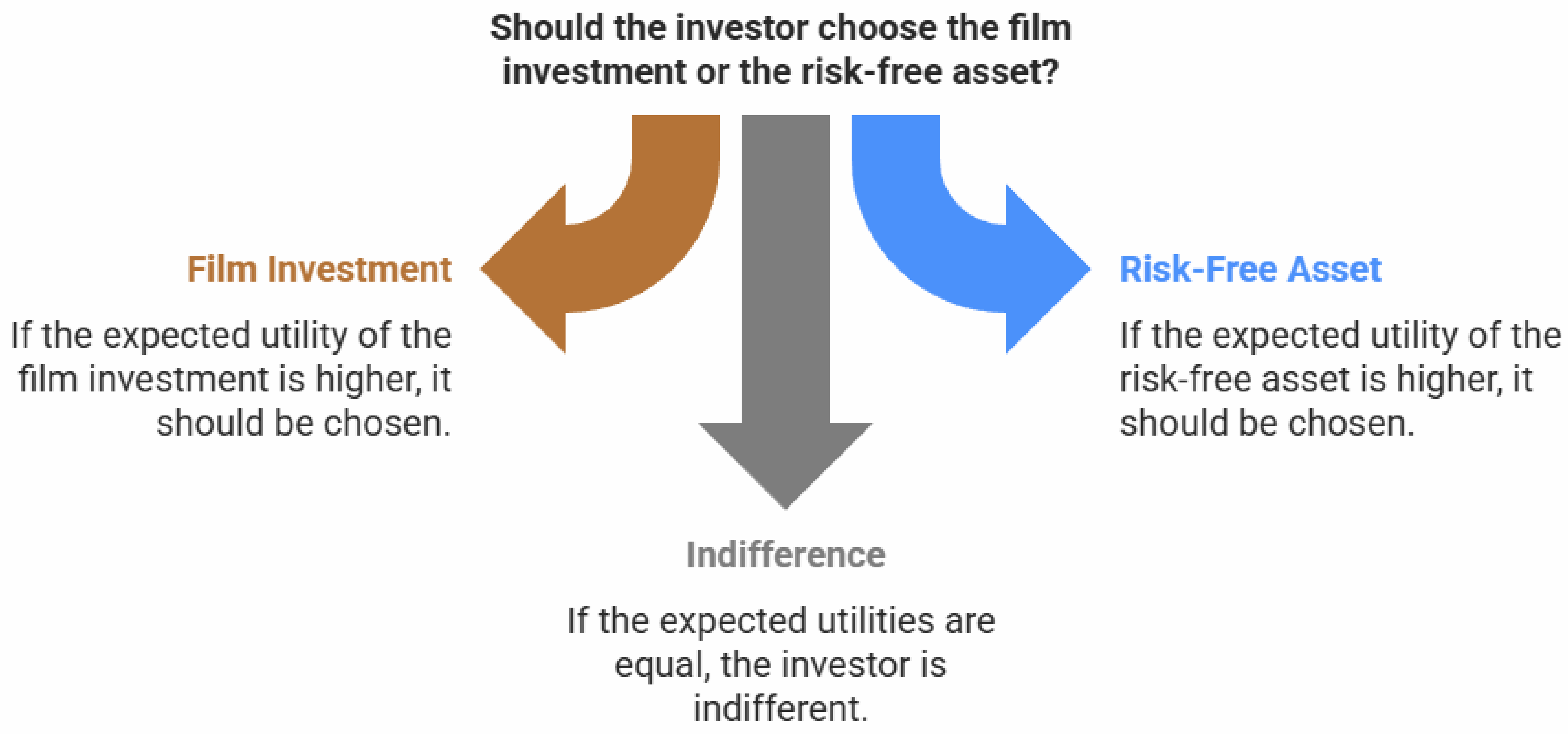
6.1 Evaluating the von Neumann-Morgenstern Framework: Descriptive and Normative

Aspect	Descriptive Perspective	Normative Perspective
Key Points	Axioms formalize rational decision-making; preferences are represented as utility.	Axioms ensure consistency in choices; theorem defines rational behavior under uncertainty.
Strengths	Clear, predictive, effective in stable and defined environments.	Ensures consistent, logical decisions; fundamental for economic theories like risk aversion.
Weaknesses	Violations of axioms (e.g., biases, Allais paradox); ignores real-world complexity.	Over-simplifies human behavior; assumes rationality, neglects social and emotional factors.
Compelling Nature	Limited: Effective in theoretical models, but doesn't fully reflect real-world complexity.	Compelling: Provides strong guidance for rational decision-making, though assumes ideal rationality.

6.2 Investment Choice: Film or 20% Risk-Free Asset

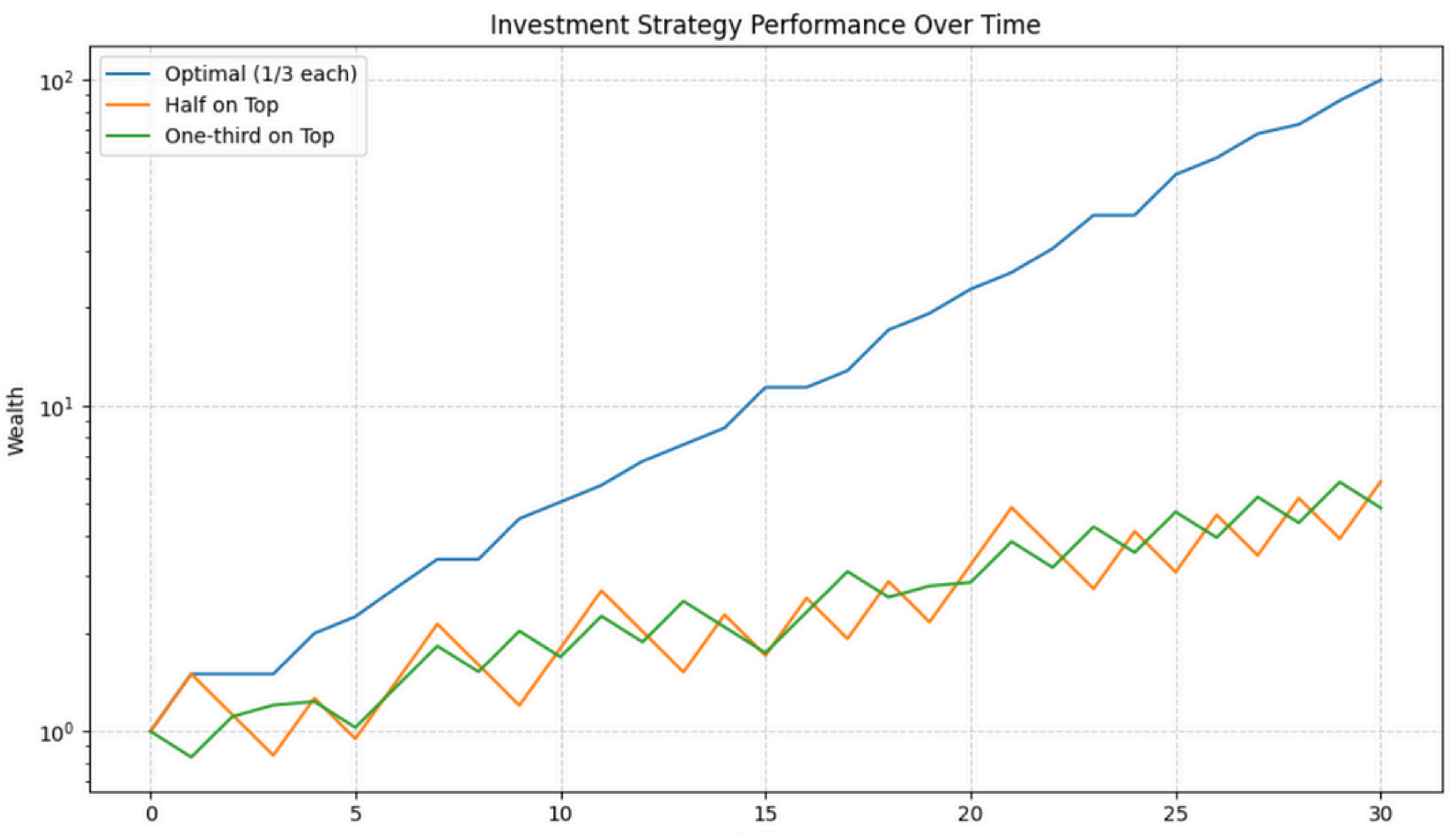
Assumption

Suppose the film investment has different outcomes x_i with probabilities p_i.
The expected utility of the film investment is $E(u(X)) = \sum_{i=1}^{n} p_i \log(x_i)$, while the utility of the risk-free investment is $\log(1.2I)$.



6.3 Optimal Investment Strategy for the Investment Wheel

Goal:	To maximise long-term growth across 3 sectors, each with 50% chance to double (×2) or halve (×0.5).
Optimal Strategy:	Maximises expected log-return, reduces volatility, and ensures consistent growth.



Simulation Results

Over 30 periods and 1000 runs:

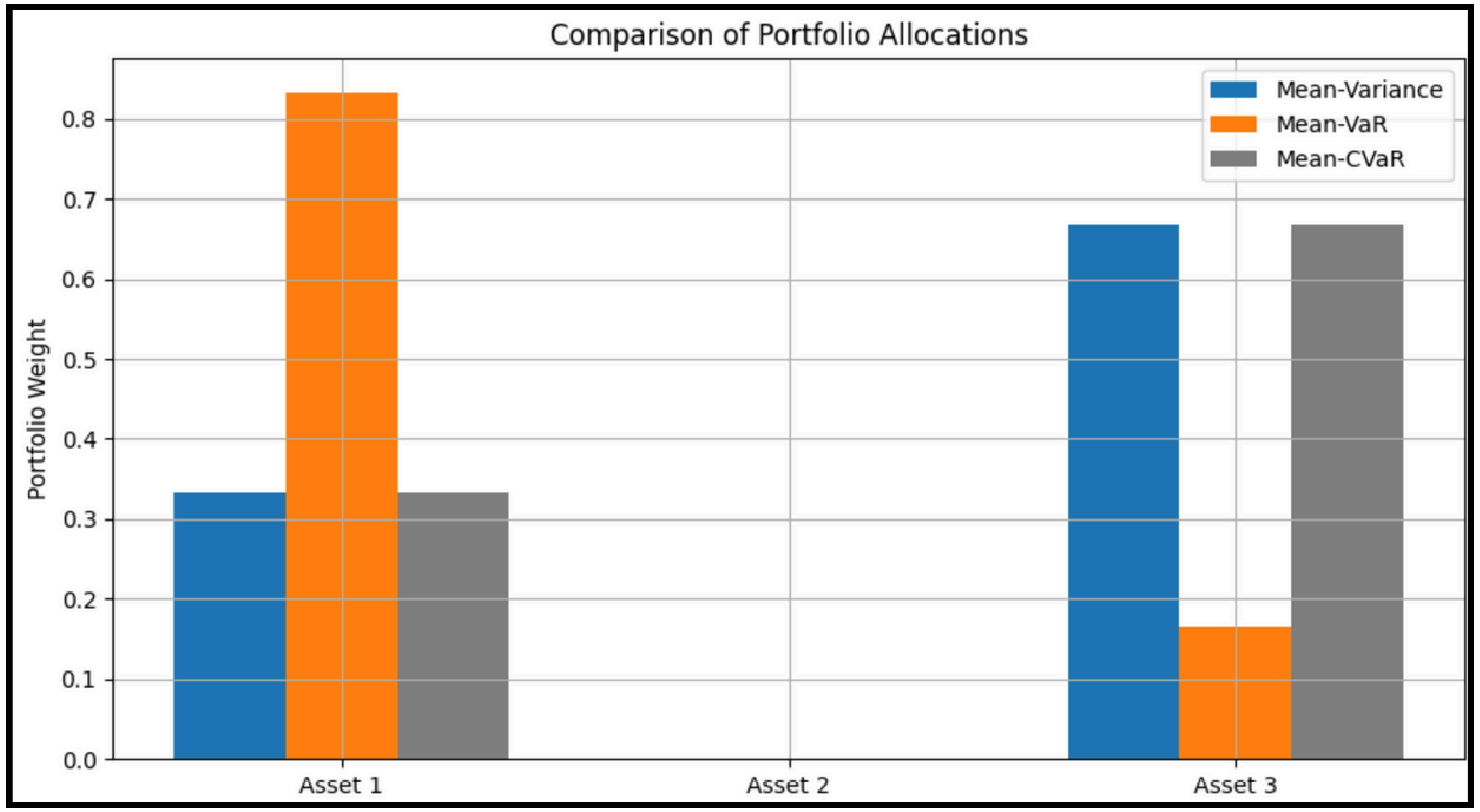
- **Equal allocation (1/3 each)** yields highest median wealth.
- **Concentrated bets** show more volatility and lower returns.
- Confirms **diversification** supports Kelly-optimal growth.

Interpretation

- Concentrated bets **increase** risk and underperform on average.
- Diversification **lowers** risk
- Diversification is key for **optimal** long-term outcomes.

7.1 Portfolio Selection: Mean-Variance, VaR, and CVaR

Method	Asset1	Asset2	Asset3	Expected return
Mean-Variance	0.3333	0.0	0.6667	1.0300
Mean-VaR	0.8333	0.0	0.1667	1.0525
Mean-CVaR	0.3333	0.0	0.6667	1.0300



Interpretation

Mean-Variance & Mean-CVaR align
Same portfolio (Asset1: 33%, Asset3: 67%) with 1.0300 return — robust, risk-balanced allocation.

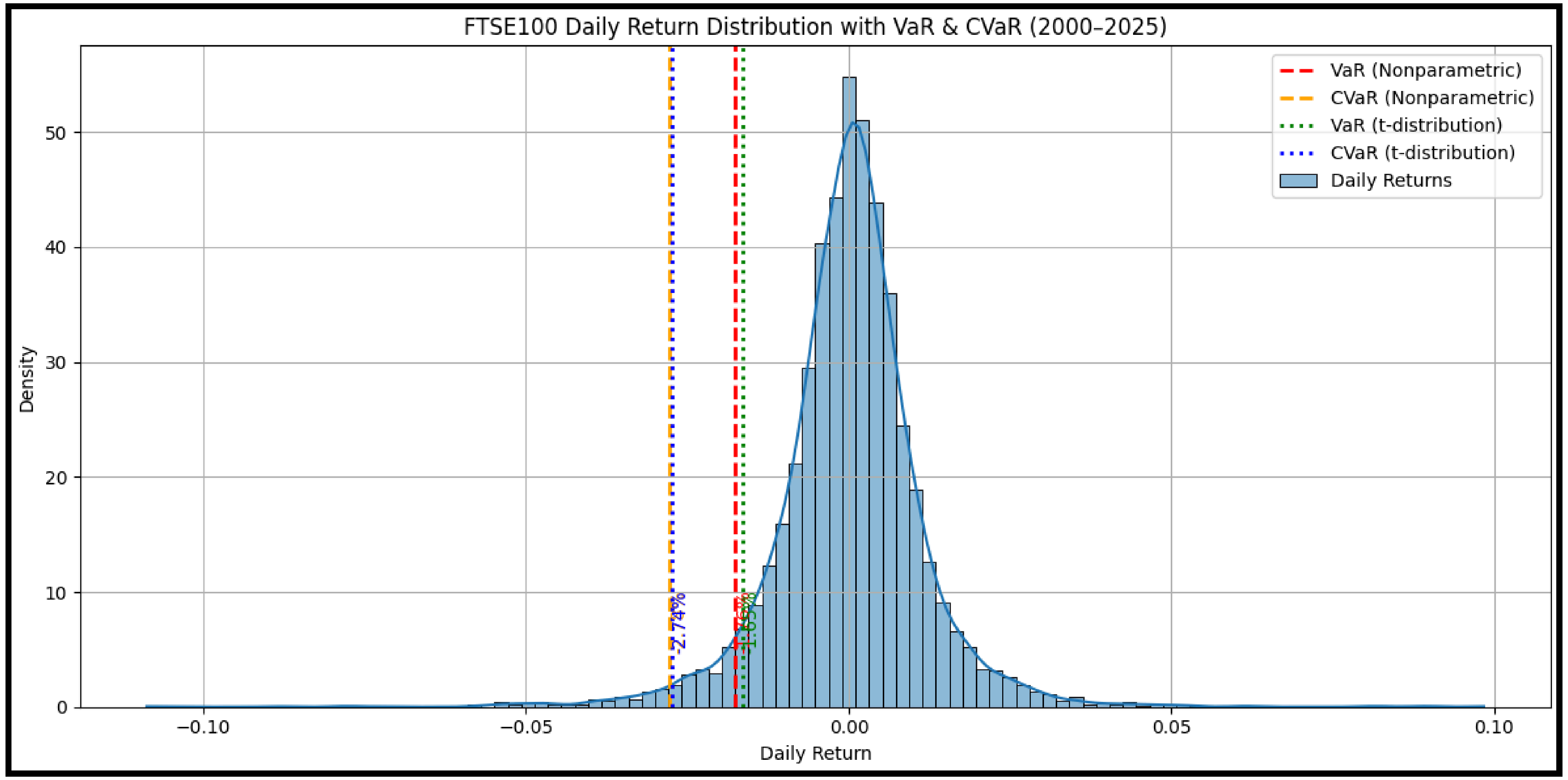
Mean-VaR maximizes return
Heavy tilt to Asset1 (83%) delivers highest return: 1.0525 — aggressive, return-driven strategy.

Asset2 universally rejected
0% allocation in all models — consistently inefficient asset.

Risk-aware ≠ underperformance
Mean-CVaR matches Mean-Variance — strong proof that conservative strategies stay competitive.

7.2 FTSE 100: Non-Parametric vs Parametric Risk Measures

Method	VaR(95%)	CVaR(95%)
Non - Parametric	-1.76%	-2.78%
Parametric (t-distribution)	-1.65%	-2.74%



Interpretation

The historical (nonparametric) method shows a VaR of −1.76% and CVaR of −2.78%, indicating that on the worst 5% of days, losses exceeded 1.76%, and averaged 2.78% when they did.

The parametric method (t-distribution) gives slightly less extreme estimates (VaR −1.65%, CVaR −2.74%), suggesting it may have underestimated the severity of actual tail events in the FTSE100.

In both methods, CVaR exceeds VaR, highlighting the importance of considering not just the probability of extreme losses, but also their potential magnitude.

This analysis suggests that relying solely on model-based assumptions may understate real-world risks, while historical data captures the true extent of past market turmoil.

Thank you!